BIPOLAR INTUITIONISTIC FUZZY α-IDEAL OF A BP-ALGEBRA

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Abstract:*The concept of a bipolar intuitionistic fuzzy ideal, bipolar intuitionistic anti fuzzy ideal, bipolar intuitionistic fuzzy α-ideal and bipolar intuitionistic anti fuzzyα-idealare a new algebraicstructure of BP-algebra are defined and some related properties are investigated. The purpose of this study is to implement the fuzzy set theory and ideal theory of a BP-algebra.*

Keywords:BP-algebra, fuzzy ideal, bipolar fuzzy ideal, bipolar intuitionistic fuzzy ideal, bipolar intuitionistic anti fuzzy ideal, bipolar intuitionistic fuzzy α-ideal,bipolar intuitionistic anti fuzzy α-ideal.

1. INTRODUCTION

The concept of fuzzy sets was initiated by I.A.Zadeh [14] then it has become a vigorous area of research in engineering, medical science, graph theory. S.S.Ahn [2] gave the idea of BP-algebra. Bipolar valued fuzzy sets was introduced by K.J.Lee [9] are an extension of fuzzy sets whose positive membership degree range is enlarged from the interval [0, 1] to [-1, 0]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the positive membership degree (0, 1] indicates that elements somewhat satisfy the property and the negative membership degree [-1, 0) indicates that elements somewhat satisfy the implicit counter property. The author W.R.Zhang [15] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. K.Chakrabarthy and Biswas R.Nanda [3] investigated note on union and intersection of intuitionistic fuzzy sets. A.Rajeshkumar [13] was analyzed fuzzy groups and level subgroups.M.Palanivelrajan and S.Nandakumar [12] introduced the definition and some operations of intuitionistic fuzzy primary and semiprimary ideal.

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2. PRELIMINARIES

Definition: 1

An algebra (X, *, 0) is called BP-algebra if it satisfies the following axioms:

(i) x*x = 0

(ii) x*(x*y) = y

(iii) (x*z)*(y*z) = x*y, for all x, y, z ∈ X.

In X, we can define a binary relation " ≤ " by x ≤ y if and only if x*y = 0.

Example:

Let X = {0, a, b, c}. Define * on X as the following table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then (X, *, 0) is a BP-algebra.

Definition: 2

A nonempty subset S of a BP-algebra X is called asubalgebra of X if x*y ∈ S, for all x, y∈ S.

Definition: 3

A nonempty subset I of a BP-algebra X is called an ideal of X if for all x, y ∈ X,

(i) 0 ∈I

(ii) x*y∈ I and y∈ I ⇒ x∈ I.

Definition: 4

Let (X, *, 0) be a BP-algebra. A fuzzy set μin X is called a fuzzy ideal of X ifit satisfies:

(i) $\mu(0) \geq \mu(x)$

(ii) $\mu(x) \geq$min { $\mu(x * y), \mu(y)$ }, for all x, y ∈ X.

Example:

Consider a BP-algebra X= {0, a, b, c} in which the operation * is given by example. Let $n_1, n_2, n_3 \in [0, 1]$ be such that $n_1 \geq n_2 > n_3$. Define the mapping μ: X ⟶[0, 1] by $\mu(0) = n_1$, $\mu(a) = n_2$ and $\mu(b) = \mu(c) = n_3$. Then usual calculation gives that μ is a fuzzy ideal of X.

Definition: 5

A nonempty subset I of a BP-algebra X is called aα-ideal of X if for all x, y, z∈ X,

(i) 0 ∈ I

(ii) x*z ∈ I and x*y ∈ I ⇒ y*z ∈ I.

Definition: 6

Let (X, *, 0) be a BP-algebra. A fuzzy set $\mu_\alpha$in X is called a fuzzy α-ideal of X ifit satisfies:

(i) $\mu_\alpha(0) \geq \mu_\alpha(x)$

(ii) $\mu_\alpha(y * z) \geq$min { $\mu_\alpha(x * z),\ \mu_\alpha(x * y)$}, for all x, y, z ∈ X.

Definition: 7

A bipolar fuzzy set A of BP-algebra X is defined as A={(x,$\mu_A^P(x), \mu_A^N(x)$) /x ∈ X)},whereμ_A^P : X ⟶ [0, 1] and μ_A^N : X ⟶ [-1, 0] are mappings and $\mu_A^P(x)$ is the degree of positive membership function and $\mu_A^N(x)$is the degree of negative membership function.

Definition: 8

Let A and B be any two bipolar fuzzy set A = (μ_A^P, μ_A^N) and B = (μ_B^P, μ_B^N) in X, we define

(i) A ∩ B ={(x, min($\mu_A^P(x), \mu_B^P(x)$), max($\mu_A^N(x), \mu_B^N(x)$))/x ∈ X}

(ii) A ∪ B ={ (x, max($\mu_A^P(x), \mu_B^P(x)$), min($\mu_A^N(x), \mu_B^N(x)$))/ x ∈ X}

Definition:9

A bipolar fuzzy set A = (μ_A^P, μ_A^N) of BP-algebra X is called a bipolar fuzzy ideal of X if it satisfies the following conditions:

(i) $\mu_A^P(0) \geq \mu_A^P(x)$ and $\mu_A^N(0) \leq \mu_A^N(x)$)

(ii) $\mu_A^P(x) \geq$ min { $\mu_A^P(x * y), \mu_A^P(y)$ }

(iii) $\mu_A^N(x) \leq$ max { $\mu_A^N(x * y), \mu_A^N(y)$}, for all x, y ∈ X.

Example:

Consider a BP-algebra X= {0, a, b, c} in which the operation * is given by example. Define a bipolar fuzzy set A = (μ_A^P, μ_A^N) by $\mu_A^P : \begin{pmatrix} 0 & a & b & c \\ 0.7 & 0.7 & 0.3 & 0.3 \end{pmatrix}$ and $\mu_A^N : \begin{pmatrix} 0 & a & b & c \\ -0.8 & -0.8 & -0.6 & -0.6 \end{pmatrix}$. Then usual calculation gives that A = (μ_A^P, μ_A^N) is a bipolarfuzzy ideal of X.

Definition:10

A bipolar fuzzy set A = ($\mu_{\alpha_A}^P$, $\mu_{\alpha_A}^N$) of BP-algebra X is called a bipolar fuzzy α-ideal of X if it satisfies the following conditions:

(i) $\mu_{\alpha_A}^P(0) \geq \mu_{\alpha_A}^P(x)$ and $\mu_{\alpha_A}^N(0) \leq \mu_{\alpha_A}^N(x)$

(ii) $\mu_{\alpha_A}^P(y * z) \geq$ min { $\mu_{\alpha_A}^P(x * z), \mu_{\alpha_A}^P(x * y)$ }

(iii) $\mu_{\alpha_A}^N(y * z) \leq$ max { $\mu_{\alpha_A}^N(x * z), \mu_{\alpha_A}^N(x * y)$}, for all x, y, z ∈ X.

Definition: 11

Abipolar intuitionistic fuzzyset AofBP-algebraX is defined as A= {(x,$\mu_A^P(x), \mu_A^N(x), \nu_A^P(x), \nu_A^N(x)$) / x ∈ X}, where$\mu_A^P: X \to [0, 1]$,$\mu_A^N: X \to [-1, 0]$,$\nu_A^P: X \to [0, 1]$, $\nu_A^N: X \to [-1, 0]$are mappings and $\mu_A^P(x), \mu_A^N(x), \nu_A^P(x)and\nu_A^N(x)$ arecalled degree ofpositive membership, negative membership,positive non-membershipand negative non-membershiprespectively and satisfying $0 \leq \mu_A^P(x) + \nu_A^P(x) \leq 1$and $-1 \leq \mu_A^N(x) + \nu_A^N(x) \leq 0$.

Definition: 12

Let A and B be any two bipolar intuitionistic fuzzy set A = ($\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$) and B = ($\mu_B^P, \mu_B^N, \nu_B^P, \nu_B^N$) in X, we define

(i) A ∩ B ={(x, min($\mu_A^P(x), \mu_B^P(x)$), max($\mu_A^N(x), \mu_B^N(x)$), max($\nu_A^P(x), \nu_B^P(x)$), min($\nu_A^N(x), \nu_B^N(x)$)) /x ∈ X}

(ii) A ∪ B ={(x, max($\mu_A^P(x), \mu_B^P(x)$), min($\mu_A^N(x), \mu_B^N(x)$), min($\nu_A^P(x), \nu_B^P(x)$), max($\nu_A^N(x), \nu_B^N(x)$)) /x ∈ X}

(iii) $\bar{A}$= {(x,$\nu_A^P(x), \nu_A^N(x), \mu_A^P(x), \mu_A^N(x)$) / x ∈ X}.

Definition: 13

A bipolar intuitionistic fuzzy set A = { $\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$ / x ∈X } of BP-algebra X is called a bipolar intuitionistic fuzzy ideal of X if it satisfies the following conditions:

(i) $\mu_A^P(0) \geq \mu_A^P(x)$ and $\mu_A^N(0) \leq \mu_A^N(x)$

(ii) $\mu_A^P(x) \geq \min\{\mu_A^P(x*y), \mu_A^P(y)\}$

(iii) $\mu_A^N(x) \leq \max\{\mu_A^N(x*y), \mu_A^N(y)\}$

(iv) $\nu_A^P(0) \leq \nu_A^P(x)$ and $\nu_A^N(0) \geq \nu_A^N(x)$

(v) $\nu_A^P(x) \leq \max\{\nu_A^P(x*y), \nu_A^P(y)\}$

(vi) $\nu_A^N(x) \geq \min\{\nu_A^N(x*y), \nu_A^N(y)\}$, for all x, y ∈ X.

