# Latetimeevaluationoftheexpandinguniverseinthe framework of $f(R, T)$ gravity

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**Abstract**
A cosmological model has been analyzed in the framework of FRW model in the modified theory of gravity. In order to solve Einstein field equation in $f(R, T)$ gravity, by taking $f(R, T) = R + 2f(T)$ and Hubble parameter $H$ is a suitable function of cosmic time $t$ which yields model is transition phase. The kinematical and physical parameters of the models are studied.

## 1. Introduction

According to a large number of recent astrophysical observations, it has been very firmly determined that the universe is currently in the stage of accelerating ex- pansion [Bennett et al. [1]; Riess et al. [2]; Perlmutter et al. [3]). countless cosmological studies have shown that about 70% of the energy content of the all the matter in the universe has a large negative pressure. This sector is called dark energy which is typically the reason behind accelerating the expansion of the uni- verse. The recorded late time acceleration of the universe amongst the one of the major obscurities of the universe, theoretical physics and the instigating mecha- nism for this accelerated expansion are still an open question. countless models have been proposed in the literature in order to explain this recent acceleration. Basically, we can take either of two approaches to explain the recentacceleration of the universe. Our first option is to speculatethe presence of a mysterious force with large negative pressure, the alleged dark energy (DE), is accountable for the current acceleration of universe. The most apt candidate for dark energy is the cosmological constant Λ (or vacuum energy), which is fluid responsible for achiev- ing an effective negative pressure combined with a constant energy density. This model is illustrated by the equation of state constant (EoS) parameter $\omega_\Lambda = 1$, and is called the ΛCDM model [4, 5]. Even though the model closely matches the observed data, it still has its faults. The two main the disadvantages of this model are the coincidence of the universe and the problem of fine-tuning [6]. In present universe, given that the dark energy and non-relativistic matter have in terms of their densities, their magnitudes on the other hand, happened to be the same order of magnitude. This coincidence is called the cosmic coincidence prob- lem. The fine-tuning problem discusses the inconsistency between the observed and the theoretically predicted values of the cosmological constant. With the intention of overcoming these problems, a time-varying energy model is proposed in models such as quintessence [7, 8], k-essence [9, 10] and perfect fluid models (such as Chaplygin gas Model) [11, 12].

The second approach to explain the current acceleration of the universe is to modify space-time geometry. We can achieve this by modifying the left side of Einstein's equation. Improved theory of gravity is nothing but geometric generalization of Einstein's general theory of relativity, which implies the acceleration of the universe can be the effect obtained by modifying the Einstein-Hilbert action of GR. Lately, the improved theory of gravity has fascinated a lot of minds. Cosmologists are interested in realizing the role of dark energy. In the correction of gravity, the origin of dark energy is considered to be the correction of gravity. Numerous studies have shown that explanation for the early and late accelerations of the universe can be obtained using revised gravity theory. Therefore, there are many motivations to discover theories go beyond the standard format of GR. Several modified theories have been proposed in the literature such as f(R) theory [13-15], $f(T)$ theory [16-18], $f(T, B)$ theory [19], $f(R, T)$ theory [20, 21], $f(Q, T)$ Theory [22, 23], $f(G)$ theory [24], $f(R, G)$ theory [25, 26] and so on. These days, a detailed study of $f(R, T)$ gravity theory is being done. In this paper we look at the basic formalism of the $f(R, T)$ gravity field equations accompanied by some essential review. The field equations in $f(R, T)$ gravity are worked on by assuming that $f(R, T) = R + 2f(T)$ and relation between Hubble parameter $H$ and cosmic time $t$ as [27].

$$H = m + n\coth t, \tag{1}$$

here $m$ and $n$ are positive constants. It gives rise to a model of universe which shows an initial deceleration with late time acceleration.

**2. Basic equations and solution:**

The action of $f(R, T)$ gravity can be given as

$$S = \int \left( \frac{1}{2\kappa} f(R, T) + S_m \right) \sqrt{-g}\, dx^4. \tag{2}$$

Here $f(R, T)$ is said to be an arbitrary function of Ricci tensor $R$ and trace energy momentum tensor $T$.