Example:

Consider a BP-algebra X ={0,a,b,c} in which the operation * is given by example. Define a bipolar intuitionistic fuzzy set A = { $\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$}, where

$$\mu_A^P : \begin{pmatrix} 0 & a & b & c \\ 0.7 & 0.7 & 0.3 & 0.3 \end{pmatrix} \nu_A^P : \begin{pmatrix} 0 & a & b & c \\ 0.2 & 0.2 & 0.5 & 0.5 \end{pmatrix}$$

$$\mu_A^N : \begin{pmatrix} 0 & a & b & c \\ -0.8 & -0.8 & -0.6 & -0.6 \end{pmatrix} \nu_A^N : \begin{pmatrix} 0 & a & b & c \\ -0.1 & -0.1 & -0.3 & -0.3 \end{pmatrix}$$

Then usual calculation gives that A = { $\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$}is a bipolar intuitionistic fuzzy ideal of X.

Definition: 14

A bipolar intuitionistic fuzzy set A = { $\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$ / x ∈X } of BP-algebra X is called a bipolar intuitionistic anti fuzzy ideal of X if it satisfies the following conditions:

(i) $\mu_A^P(0) \leq \mu_A^P(x)$ and $\mu_A^N(0) \geq \mu_A^N(x)$

(ii) $\mu_A^P(x) \leq \max\{\mu_A^P(x*y), \mu_A^P(y)\}$

(iii) $\mu_A^N(x) \geq \min\{\mu_A^N(x*y), \mu_A^N(y)\}$

(iv) $\nu_A^P(0) \geq \nu_A^P(x)$ and $\nu_A^N(0) \leq \nu_A^N(x)$

(v) $\nu_A^P(x) \geq \min\{\nu_A^P(x*y), \nu_A^P(y)\}$

(vi) $\nu_A^N(x) \leq \max\{\nu_A^N(x*y), \nu_A^N(y)\}$, for all x, y ∈ X.

Example:

Consider a BP-algebra X = {0, a, b, c} in which the operation * is given by example. Define a bipolar intuitionistic fuzzy set A = { $\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$}, where

$$\mu_A^P : \begin{pmatrix} 0 & a & b & c \\ 0.3 & 0.3 & 0.7 & 0.7 \end{pmatrix} \nu_A^P : \begin{pmatrix} 0 & a & b & c \\ 0.5 & 0.5 & 0.2 & 0.2 \end{pmatrix}$$

$$\mu_A^N : \begin{pmatrix} 0 & a & b & c \\ -0.6 & -0.6 & -0.8 & -0.8 \end{pmatrix} \nu_A^N : \begin{pmatrix} 0 & a & b & c \\ -0.3 & -0.3 & -0.1 & -0.1 \end{pmatrix}$$

Then usual calculation gives that A = { $\mu_A^P, \mu_A^N, \nu_A^P, \nu_A^N$}is a bipolar intuitionistic antifuzzy ideal of X.

Definition: 15

A bipolar intuitionistic fuzzy set A = {(x, $\mu_{\alpha_A}^P(x), \mu_{\alpha_A}^N(x), \nu_{\alpha_A}^P(x), \nu_{\alpha_A}^N(x)$)/ x ∈ X}, of BP-algebra X is called a bipolar intuitionistic fuzzy α-ideal of X if it satisfies the following conditions:

(i) $\mu_{\alpha_A}^P(0) \geq \mu_{\alpha_A}^P(x)$ and $\mu_{\alpha_A}^N(0) \leq \mu_{\alpha_A}^N(x)$

(ii) $\mu_{\alpha_A}^P(y*z) \geq \min\{\mu_{\alpha_A}^P(x*z), \mu_{\alpha_A}^P(x*y)\}$

(iii) $\mu_{\alpha_A}^N(y*z) \leq \max\{\mu_{\alpha_A}^N(x*z), \mu_{\alpha_A}^N(x*y)\}$

(iv) $\nu^P_{\alpha_A}(0) \leq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \geq \nu^N_{\alpha_A}(x)$
(v) $\nu^P_{\alpha_A}(y * z) \leq \max\{\nu^P_{\alpha_A}(x * z),\ \nu^P_{\alpha_A}(x * y)\}$
(vi) $\nu^N_{\alpha_A}(y * z) \geq \min\{\nu^N_{\alpha_A}(x * z),\ \nu^N_{\alpha_A}(x * y)\}$, for all x, y, z ∈ X.

Definition: 16

A bipolar intuitionistic fuzzy set A = {(x, $\mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x)$)/ x ∈ X}, of BP-algebra X is called a bipolar intuitionistic anti fuzzy α-idealof X if it satisfies the following conditions:

(i) $\mu^P_{\alpha_A}(0) \leq \mu^P_{\alpha_A}(x)$ and $\mu^N_{\alpha_A}(0) \geq \mu^N_{\alpha_A}(x)$
(ii) $\mu^P_{\alpha_A}(y * z) \leq \max\{\mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_A}(x * y)\}$
(iii) $\mu^N_{\alpha_A}(y * z) \geq \min\{\mu^N_{\alpha_A}(x * z),\ \mu^N_{\alpha_A}(x * y)\}$
(iv) $\nu^P_{\alpha_A}(0) \geq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \leq \nu^N_{\alpha_A}(x)$
(v) $\nu^P_{\alpha_A}(y * z) \geq \min\{\nu^P_{\alpha_A}(x * z),\ \nu^P_{\alpha_A}(x * y)\}$
(vi) $\nu^N_{\alpha_A}(y * z) \leq \max\{\nu^N_{\alpha_A}(x * z),\ \nu^N_{\alpha_A}(x * y)\}$, for all x, y, z ∈ X.

3. SOME OPERATIONS ON BIPOLAR INTUITIONISTIC FUZZY IDEAL AND BIPOLAR INTUITIONISTIC FUZZY α-IDEAL

Theorem: 1

If A is a bipolar intuitionistic fuzzy ideal of BP-algebra X, then $\bar{\bar{A}}$ = A is a bipolar intuitionistic fuzzy ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy ideal of X.

Consider 0,x,y∈ A.

(i) Now $\mu^P_{\bar{\bar{A}}}(0) = \nu^P_{\bar{A}}(0)$
$= \mu^P_A(0)$
$\geq \mu^P_A(x)$

Therefore $\mu^P_{\bar{\bar{A}}}(0) \geq \mu^P_A(x)$

Now $\mu^N_{\bar{\bar{A}}}(0) = \nu^N_{\bar{A}}(0)$
$= \mu^N_A(0)$
$\leq \mu^N_A(x)$

Therefore $\mu^N_{\bar{\bar{A}}}(0) \leq \mu^N_A(x)$

(ii) Now $\mu^P_{\bar{\bar{A}}}(x) = \nu^P_{\bar{A}}(x)$
$= \mu^P_A(x)$
$\geq \min\{\mu^P_A(x * y), \mu^P_A(y)\}$

Therefore $\mu^P_{\bar{\bar{A}}}(x) \geq \min\{\mu^P_A(x * y),\ \mu^P_A(y)\}$

(iii) Now $\mu^N_{\bar{\bar{A}}}(x) = \nu^N_{\bar{A}}(x)$
$= \mu^N_A(x)$
$\leq \max\{\mu^N_A(x * y),\ \mu^N_A(y)\}$

Therefore $\mu^N_{\bar{\bar{A}}}(x) \leq \max\{\mu^N_A(x * y),\ \mu^N_A(y)\}$

(iv) Now $\nu^P_{\bar{\bar{A}}}(0) = \mu^P_{\bar{A}}(0)$
$= \nu^P_A(0)$
$\leq \nu^P_A(x)$

Therefore $\nu^P_{\bar{\bar{A}}}(0) \leq \nu^P_A(x)$

Now $\nu_{\bar{\bar{A}}}^{N}(0) = \mu_{\bar{A}}^{N}(0)$

$= \nu_{A}^{N}(0)$

$\geq \nu_{A}^{N}(x)$

Therefore $\nu_{\bar{\bar{A}}}^{N}(0) \geq \nu_{A}^{N}(x)$

(v) Now $\nu_{\bar{\bar{A}}}^{P}(x) = \mu_{\bar{A}}^{P}(x)$

$= \nu_{A}^{P}(x)$

$\leq \max\{\nu_{A}^{P}(x * y), \quad \nu_{A}^{P}(y)$

Therefore $\nu_{\bar{\bar{A}}}^{P}(x) \leq \max\{\nu_{A}^{P}(x * y), \nu_{A}^{P}(y)\}$

(vi) Now $\nu_{\bar{\bar{A}}}^{N}(x) = \mu_{\bar{A}}^{N}(x)$

$= \nu_{A}^{N}(x)$

$\geq \min\{\nu_{A}^{N}(x * y), \nu_{A}^{N}(y)\}$

Therefore $\nu_{\bar{\bar{A}}}^{N}(x) \geq \min\{\nu_{A}^{N}(x * y), \nu_{A}^{N}(y)\}$

Therefore $\bar{\bar{A}}$ = A is also a bipolar intuitionistic fuzzy ideal of X.

Theorem: 2

Intersection of any two bipolar intuitionistic fuzzy ideal of BP-algebra X is again a bipolar intuitionistic fuzzy ideal of X.

Proof: Let Aand B be two bipolar intuitionistic fuzzy ideal of X.

Consider0,x,y∈A ∩ B⟹0,x,y∈A and 0,x,y∈B.

(i) Now $\mu_{A\cap B}^{P}(0) = \min\{\mu_{A}^{P}(0), \mu_{B}^{P}(0)\}$

$\geq \min\{\mu_{A}^{P}(x), \mu_{B}^{P}(x)\}$

$= \mu_{A\cap B}^{P}(x)$

Therefore $\mu_{A\cap B}^{P}(0) \geq \mu_{A\cap B}^{P}(x)$

Now $\mu_{A\cap B}^{N}(0) = \max\{\mu_{A}^{N}(0), \mu_{B}^{N}(0)\}$

$\leq \max\{\mu_{A}^{N}(x), \mu_{B}^{N}(x)\}$

$= \mu_{A\cap B}^{N}(x)$

Therefore $\mu_{A\cap B}^{N}(0) \leq \mu_{A\cap B}^{N}(x)$

(ii) Now $\mu_{A\cap B}^{P}(x) = \min\{\mu_{A}^{P}(x), \mu_{B}^{P}(x)\}$

$\geq \min\{\min\{\mu_{A}^{P}(x * y), \mu_{A}^{P}(y)\}, \min\{\mu_{B}^{P}(x * y), \mu_{B}^{P}(y)\}\}$

$= \min\{\min\{\mu_{A}^{P}(x * y), \mu_{B}^{P}(x * y)\}, \min\{\mu_{A}^{P}(y), \mu_{B}^{P}(y)\}\}$

$= \min\{\mu_{A\cap B}^{P}(x * y), \mu_{A\cap B}^{P}(y)\}$

Therefore $\mu_{A\cap B}^{P}(x) \geq \min\{\mu_{A\cap B}^{P}(x * y), \mu_{A\cap B}^{P}(y)\}$

(iii) Now $\mu_{A\cap B}^{N}(x) = \max\{\mu_{A}^{N}(x), \mu_{B}^{N}(x)\}$

$\leq \max\{\max\{\mu_{A}^{N}(x * y), \mu_{A}^{N}(y)\}, \max\{\mu_{B}^{N}(x * y), \mu_{B}^{N}(y)\}\}$

$= \max\{\max\{\mu_{A}^{N}(x * y), \mu_{B}^{N}(x * y)\}, \max\{\mu_{A}^{N}(y), \mu_{B}^{N}(y)\}\}$

$= \max\{\mu_{A\cap B}^{N}(x * y), \mu_{A\cap B}^{N}(y)\}$

Therefore $\mu_{A\cap B}^{N}(x) \leq \max\{\mu_{A\cap B}^{N}(x * y), \mu_{A\cap B}^{N}(y)\}$

(iv) Now $\nu_{A\cap B}^{P}(0) = \max\{\nu_{A}^{P}(0), \nu_{B}^{P}(0)\}$

$\leq \max\{\nu_{A}^{P}(x), \nu_{B}^{P}(x)\}$

$= \nu_{A\cap B}^{P}(x)$

Therefore $\nu_{A\cap B}^{P}(0) \leq \nu_{A\cap B}^{P}(x)$

Now $\nu_{A\cap B}^{N}(0) = \min\{\nu_{A}^{N}(0), \nu_{B}^{N}(0)\}$

$\geq \min\{\nu_{A}^{N}(x), \nu_{B}^{N}(x)\}$

$= \nu_{A\cap B}^{N}(x)$

Therefore $\nu_{A\cap B}^{N}(0) \geq \nu_{A\cap B}^{N}(x)$

(v) Now $\nu^{P}_{A\cap B}(x) = \max\{\nu^{P}_{A}(x), \nu^{P}_{B}(x)\}$

$\leq \max\{\max\{\nu^{P}_{A}(x*y), \nu^{P}_{A}(y)\}, \max\{\nu^{P}_{B}(x*y), \nu^{P}_{B}(y)\}\}$

$= \max\{\max\{\nu^{P}_{A}(x*y), \nu^{P}_{B}(x*y)\}, \max\{\nu^{P}_{A}(y), \nu^{P}_{B}(y)\}\}$

$= \max\{\nu^{P}_{A\cap B}(x*y), \nu^{P}_{A\cap B}(y)\}$

Therefore $\nu^{P}_{A\cap B}(x) \leq \max\{\nu^{P}_{A\cap B}(x*y), \nu^{P}_{A\cap B}(y)\}$

(vi) Now $\nu^{N}_{A\cap B}(x) = \min\{\nu^{N}_{A}(x), \nu^{N}_{B}(x)\}$

$\geq \min\{\min\{\nu^{N}_{A}(x*y), \nu^{N}_{A}(y)\}, \min\{\nu^{N}_{B}(x*y), \nu^{N}_{B}(y)\}\}$

$= \min\{\min\{\nu^{N}_{A}(x*y), \nu^{N}_{B}(x*y)\}, \min\{\nu^{N}_{A}(y), \nu^{N}_{B}(y)\}\}$

$= \min\{\nu^{N}_{A\cap B}(x*y), \nu^{N}_{A\cap B}(y)\}$

Therefore $\nu^{N}_{A\cap B}(x) \geq \min\{\nu^{N}_{A\cap B}(x*y), \nu^{N}_{A\cap B}(y)\}$

Henceintersection of any two bipolar intuitionistic fuzzy ideal of X is again a bipolar intuitionistic fuzzy ideal of X.

Theorem:3

Union of any two bipolar intuitionistic fuzzy ideal of BP-algebra X is again a bipolar intuitionistic fuzzy ideal of X if either is contained in the other.

Proof: LetA and B be twobipolar intuitionistic fuzzy ideal of X.

If A ⊆ B ⟹A∪ B = B and if B ⊆ A ⟹A ∪ B = A.

Consider0,x,y∈A ∪ B⟹0,x,y∈A or0,x,y∈B

(i) Now $\mu^{P}_{A\cup B}(0) = \max\{\mu^{P}_{A}(0), \mu^{P}_{B}(0)\}$

$\geq \max\{\mu^{P}_{A}(x), \mu^{P}_{B}(x)\}$

$= \mu^{P}_{A\cup B}(x)$

Therefore$\mu^{P}_{A\cup B}(0) \geq \mu^{P}_{A\cup B}(x)$

Now $\mu^{N}_{A\cup B}(0) = \min\{\mu^{N}_{A}(0), \mu^{N}_{B}(0)\}$

$\leq \min\{\mu^{N}_{A}(x), \mu^{N}_{B}(x)\}$

$= \mu^{N}_{A\cup B}(x)$

Therefore$\mu^{N}_{A\cup B}(0) \leq \mu^{N}_{A\cup B}(x)$

(ii) Now $\mu^{P}_{A\cup B}(x) = \max\{\mu^{P}_{A}(x), \mu^{P}_{B}(x)\}$

$\geq \max\{\min\{\mu^{P}_{A}(x*y), \mu^{P}_{A}(y)\}, \min\{\mu^{P}_{B}(x*y), \mu^{P}_{B}(y)\}\}$

$= \min\{\max\{\mu^{P}_{A}(x*y), \mu^{P}_{B}(x*y)\}, \max\{\mu^{P}_{A}(y), \mu^{P}_{B}(y)\}\}$

$= \min\{\mu^{P}_{A\cup B}(x*y), \mu^{P}_{A\cup B}(y)\}$

Therefore$\mu^{P}_{A\cup B}(x) \geq \min\{\mu^{P}_{A\cup B}(x*y), \mu^{P}_{A\cup B}(y)\}$

(iii) Now $\mu^{N}_{A\cup B}(x) = \min\{\mu^{N}_{A}(x), \mu^{N}_{B}(x)\}$

$\leq \min\{\max\{\mu^{N}_{A}(x*y), \mu^{N}_{A}(y)\}, \max\{\mu^{N}_{B}(x*y), \mu^{N}_{B}(y)\}\}$

$= \max\{\min\{\mu^{N}_{A}(x*y), \mu^{N}_{B}(x*y)\}, \min\{\mu^{N}_{A}(y), \mu^{N}_{B}(y)\}\}$

$= \max\{\mu^{N}_{A\cup B}(x*y), \mu^{N}_{A\cup B}(y)\}$

Therefore$\mu^{N}_{A\cup B}(x) \leq \max\{\mu^{N}_{A\cup B}(x*y), \mu^{N}_{A\cup B}(y)\}$