Harko et al. [20] took the succeeding conditions into consideration

$$f(R, T) = \begin{cases} R + 2f(T) \\ f_1(R) + f_2(T) \\ f_1(R) + f_2(R) f_3(T) \end{cases} \tag{3}$$

In case of this paper, we assumed that $f(R, T) = R + 2f(T)$, and take $f(T) = \lambda T$, where $\lambda$ is known as coupling constant of $f(R, T)$ gravity.

We can rewrite equation (2) in terms of aforementioned condition as

$$f_R(R,T)R_{ij}-\frac{1}{2}f(R,T)g_{ij}+(g_{ij}Q-\nabla_i\nabla_j)f_R(R,T)=\kappa T_{ij}-f_T(R,T)T_{ij}-f_T(R,T)\Theta_{ij}, \tag{4}$$

where Q is D'Alembertian operator and defined as ($Q = \nabla^i\nabla_j$), and also $\Theta_{ij}$ is defined as

$$\Theta_{ij} = g^{lm}\frac{\delta T_{lm}}{\delta g^{ij}}. \tag{5}$$

We assume that the matter content in universe is a perfect fluid $\Theta_{ij}$ becomes

$$\Theta_{ij} = -2T_{ij} - pg_{ij}, \tag{6}$$

here $T_{ij}$ is energy momentum tensor with perfect fluid demarcated as

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \tag{7}$$

where $\rho$ and $p$ are ED and cosmic pressure. $u^i$ is four velocity vector which satisfies the condition $u^i u_i = 1$.
The field equation thus attains the following form

$$R_{ij} - \frac{1}{2}Rg_{ij} = \kappa T_{ij} + 2f_T T_{ij} + [f(T) + 2pf_T]g_{ij}. \tag{8}$$

Taking in consideration the flat FRW metric

$$ds^2 = dt^2 - \Sigma^3_{i=1}\, a^2(t)(dx^2), \tag{9}$$

here $a(t)$ is scale factor.
The resulting field equations are

$$3H^2 = (1+3\lambda)\rho - \lambda p, \tag{10}$$

$$2\dot{H} + 3H^2 = -(1+3\lambda)p + \lambda\rho. \tag{11}$$

We elected $\kappa = 1$ and $H$ is Hubble parameter defined as

$$H = \frac{\dot{a}}{a}. \tag{12}$$

We calculate the energy density and cosmic pressure in the terms of Hubble pa- rameter

$$\rho = \frac{1}{[(1+3\lambda)^2-\lambda^2]}\left[(3+6\lambda)H^2 - 2\lambda\dot{H}\right] \tag{13}$$

$$p = \frac{-1}{[(1+3\lambda)^2-\lambda^2]}\left[(3+6\lambda)H^2 + 2(1+3\lambda)\dot{H}\right]. \tag{14}$$

**3. Solution of the field equations**

In case of assumption (1), the parameters, scalar factor ($a$), expansion scalar ($\theta$), and deceleration parameter ($q$)care given by