(iv) Now $\nu^{P}_{A\cup B}(0) = \min\{\nu^{P}_{A}(0), \nu^{P}_{B}(0)\}$

$\leq \min\{\nu^{P}_{A}(x), \nu^{P}_{B}(x)\}$

$= \nu^{P}_{A\cup B}(x)$

Therefore $\nu^{P}_{A\cup B}(0) \leq \nu^{P}_{A\cup B}(x)$

Now $\nu^{N}_{A\cup B}(0) = \max\{\nu^{N}_{A}(0), \nu^{N}_{B}(0)\}$

$\geq \max\{\nu^{N}_{A}(x), \nu^{N}_{B}(x)\}$

$= \nu^{N}_{A\cup B}(x)$

Therefore $\nu^{N}_{A\cup B}(0) \geq \nu^{N}_{A\cup B}(x)$

(v) Now $\nu^{P}_{A\cup B}(x) = \min\{\nu^{P}_{A}(x), \nu^{P}_{B}(x)\}$

$\leq \min\{\max\{\nu_A^P(x*y),\ \nu_A^P(y)\},\ \max\{\nu_B^P(x*y),\ \nu_B^P(y)\}\}$
$= \max\{\min\{\nu_A^P(x*y), \nu_B^P(x*y)\},\ \min\{\nu_A^P(y),\ \nu_B^P(y)\}\}$
$= \max\{\nu_{A\cup B}^P(x*y),\ \nu_{A\cup B}^P(y)\}$

Therefore $\nu_{A\cup B}^P(x) \leq \max\{\nu_{A\cup B}^P(x*y),\ \nu_{A\cup B}^P(y)\}$

(vi) Now $\nu_{A\cup B}^N(x) = \max\{\nu_A^N(x),\ \nu_B^N(x)\}$
$\geq \max\{\min\{\nu_A^N(x*y),\ \nu_A^N(y)\},\ \min\{\nu_B^N(x*y),\ \nu_B^N(y)\}\}$
$= \min\{\max\{\nu_A^N(x*y), \nu_B^N(x*y)\},\ \max\{\nu_A^N(y),\ \nu_B^N(y)\}\}$
$= \min\{\nu_{A\cup B}^N(x*y),\ \nu_{A\cup B}^N(y)\}$

Therefore $\nu_{A\cup B}^N(x) \geq \min\{\nu_{A\cup B}^N(x*y),\ \nu_{A\cup B}^N(y)\}$

ThereforeA ∪ B is a bipolar intuitionistic fuzzy ideal of X.

Theorem: 4

If A is a bipolar intuitionistic anti fuzzy ideal of BP-algebra X, then $\bar{\bar{A}}$ = A is a bipolar intuitionistic anti fuzzy ideal of X.

Proof: Given A is a bipolar intuitionistic anti fuzzy ideal of X.

Consider 0,x,y∈ A.

(i) Now $\mu_{\bar{\bar{A}}}^P(0) = \nu_{\bar{A}}^P(0)$
$= \mu_A^P(0)$
$\leq \mu_A^P(x)$

Therefore $\mu_{\bar{\bar{A}}}^P(0) \leq \mu_A^P(x)$

Now $\mu_{\bar{\bar{A}}}^N(0) = \nu_{\bar{A}}^N(0)$
$= \mu_A^N(0)$
$\geq \mu_A^N(x)$

Therefore $\mu_{\bar{\bar{A}}}^N(0) \geq \mu_A^N(x)$

(ii) Now $\mu_{\bar{\bar{A}}}^P(x) = \nu_{\bar{A}}^P(x)$
$= \mu_A^P(x)$
$\leq \max\{\mu_A^P(x*y),\ \mu_A^P(y)\}$

Therefore $\mu_{\bar{\bar{A}}}^P(x) \leq \max\{\mu_A^P(x*y),\ \mu_A^P(y)\}$

(iii) Now $\mu_{\bar{\bar{A}}}^N(x) = \nu_{\bar{A}}^N(x)$
$= \mu_A^N(x)$
$\geq \min\{\mu_A^N(x*y),\ \mu_A^N(y)\}$

Therefore $\mu_{\bar{\bar{A}}}^N(x) \geq \min\{\mu_A^N(x*y),\ \mu_A^N(y)\}$

(iv) Now $\nu_{\bar{\bar{A}}}^P(0) = \mu_{\bar{A}}^P(0)$
$= \nu_A^P(0)$
$\geq \nu_A^P(x)$

Therefore $\nu_{\bar{\bar{A}}}^P(0) \geq \nu_A^P(x)$

Now $\nu_{\bar{\bar{A}}}^N(0) = \mu_{\bar{A}}^N(0)$
$= \nu_A^N(0)$
$\leq \nu_A^N(x)$

Therefore $\nu_{\bar{\bar{A}}}^N(0) \leq \nu_A^N(x)$

(v) Now $\nu_{\bar{\bar{A}}}^P(x) = \mu_{\bar{A}}^P(x)$
$= \nu_A^P(x)$

$\geq \min\{\nu_A^P(x*y),\ \nu_A^P(y)\}$

Therefore $\nu^P_{\bar{\bar{A}}}(x) \geq \min\{\nu^P_A(x*y),\ \nu^P_A(y)\}$

(vi) Now $\nu^N_{\bar{\bar{A}}}(x) = \mu^N_{\bar{A}}(x)$

$= \nu^N_A(x)$

$\leq \max\ \{\nu^N_A(x*y),\ \nu^N_A(y)\}$

Therefore $\nu^N_{\bar{\bar{A}}}(x) \leq \max\{\nu^N_A(x*y),\ \nu^N_A(y)\}$

Therefore $\bar{\bar{A}}$ = A is also a bipolar intuitionistic anti fuzzy ideal of X.

Theorem: 5

Intersection of any two bipolar intuitionistic anti fuzzy ideal of BP-algebra X is again a bipolar intuitionistic anti fuzzy ideal of X.

Proof: Let A and B be two bipolar intuitionistic anti fuzzy ideal of X.

Consider $0,x,y \in A \cap B \Longrightarrow 0,x,y \in A$ and $0,x,y \in B$.

(i) Now $\mu^P_{A\cap B}(0) = \min\{\mu^P_A(0),\ \mu^P_B(0)\}$

$\leq \min\{\mu^P_A(x),\ \mu^P_B(x)\}$

$= \mu^P_{A\cap B}(x)$

Therefore $\mu^P_{A\cap B}(0) \leq \mu^P_{A\cap B}(x)$

Now $\mu^N_{A\cap B}(0) = \max\{\mu^N_A(0),\ \mu^N_B(0)\}$

$\geq \max\{\mu^N_A(x),\ \mu^N_B(x)\}$

$= \mu^N_{A\cap B}(x)$

Therefore $\mu^N_{A\cap B}(0) \geq \mu^N_{A\cap B}(x)$

(ii) Now $\mu^P_{A\cap B}(x) = \min\{\mu^P_A(x),\ \mu^P_B(x)\}$

$\leq \min\{\max\{\mu^P_A(x*y), \mu^P_A(y)\},\ \max\{\mu^P_B(x*y), \mu^P_B(y)\}\}$

$= \max\{\min\{\mu^P_A(x*y),\ \mu^P_B(x*y)\},\ \min\{\mu^P_A(y), \mu^P_B(y)\}\}$

$= \max\{\mu^P_{A\cap B}(x*y),\ \mu^P_{A\cap B}(y)\}$

Therefore $\mu^P_{A\cap B}(x) \leq \max\{\mu^P_{A\cap B}(x*y),\ \mu^P_{A\cap B}(y)\}$

(iii) Now $\mu^N_{A\cap B}(x) = \max\{\mu^N_A(x),\ \mu^N_B(x)\}$

$\geq \max\{\min\{\mu^N_A(x*y), \mu^N_A(y)\},\ \min\{\mu^N_B(x*y), \mu^N_B(y)\}\}$

$= \min\{\max\{\mu^N_A(x*y),\ \mu^N_B(x*y)\},\ \max\{\mu^N_A(y), \mu^N_B(y)\}\}$

$= \min\{\mu^N_{A\cap B}(x*y),\ \mu^N_{A\cap B}(y)\}$

Therefore $\mu^N_{A\cap B}(x) \geq \min\{\mu^N_{A\cap B}(x*y),\ \mu^N_{A\cap B}(y)\}$

(iv) Now $\nu^P_{A\cap B}(0) = \max\{\nu^P_A(0),\ \nu^P_B(0)\}$

$\geq \max\{\nu^P_A(x),\ \nu^P_B(x)\}$

$= \nu^P_{A\cap B}(x)$

Therefore $\nu^P_{A\cap B}(0) \geq \nu^P_{A\cap B}(x)$

Now $\nu^N_{A\cap B}(0) = \min\{\nu^N_A(0),\ \nu^N_B(0)\}$

$\leq \min\{\nu^N_A(x),\ \nu^N_B(x)\}$

$= \nu^N_{A\cap B}(x)$

Therefore $\nu^N_{A\cap B}(0) \leq \nu^N_{A\cap B}(x)$

(v) Now $\nu^P_{A\cap B}(x) = \max\{\nu^P_A(x),\ \nu^P_B(x)\}$

$\geq \max\{\min\{\nu^P_A(x*y),\ \nu^P_A(y)\},\ \min\{\nu^P_B(x*y),\ \nu^P_B(y)\}\}$

$= \min\{\max\{\nu^P_A(x*y),\ \nu^P_B(x*y)\},\ \max\{\nu^P_A(y),\ \nu^P_B(y)\}\}$

$= \min\{\nu^P_{A\cap B}(x*y),\ \nu^P_{A\cap B}(y)\}$

Therefore $\nu^P_{A\cap B}(x) \geq \min\{\nu^P_{A\cap B}(x*y),\ \nu^P_{A\cap B}(y)\}$

(vi) Now $\nu^N_{A\cap B}(x) = \min\{\nu^N_A(x),\ \nu^N_B(x)\}$

$\leq \min\{\max\{\nu^N_A(x*y),\ \nu^N_A(y)\},\ \max\{\nu^N_B(x*y),\ \nu^N_B(y)\}\}$

$= \max \{ \min \{ \nu_A^N(x*y), \nu_B^N(x*y) \}, \min \{ \nu_A^N(y), \nu_B^N(y) \} \}$

$= \max \{ \nu_{A\cap B}^N(x*y), \nu_{A\cap B}^N(y) \}$

Therefore $\nu_{A\cap B}^N(x) \le \max \{ \nu_{A\cap B}^N(x*y), \nu_{A\cap B}^N(y) \}$

Hence intersection of any two bipolar intuitionistic anti fuzzy ideal of X is again a bipolar intuitionistic anti fuzzy ideal of X.