$$a = e^{mt}(\sinh t)^n, \qquad (15)$$

$$\theta = 3(m + n\coth t), \qquad (16)$$

$$q = -1 + \frac{n}{(m\sinh t + n\cosh t)^2}. \qquad (17)$$

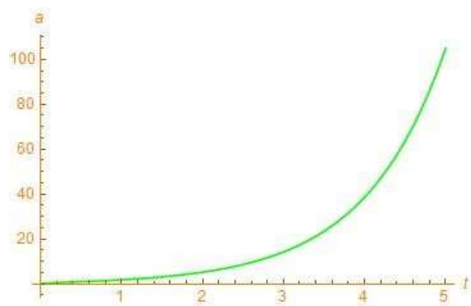


**Fig-1.** Scale factor vs time

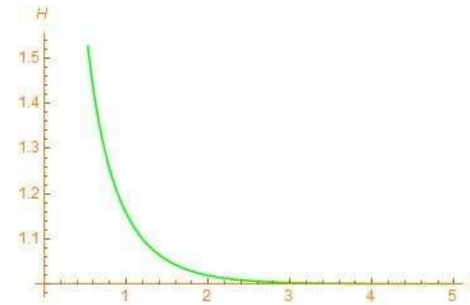


**Fig.-2.** Hubble parameter vs time

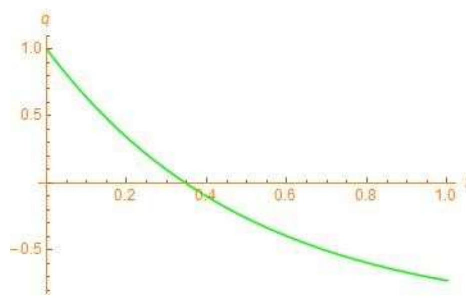


**Fig-3.** Deceleration parameter vs time .

We observed as $t = 0$,, the scale factor is zero and expansion scalar is infinity and $t \to \infty$ the scalar factor is infinity and expansion scalar are zero. The model is supporting the big-bang theory. The deceleration parameter $q = -1 + \frac{1}{n} > 0$ where $0 < n < 1$ at $t = 0$ and also $t = \infty$, $q = -1$. Therefore, this modelrepresents initial deceleration phase and late time acceleration phase of expansion.

Using equation (13) and (14), we calculate the energy density and pressure

$$\rho = \frac{1}{[(1+3\lambda)^2 - \lambda^2]}\left[(3+6\lambda)[m + n\coth t]^2 - 2\lambda n^2 cosech^2 t\right], \qquad (18)$$

$$p = \frac{-1}{[(1+3\lambda)^2 - \lambda^2]}\left[(3+6\lambda)[m + n\coth t]^2 + 2(1+3\lambda) n^2 cosech^2 t\right]. \qquad (19)$$

The equation of state parameter, we have

$$\omega = -\frac{[(3+6\lambda)[m+n\coth t]^2 + 2(1+3\lambda)n^2 cosech^2 t]}{[(3+6\lambda)[m+n\coth t]^2 - 2\lambda n^2 cosech^2 t]}. \qquad (20)$$

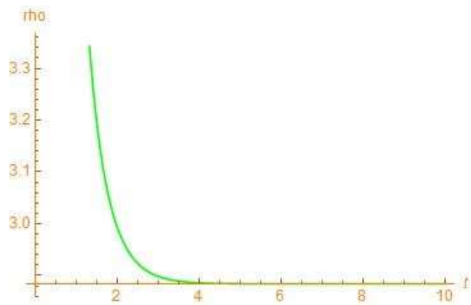


**Fig-4.** Energy density vs time

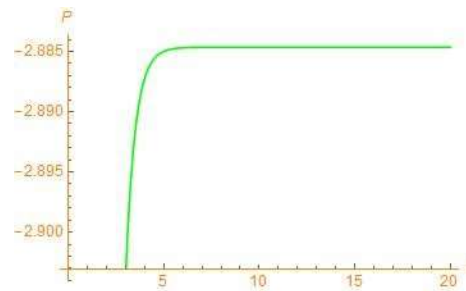


**Fig.-5.** Pressure vs time

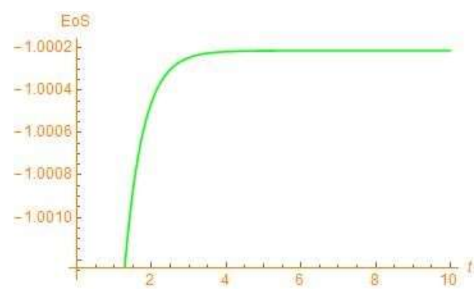


**Fig-6.** EoS vs time

We observe that the energy density is positive and pressure is negative, the negative pressure is responsible for accelerating universe in $f(R, T)$ theory.It was observed that equation of state parameter, presently has a value equal to -1. The variable of equation of state parameter of dark energy model are provided quintessence and phantom models of dark energy. The quintessence model ranges between 1 $\omega < 0$ and phantom model $\omega$ 1 $\leq$ Various like Super novae legacy survey, gold sample $\leq$ of Hubble space telescope [27-28], CMB [29-30] and large-scale structure data [31], found that the value of equation of state parameter $\omega < 1$ but might be slightly less than -1.