Theorem:6

Union of any two bipolar intuitionistic anti fuzzy ideal of BP-algebra X is again a bipolar intuitionistic anti fuzzy ideal of X if either is contained in the other.

Proof: LetA and B be two bipolar intuitionistic anti fuzzy ideal of X.

If $A \subseteq B \Longrightarrow A\cup B = B$ and if $B \subseteq A \Longrightarrow A \cup B = A$.

Consider$0,x,y \in A \cup B \Longrightarrow 0,x,y \in A$ or$0,x,y \in B$

(i) Now $\mu_{A\cup B}^P(0) = \max \{ \mu_A^P(0), \mu_B^P(0) \}$

$\le \max \{ \mu_A^P(x), \mu_B^P(x) \}$

$= \mu_{A\cup B}^P(x)$

Therefore $\mu_{A\cup B}^P(0) \le \mu_{A\cup B}^P(x)$

Now $\mu_{A\cup B}^N(0) = \min \{ \mu_A^N(0), \mu_B^N(0) \}$

$\ge \min \{ \mu_A^N(x), \mu_B^N(x) \}$

$= \mu_{A\cup B}^N(x)$

Therefore $\mu_{A\cup B}^N(0) \ge \mu_{A\cup B}^N(x)$

(ii) Now $\mu_{A\cup B}^P(x) = \max \{ \mu_A^P(x), \mu_B^P(x) \}$

$\le \max \{ \max\{ \mu_A^P(x*y), \mu_A^P(y) \}, \max \{ \mu_B^P(x*y), \mu_B^P(y) \} \}$

$= \max \{ \max\{ \mu_A^P(x*y), \mu_B^P(x*y) \}, \max \{ \mu_A^P(y), \mu_B^P(y) \} \}$

$= \max \{ \mu_{A\cup B}^P(x*y), \mu_{A\cup B}^P(y) \}$

Therefore $\mu_{A\cup B}^P(x) \le \max \{ \mu_{A\cup B}^P(x*y), \mu_{A\cup B}^P(y) \}$

(iii) Now $\mu_{A\cup B}^N(x) = \min \{ \mu_A^N(x), \mu_B^N(x) \}$

$\ge \min \{ \min\{ \mu_A^N(x*y), \mu_A^N(y) \}, \min \{ \mu_B^N(x*y), \mu_B^N(y) \} \}$

$= \min \{ \min\{ \mu_A^N(x*y), \mu_B^N(x*y) \}, \min \{ \mu_A^N(y), \mu_B^N(y) \} \}$

$= \min \{ \mu_{A\cup B}^N(x*y), \mu_{A\cup B}^N(y) \}$

Therefore $\mu_{A\cup B}^N(x) \ge \min \{ \mu_{A\cup B}^N(x*y), \mu_{A\cup B}^N(y) \}$

(iv) Now $\nu_{A\cup B}^P(0) = \min \{ \nu_A^P(0), \nu_B^P(0) \}$

$\ge \min \{ \nu_A^P(x), \nu_B^P(x) \}$

$= \nu_{A\cup B}^P(x)$

Therefore $\nu_{A\cup B}^P(0) \ge \nu_{A\cup B}^P(x)$

Now $\nu_{A\cup B}^N(0) = \max \{ \nu_A^N(0), \nu_B^N(0) \}$

$\le \max \{ \nu_A^N(x), \nu_B^N(x) \}$

$= \nu_{A\cup B}^N(x)$

Therefore $\nu_{A\cup B}^N(0) \le \nu_{A\cup B}^N(x)$

(v) Now $\nu_{A\cup B}^P(x) = \min \{ \nu_A^P(x), \nu_B^P(x) \}$

$\ge \min \{ \min \{ \nu_A^P(x*y), \nu_A^P(y) \}, \min \{ \nu_B^P(x*y), \nu_B^P(y) \} \}$

$= \min \{ \min \{ \nu_A^P(x*y), \nu_B^P(x*y) \}, \min \{ \nu_A^P(y), \nu_B^P(y) \} \}$

$= \min \{ \nu_{A\cup B}^P(x*y), \nu_{A\cup B}^P(y) \}$

Therefore $\nu_{A\cup B}^P(x) \ge \min \{ \nu_{A\cup B}^P(x*y), \nu_{A\cup B}^P(y) \}$

(vi) Now $\nu_{A\cup B}^N(x) = \max \{ \nu_A^N(x), \nu_B^N(x) \}$

$\le \max \{ \max \{ \nu_A^N(x*y), \nu_A^N(y) \}, \max \{ \nu_B^N(x*y), \nu_B^N(y) \} \}$

$= \max \{ \max \{ \nu_A^N(x*y), \nu_B^N(x*y) \}, \max \{ \nu_A^N(y), \nu_B^N(y) \} \}$

$= \max\{ \nu^N_{A\cup B}(x*y),\ \nu^N_{A\cup B}(y)\}$

Therefore $\nu^N_{A\cup B}(x) \leq \max\{ \nu^N_{A\cup B}(x*y),\ \nu^N_{A\cup B}(y)\}$

Therefore A ∪ B is a bipolar intuitionistic anti fuzzy ideal of X.

Theorem: 7

If A is a bipolar intuitionistic fuzzy α-ideal of BP-algebra X, then $\bar{\bar{A}}$ = A is a bipolar intuitionistic fuzzy α-idealof X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0,x,y,z∈ A.

(i) Now $\mu^P_{\alpha_{\bar{\bar{A}}}}(0) = \nu^P_{\alpha_{\bar{A}}}(0)$

$= \mu^P_{\alpha_A}(0)$

$\geq \mu^P_{\alpha_A}(x)$

Therefore $\mu^P_{\alpha_{\bar{\bar{A}}}}(0) \geq \mu^P_{\alpha_A}(x)$

Now $\mu^N_{\alpha_{\bar{\bar{A}}}}(0) = \nu^N_{\alpha_{\bar{A}}}(0)$

$= \mu^N_{\alpha_A}(0)$

$\leq \mu^N_{\alpha_A}(x)$

Therefore $\mu^N_{\alpha_{\bar{\bar{A}}}}(0) \leq \mu^N_{\alpha_A}(x)$

(ii) Now $\mu^P_{\alpha_{\bar{\bar{A}}}}(y*z) = \nu^P_{\alpha_{\bar{A}}}(y*z)$

$= \mu^P_{\alpha_A}(y*z)$

$\geq \min\{ \mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_A}(x*y)\}$

Therefore $\mu^P_{\alpha_{\bar{\bar{A}}}}(y*z) \geq \min\{ \mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_A}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\bar{\bar{A}}}}(y*z) = \nu^N_{\alpha_{\bar{A}}}(y*z)$

$= \mu^N_{\alpha_A}(y*z)$

$\leq \max\{ \mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_A}(x*y)\}$

Therefore $\mu^N_{\alpha_{\bar{\bar{A}}}}(y*z) \leq \max\{ \mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_A}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\bar{\bar{A}}}}(0) = \mu^P_{\alpha_{\bar{A}}}(0)$

$= \nu^P_{\alpha_A}(0)$

$\leq \nu^P_{\alpha_A}(x)$

Therefore $\nu^P_{\alpha_{\bar{\bar{A}}}}(0) \leq \nu^P_{\alpha_A}(x)$

Now $\nu^N_{\alpha_{\bar{\bar{A}}}}(0) = \mu^N_{\alpha_{\bar{A}}}(0)$

$= \nu^N_{\alpha_A}(0)$

$\geq \nu^N_{\alpha_A}(x)$

Therefore $\nu^N_{\alpha_{\bar{\bar{A}}}}(0) \geq \nu^N_{\alpha_A}(x)$

(v) Now $\nu^P_{\alpha_{\bar{\bar{A}}}}(y*z) = \mu^P_{\alpha_{\bar{A}}}(y*z)$

$= \nu^P_{\alpha_A}(y*z)$

$\leq \max\{ \nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\}$

Therefore $\nu^P_{\alpha_{\bar{\bar{A}}}}(y*z) \leq \max\{ \nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\bar{\bar{A}}}}(y*z) = \mu^N_{\alpha_{\bar{A}}}(y*z)$

$= \nu^N_{\alpha_A}(y*z)$

$\geq \min\{ \nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}$

Therefore $\nu^N_{\alpha_{\bar{\bar{A}}}}(y*z) \geq \min\{ \nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}$

Therefore $\bar{\bar{A}}$ = A is also a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 8

Intersection of any two bipolar intuitionistic fuzzy α-ideal of BP-algebra X is again a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Let A and B be two bipolar intuitionistic fuzzy α-ideal of X.

Consider 0,x,y,z∈A ∩ B⟹0,x,y,z∈A and 0,x,y,z∈B .

(i) Now $\mu^P_{\alpha_{A\cap B}}(0) = \min\{\mu^P_{\alpha_A}(0), \mu^P_{\alpha_B}(0)\}$

$\geq \min\{\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)\}\}$

$= \mu^P_{\alpha_{A\cap B}}(x)$

Therefore $\mu^P_{\alpha_{A\cap B}}(0) \geq \mu^P_{\alpha_{A\cap B}}(x)$

Now $\mu^N_{\alpha_{A\cap B}}(0) = \max\{\mu^N_{\alpha_A}(0), \mu^N_{\alpha_B}(0)\}$

$\leq \max\{\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x)\}\}$

$= \mu^N_{\alpha_{A\cap B}}(x)$

Therefore $\mu^N_{\alpha_{A\cap B}}(0) \leq \mu^N_{\alpha_{A\cap B}}(x)$

(ii) Now $\mu^P_{\alpha_{A\cap B}}(y*z) = \min\{\mu^P_{\alpha_A}(y*z), \mu^P_{\alpha_B}(y*z)\}$

$\geq \min\{\min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_A}(x*y)\}, \min\{\mu^P_{\alpha_B}(x*z), \mu^P_{\alpha_B}(x*y)\}\}$

$= \min\{\min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_B}(x*z)\}, \min\{\mu^P_{\alpha_A}(x*y), \mu^P_{\alpha_B}(x*y)\}\}$

$= \min\{\mu^P_{\alpha_{A\cap B}}(x*z), \mu^P_{\alpha_{A\cap B}}(x*y)\}$

Therefore $\mu^P_{\alpha_{A\cap B}}(y*z) \geq \min\{\mu^P_{\alpha_{A\cap B}}(x*z), \mu^P_{\alpha_{A\cap B}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{A\cap B}}(y*z) = \max\{\mu^N_{\alpha_A}(y*z), \mu^N_{\alpha_B}(y*z)\}$

$\leq \max\{\max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\}, \max\{\mu^N_{\alpha_B}(x*z), \mu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_B}(x*z)\}, \max\{\mu^N_{\alpha_A}(x*y), \mu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\mu^N_{\alpha_{A\cap B}}(x*z), \mu^N_{\alpha_{A\cap B}}(x*y)\}$

Therefore $\mu^N_{\alpha_{A\cap B}}(y*z) \leq \max\{\mu^N_{\alpha_{A\cap B}}(x*z), \mu^N_{\alpha_{A\cap B}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{A\cap B}}(0) = \max\{\nu^P_{\alpha_A}(0), \nu^P_{\alpha_B}(0)\}$

$\leq \max\{\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)\}$

$= \nu^P_{\alpha_{A\cap B}}(x)$

Therefore $\nu^P_{\alpha_{A\cap B}}(0) \leq \nu^P_{\alpha_{A\cap B}}(x)$

Now $\nu^N_{\alpha_{A\cap B}}(0) = \min\{\nu^N_{\alpha_A}(0), \nu^N_{\alpha_B}(0)\}$

$\geq \min\{\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x)\}$

$= \nu^N_{\alpha_{A\cap B}}(x)$

Therefore $\nu^N_{\alpha_{A\cap B}}(0) \geq \nu^N_{\alpha_{A\cap B}}(x)$

(v) Now $\nu^P_{\alpha_{A\cap B}}(y*z) = \max\{\nu^P_{\alpha_A}(y*z), \nu^P_{\alpha_B}(y*z)\}$

$\leq \max\{\max\{\nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_A}(x*y)\}, \max\{\nu^P_{\alpha_B}(x*z), \nu^P_{\alpha_B}(x*y)\}\}$

$= \max\{\max\{\nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_B}(x*z)\}, \max\{\nu^P_{\alpha_A}(x*y), \nu^P_{\alpha_B}(x*y)\}\}$

$= \max\{\nu^P_{\alpha_{A\cap B}}(x*z), \nu^P_{\alpha_{A\cap B}}(x*y)\}$

Therefore $\nu^P_{\alpha_{A\cap B}}(y*z) \leq \max\{\nu^P_{\alpha_{A\cap B}}(x*z), \nu^P_{\alpha_{A\cap B}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{A\cap B}}(y*z) = \min\{\nu^N_{\alpha_A}(y*z), \nu^N_{\alpha_B}(y*z)\}$

$\geq \min\{\min\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_A}(x*y)\}, \min\{\nu^N_{\alpha_B}(x*z), \nu^N_{\alpha_B}(x*y)\}\}$

$= \min\{\min\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_B}(x*z)\}, \min\{\nu^N_{\alpha_A}(x*y), \nu^N_{\alpha_B}(x*y)\}\}$

$= \min\{\nu^N_{\alpha_{A\cap B}}(x*z), \nu^N_{\alpha_{A\cap B}}(x*y)\}$

Therefore $\nu^N_{\alpha_{A\cap B}}(y*z) \geq \min\{\nu^N_{\alpha_{A\cap B}}(x*z), \nu^N_{\alpha_{A\cap B}}(x*y)\}$

Hence intersection of any two bipolar intuitionistic fuzzy α-ideal of X is again a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 9

Union of any two bipolar intuitionistic fuzzy α-idealof BP-algebra X is again a bipolar intuitionistic fuzzy α-ideal of X if either is contained in the other.

Proof: LetA and B be two bipolar intuitionistic fuzzy α-ideal of X.

If $A \subseteq B \Rightarrow A\cup B = B$ and if $B \subseteq A \Rightarrow A \cup B = A$.

Consider$0,x,y,z\in A \cup B \Rightarrow 0,x,y,z\in A$ or$0,x,y,z\in B$

(i) Now $\mu^P_{\alpha_{A\cup B}}(0) = \max\{\mu^P_{\alpha_A}(0), \mu^P_{\alpha_B}(0)\}$

$\geq \max\{\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x)\}\}$

$= \mu^P_{\alpha_{A\cup B}}(x)$

Therefore $\mu^P_{\alpha_{A\cup B}}(0) \geq \mu^P_{\alpha_{A\cup B}}(x)$

Now $\mu^N_{\alpha_{A\cup B}}(0) = \min\{\mu^N_{\alpha_A}(0), \mu^N_{\alpha_B}(0)\}$

$\leq \min\{\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x)\}$

$= \mu^N_{\alpha_{A\cup B}}(x)$

Therefore $\mu^N_{\alpha_{A\cup B}}(0) \leq \mu^N_{\alpha_{A\cup B}}(x)$

(ii) Now $\mu^P_{\alpha_{A\cup B}}(y*z) = \max\{\mu^P_{\alpha_A}(y*z), \mu^P_{\alpha_B}(y*z)\}$

$\geq \max\{\min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_A}(x*y)\}, \min\{\mu^P_{\alpha_B}(x*z), \mu^P_{\alpha_B}(x*y)\}\}$

$= \min\{\max\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_B}(x*z)\}, \max\{\mu^P_{\alpha_A}(x*y), \mu^P_{\alpha_B}(x*y)\}\}$

$= \min\{\mu^P_{\alpha_{A\cup B}}(x*z), \mu^P_{\alpha_{A\cup B}}(x*y)\}$

Therefore $\mu^P_{\alpha_{A\cup B}}(y*z) \geq \min\{\mu^P_{\alpha_{A\cup B}}(x*z), \mu^P_{\alpha_{A\cup B}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{A\cup B}}(y*z) = \min\{\mu^N_{\alpha_A}(y*z), \mu^N_{\alpha_B}(y*z)\}$

$\leq \min\{\max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\}, \max\{\mu^N_{\alpha_B}(x*z), \mu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\min\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_B}(x*z)\}, \min\{\mu^N_{\alpha_A}(x*y), \mu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\mu^N_{\alpha_{A\cup B}}(x*z), \mu^N_{\alpha_{A\cup B}}(x*y)\}$

Therefore $\mu^N_{\alpha_{A\cup B}}(y*z) \leq \max\{\mu^N_{\alpha_{A\cup B}}(x*z), \mu^N_{\alpha_{A\cup B}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{A\cup B}}(0) = \min\{\nu^P_{\alpha_A}(0), \nu^P_{\alpha_B}(0)\}$

$\leq \min\{\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)\}$

$= \nu^P_{\alpha_{A\cup B}}(x)$

Therefore $\nu^P_{\alpha_{A\cup B}}(0) \leq \nu^P_{\alpha_{A\cup B}}(x)$

Now $\nu^N_{\alpha_{A\cup B}}(0) = \max\{\nu^N_{\alpha_A}(0), \nu^N_{\alpha_B}(0)\}$

$\geq \max\{\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x)\}$

$= \nu^N_{\alpha_{A\cup B}}(x)$

Therefore $\nu^N_{\alpha_{A\cup B}}(0) \geq \nu^N_{\alpha_{A\cup B}}(x)$

(v) Now $\nu^P_{\alpha_{A\cup B}}(y*z) = \min\{\nu^P_{\alpha_A}(y*z), \nu^P_{\alpha_B}(y*z)\}$

$\leq \min\{\max\{\nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_A}(x*y)\}, \max\{\nu^P_{\alpha_B}(x*z), \nu^P_{\alpha_B}(x*y)\}\}$

$= \max\{\min\{\nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_B}(x*z)\}, \min\{\nu^P_{\alpha_A}(x*y), \nu^P_{\alpha_B}(x*y)\}\}$

$= \max\{\nu^P_{\alpha_{A\cup B}}(x*z), \nu^P_{\alpha_{A\cup B}}(x*y)$

Therefore $\nu^P_{\alpha_{A\cup B}}(y*z) \leq \max\{\nu^P_{\alpha_{A\cup B}}(x*z), \nu^P_{\alpha_{A\cup B}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{A\cup B}}(y*z) = \max\{\nu^N_{\alpha_A}(y*z),\ \nu^N_{\alpha_B}(y*z)\}$

$\geq \max\{\min\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}, \min\{\nu^N_{\alpha_B}(x*z),\ \nu^N_{\alpha_B}(x*y)\}\}$

$= \min\{\max\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_B}(x*z)\},\ \max\{\nu^N_{\alpha_A}(x*y),\ \nu^N_{\alpha_B}(x*y)\}\}$

$= \min\{\nu^N_{\alpha_{A\cup B}}(x*z),\ \nu^N_{\alpha_{A\cup B}}(x*y)\}$

Therefore $\nu^N_{\alpha_{A\cup B}}(y*z) \geq \min\{\nu^N_{\alpha_{A\cup B}}(x*z),\ \nu^N_{\alpha_{A\cup B}}(x*y)\}$

Therefore A ∪ B is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 10

If A is a bipolar intuitionistic anti fuzzy α-ideal of BP-algebra X, then $\bar{\bar{A}}$ = A is a bipolar intuitionistic anti fuzzy α-idealof X.

Proof: Given A is a bipolar intuitionistic anti fuzzy α-ideal of X.

Consider 0,x,y,z∈ A.

(i) Now $\mu^P_{\alpha_{\bar{\bar{A}}}}(0) = \nu^P_{\alpha_{\bar{A}}}(0)$

$= \mu^P_{\alpha_A}(0)$

$\leq \mu^P_{\alpha_A}(x)$

Therefore $\mu^P_{\alpha_{\bar{\bar{A}}}}(0) \leq \mu^P_{\alpha_A}(x)$

Now $\mu^N_{\alpha_{\bar{\bar{A}}}}(0) = \nu^N_{\alpha_{\bar{A}}}(0)$

$= \mu^N_{\alpha_A}(0)$

$\geq \mu^N_{\alpha_A}(x)$

Therefore $\mu^N_{\alpha_{\bar{\bar{A}}}}(0) \geq \mu^N_{\alpha_A}(x)$

(ii) Now $\mu^P_{\alpha_{\bar{\bar{A}}}}(y*z) = \nu^P_{\alpha_{\bar{A}}}(y*z)$

$= \mu^P_{\alpha_A}(y*z)$

$\leq \max\{\mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_A}(x*y)\}$

Therefore $\mu^P_{\alpha_{\bar{\bar{A}}}}(y*z) \leq \max\{\mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_A}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\bar{\bar{A}}}}(y*z) = \nu^N_{\alpha_{\bar{A}}}(y*z)$

$= \mu^N_{\alpha_A}(y*z)$

$\geq \min\{\mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_A}(x*y)\}$

Therefore $\mu^N_{\alpha_{\bar{\bar{A}}}}(y*z) \geq \min\{\mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_A}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\bar{\bar{A}}}}(0) = \mu^P_{\alpha_{\bar{A}}}(0)$

$= \nu^P_{\alpha_A}(0)$

$\geq \nu^P_{\alpha_A}(x)$

Therefore $\nu^P_{\alpha_{\bar{\bar{A}}}}(0) \geq \nu^P_{\alpha_A}(x)$

Now $\nu^N_{\alpha_{\bar{\bar{A}}}}(0) = \mu^N_{\alpha_{\bar{A}}}(0)$

$= \nu^N_{\alpha_A}(0)$

$\leq \nu^N_{\alpha_A}(x)$

Therefore $\nu^N_{\alpha_{\bar{\bar{A}}}}(0) \leq \nu^N_{\alpha_A}(x)$

(v) Now $\nu^P_{\alpha_{\bar{\bar{A}}}}(y*z) = \mu^P_{\alpha_{\bar{A}}}(y*z)$

$= \nu^P_{\alpha_A}(y*z)$

$\geq \min\{\nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\}$

Therefore $\nu^P_{\alpha_{\bar{\bar{A}}}}(y*z) \geq \min\{\nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\bar{\bar{A}}}}(y*z) = \mu^N_{\alpha_{\bar{A}}}(y*z)$

$= \nu^N_{\alpha_A}(y * z)$

$\leq \max\{\nu^N_{\alpha_A}(x * z),\ \nu^N_{\alpha_A}(x * y)\}$

Therefore $\nu^N_{\alpha_{\bar{\bar{A}}}}(y * z) \leq \max\{\nu^N_{\alpha_A}(x * z),\ \nu^N_{\alpha_A}(x * y)\}$

Therefore $\bar{\bar{A}}$ = A is also a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem: 11

Intersection of any two bipolar intuitionistic anti fuzzy α-ideal of BP-algebra X is again a bipolar intuitionistic anti fuzzy α-ideal of X.

Proof: Let A and B be two bipolar intuitionistic anti fuzzy α-ideal of X.

Consider 0,x,y,z∈A ∩ B⟹0,x,y,z∈A and 0,x,y,z∈B.

(i) Now $\mu^P_{\alpha_{A\cap B}}(0) = \min\{\mu^P_{\alpha_A}(0),\ \mu^P_{\alpha_B}(0)\}$

$\leq \min\{\mu^P_{\alpha_A}(x),\ \mu^P_{\alpha_B}(x)\}\}$

$= \mu^P_{\alpha_{A\cap B}}(x)$

Therefore $\mu^P_{\alpha_{A\cap B}}(0) \leq \mu^P_{\alpha_{A\cap B}}(x)$

Now $\mu^N_{\alpha_{A\cap B}}(0) = \max\{\mu^N_{\alpha_A}(0),\ \mu^N_{\alpha_B}(0)\}$

$\geq \max\{\mu^N_{\alpha_A}(x),\ \mu^N_{\alpha_B}(x)\}\}$

$= \mu^N_{\alpha_{A\cap B}}(x)$

Therefore $\mu^N_{\alpha_{A\cap B}}(0) \geq \mu^N_{\alpha_{A\cap B}}(x)$

(ii) Now $\mu^P_{\alpha_{A\cap B}}(y * z) = \min\{\mu^P_{\alpha_A}(y * z),\ \mu^P_{\alpha_B}(y * z)\}$

$\leq \min\{\max\{\mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_A}(x * y)\}, \max\{\mu^P_{\alpha_B}(x * z),\ \mu^P_{\alpha_B}(x * y)\}\}$

$= \max\{\min\{\mu^P_{\alpha_A}(x * z),\ \mu^P_{\alpha_B}(x * z)\},\ \min\{\mu^P_{\alpha_A}(x * y), \mu^P_{\alpha_B}(x * y)\}\}$

$= \max\{\mu^P_{\alpha_{A\cap B}}(x * z),\ \mu^P_{\alpha_{A\cap B}}(x * y)\}$

Therefore $\mu^P_{\alpha_{A\cap B}}(y * z) \leq \max\{\mu^P_{\alpha_{A\cap B}}(x * z),\ \mu^P_{\alpha_{A\cap B}}(x * y)\}$

(iii) Now $\mu^N_{\alpha_{A\cap B}}(y * z) = \max\{\mu^N_{\alpha_A}(y * z),\ \mu^N_{\alpha_B}(y * z)\}$

$\geq \max\{\min\{\mu^N_{\alpha_A}(x * z),\ \mu^N_{\alpha_A}(x * y)\}, \min\{\mu^N_{\alpha_B}(x * z),\ \mu^N_{\alpha_B}(x * y)\}\}$

$= \min\{\max\{\mu^N_{\alpha_A}(x * z),\ \mu^N_{\alpha_B}(x * z)\},\ \max\{\mu^N_{\alpha_A}(x * y), \mu^N_{\alpha_B}(x * y)\}\}$

$= \min\{\mu^N_{\alpha_{A\cap B}}(x * z),\ \mu^N_{\alpha_{A\cap B}}(x * y)\}$

Therefore $\mu^N_{\alpha_{A\cap B}}(y * z) \geq \min\{\mu^N_{\alpha_{A\cap B}}(x * z),\ \mu^N_{\alpha_{A\cap B}}(x * y)\}$

(iv) Now $\nu^P_{\alpha_{A\cap B}}(0) = \max\{\nu^P_{\alpha_A}(0),\ \nu^P_{\alpha_B}(0)\}$

$\geq \max\{\nu^P_{\alpha_A}(x),\ \nu^P_{\alpha_B}(x)\}$

$= \nu^P_{\alpha_{A\cap B}}(x)$

Therefore $\nu^P_{\alpha_{A\cap B}}(0) \geq \nu^P_{\alpha_{A\cap B}}(x)$

Now $\nu^N_{\alpha_{A\cap B}}(0) = \min\{\nu^N_{\alpha_A}(0),\ \nu^N_{\alpha_B}(0)\}$

$\leq \min\{\nu^N_{\alpha_A}(x),\ \nu^N_{\alpha_B}(x)\}$

$= \nu^N_{\alpha_{A\cap B}}(x)$

Therefore $\nu^N_{\alpha_{A\cap B}}(0) \leq \nu^N_{\alpha_{A\cap B}}(x)$

(v) Now $\nu^P_{\alpha_{A\cap B}}(y * z) = \max\{\nu^P_{\alpha_A}(y * z),\ \nu^P_{\alpha_B}(y * z)\}$

$\geq \max\{\min\{\nu^P_{\alpha_A}(x * z),\ \nu^P_{\alpha_A}(x * y)\},\ \min\{\nu^P_{\alpha_B}(x * z),\ \nu^P_{\alpha_B}(x * y)\}\}$

$= \min\{\max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_B}(x * z)\}, \max\{\nu^P_{\alpha_A}(x * y),\ \nu^P_{\alpha_B}(x * y)\}\}$

$= \min\{\nu^P_{\alpha_{A\cap B}}(x * z),\ \nu^P_{\alpha_{A\cap B}}(x * y)\}$

Therefore $\nu^P_{\alpha_{A\cap B}}(y * z) \geq \min\{\nu^P_{\alpha_{A\cap B}}(x * z),\ \nu^P_{\alpha_{A\cap B}}(x * y)\}$

(vi) Now $\nu^N_{\alpha_{A\cap B}}(y * z) = \min\{\nu^N_{\alpha_A}(y * z),\ \nu^N_{\alpha_B}(y * z)\}$

$\leq \min \{ \max \{ \nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\},\ \max \{ \nu^N_{\alpha_B}(x*z),\ \nu^N_{\alpha_B}(x*y)\}\}$

$= \max \{\min \{ \nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_B}(x*z)\}, \min \{ \nu^N_{\alpha_A}(x*y),\ \nu^N_{\alpha_B}(x*y)\}\}$

$= \max \{ \nu^N_{\alpha_{A\cap B}}(x*z),\ \nu^N_{\alpha_{A\cap B}}(x*y)\}$

Therefore $\nu^N_{\alpha_{A\cap B}}(y*z) \leq \max \{ \nu^N_{\alpha_{A\cap B}}(x*z),\ \nu^N_{\alpha_{A\cap B}}(x*y)\}$

Hence intersection of any two bipolar intuitionistic anti fuzzy α-ideal of X is again a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem:12

Union of any two bipolar intuitionistic anti fuzzy α-idealof BP-algebra X is again a bipolar intuitionistic anti fuzzy α-ideal of X if either is contained in the other.

Proof: LetA and B be two bipolar intuitionistic anti fuzzy α-ideal of X.

If $A \subseteq B \Rightarrow A\cup B = B$ and if $B \subseteq A \Rightarrow A \cup B = A$.

Consider$0,x,y,z \in A \cup B \Rightarrow 0,x,y,z \in A$ or$0,x,y,z \in B$

(i) Now $\mu^P_{\alpha_{A\cup B}}(0) = \max \{ \mu^P_{\alpha_A}(0),\ \mu^P_{\alpha_B}(0)\}$

$\leq \max \{ \mu^P_{\alpha_A}(x),\ \mu^P_{\alpha_B}(x)\}\}$

$= \mu^P_{\alpha_{A\cup B}}(x)$

Therefore $\mu^P_{\alpha_{A\cup B}}(0) \leq \mu^P_{\alpha_{A\cup B}}(x)$

Now $\mu^N_{\alpha_{A\cup B}}(0) = \min \{ \mu^N_{\alpha_A}(0),\ \mu^N_{\alpha_B}(0)\}$

$\geq \min \{ \mu^N_{\alpha_A}(x),\ \mu^N_{\alpha_B}(x)\}$

$= \mu^N_{\alpha_{A\cup B}}(x)$

Therefore $\mu^N_{\alpha_{A\cup B}}(0) \geq \mu^N_{\alpha_{A\cup B}}(x)$

(ii) Now $\mu^P_{\alpha_{A\cup B}}(y*z) = \max \{\mu^P_{\alpha_A}(y*z),\ \mu^P_{\alpha_B}(y*z)\}$

$\leq \max \{ \max \{ \mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_A}(x*y)\},\ \max \{ \mu^P_{\alpha_B}(x*z),\ \mu^P_{\alpha_B}(x*y)\}\}$

$= \max \{ \max\{\mu^P_{\alpha_A}(x*z),\ \mu^P_{\alpha_B}(x*z)\},\ \max \{\mu^P_{\alpha_A}(x*y), \mu^P_{\alpha_B}(x*y)\}\}$

$= \max \{ \mu^P_{\alpha_{A\cup B}}(x*z),\ \mu^P_{\alpha_{A\cup B}}(x*y)\}$

Therefore $\mu^P_{\alpha_{A\cup B}}(y*z) \leq \max \{ \mu^P_{\alpha_{A\cup B}}(x*z),\ \mu^P_{\alpha_{A\cup B}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{A\cup B}}(y*z) = \min \{\mu^N_{\alpha_A}(y*z),\ \mu^N_{\alpha_B}(y*z)\}$

$\geq \min \{ \min \{ \mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_A}(x*y)\},\ \min \{ \mu^N_{\alpha_B}(x*z),\ \mu^N_{\alpha_B}(x*y)\}\}$

$= \min \{ \min\{\mu^N_{\alpha_A}(x*z),\ \mu^N_{\alpha_B}(x*z)\},\ \min \{\mu^N_{\alpha_A}(x*y), \mu^N_{\alpha_B}(x*y)\}\}$

$= \min \{ \mu^N_{\alpha_{A\cup B}}(x*z),\ \mu^N_{\alpha_{A\cup B}}(x*y)\}$

Therefore $\mu^N_{\alpha_{A\cup B}}(y*z) \geq \min \{ \mu^N_{\alpha_{A\cup B}}(x*z),\ \mu^N_{\alpha_{A\cup B}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{A\cup B}}(0) = \min \{ \nu^P_{\alpha_A}(0),\ \nu^P_{\alpha_B}(0)\}$

$\geq \min \{ \nu^P_{\alpha_A}(x),\ \nu^P_{\alpha_B}(x)\}$

$= \nu^P_{\alpha_{A\cup B}}(x)$

Therefore $\nu^P_{\alpha_{A\cup B}}(0) \geq \nu^P_{\alpha_{A\cup B}}(x)$

Now $\nu^N_{\alpha_{A\cup B}}(0) = \max \{ \nu^N_{\alpha_A}(0),\ \nu^N_{\alpha_B}(0)\}$

$\leq \max \{ \nu^N_{\alpha_A}(x),\ \nu^N_{\alpha_B}(x)\}$

$= \nu^N_{\alpha_{A\cup B}}(x)$

Therefore $\nu^N_{\alpha_{A\cup B}}(0) \leq \nu^N_{\alpha_{A\cup B}}(x)$

(v) Now $\nu^P_{\alpha_{A\cup B}}(y*z) = \min \{ \nu^P_{\alpha_A}(y*z),\ \nu^P_{\alpha_B}(y*z)\}$

$\geq \min \{ \min \{ \nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\},\ \min \{ \nu^P_{\alpha_B}(x*z),\ \nu^P_{\alpha_B}(x*y)\}\}$

$= \min \{ \min \{ \nu^P_{\alpha_A}(x*z), \nu^P_{\alpha_B}(x*z)\},\ \min \{ \nu^P_{\alpha_A}(x*y),\ \nu^P_{\alpha_B}(x*y)\}\}$

$= \min \{ \nu^P_{\alpha_{A\cup B}}(x*z),\ \nu^P_{\alpha_{A\cup B}}(x*y)$

Therefore $\nu^P_{\alpha_{A\cup B}}(y*z) \geq \min\{\nu^P_{\alpha_{A\cup B}}(x*z),\ \nu^P_{\alpha_{A\cup B}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{A\cup B}}(y*z) = \max\{\nu^N_{\alpha_A}(y*z),\ \nu^N_{\alpha_B}(y*z)\}$

$\leq \max\{\max\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}, \max\{\nu^N_{\alpha_B}(x*z),\ \nu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\max\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_B}(x*z)\},\ \max\{\nu^N_{\alpha_A}(x*y),\ \nu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\nu^N_{\alpha_{A\cup B}}(x*z),\ \nu^N_{\alpha_{A\cup B}}(x*y)\}$

Therefore $\nu^N_{\alpha_{A\cup B}}(y*z) \leq \max\{\nu^N_{\alpha_{A\cup B}}(x*z),\ \nu^N_{\alpha_{A\cup B}}(x*y)\}$

Therefore A ∪ B is a bipolar intuitionistic anti fuzzy α-ideal of X.

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