The matter density parameter ($\Omega_m$) and correction density parameter in $f(R, T)$ theory of gravity defined as

$$\Omega_m = \frac{\rho}{3H^2}, \qquad (21)$$

$$\Omega_\lambda = \frac{\lambda(3\rho - p)}{3H^2}, \qquad (22)$$

and the total density parameter ($\Omega_t$) as

$$\Omega_t = \Omega_m + \Omega_\lambda. \tag{23}$$

We obtain the value of $\Omega_m$ and $\Omega_\lambda$ as

$$\Omega_m = \frac{1}{3[(1+3\lambda)^2 - \lambda^2]}\left[3(1+2\lambda) - \frac{2\lambda n^2 cosech^2 t}{(m+n\coth t)^2}\right], \tag{24}$$

$$\Omega_\lambda = \frac{2\lambda}{3[(1+3\lambda)^2 - \lambda^2]}\left[6(1+2\lambda) + \frac{n^2 cosech^2 t}{(m+n\coth t)^2}\right]. \tag{25}$$

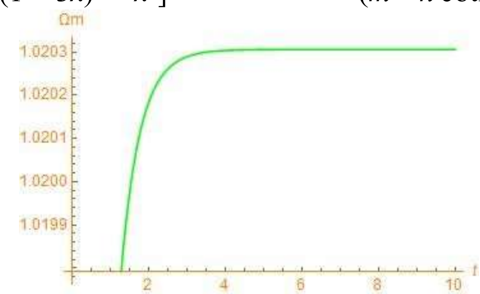


**Fig-7.** Matter density parameter vs time

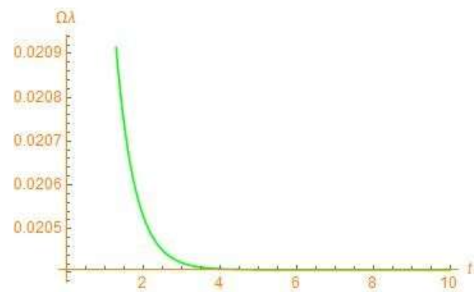


**Fig.-8.** correction term density parameter vs time

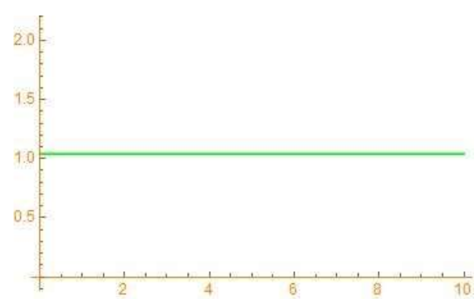


**Fig-9.** Total density parameter vs time

The geometry of universe according to the value of total density as

$$\Omega_t = 1; \text{ flat universe}$$
$$\Omega_t > 1; \text{ open universe}$$
$$\Omega_t < 1; \text{ closed universe} \quad (26)$$

**Conclusion**

In this paper, we analyzed the framework of FRW model in the modified theory of gravity. In order to find a solution of Einstein field equation in $f(R, T)$ gravity, by assuming $f(R, T) = R + 2f(T)$ and Hubble parameter $H$ is a suitable function of cosmic time $t$ which yields model is transitional phase. The universe in its initial state is a singular and Hubble parameter is positive and is a decreasing function of cosmic time t, which yields that the universe is expanding. The deceleration parameter is an early time deceleration phase and late time acceleration phase of expansion. The pressure and energy density are positive and decreasing function of cosmic time t and the equation of state parameter, presently has a value equal to -1. The variable of equation of state parameter of dark energy model are provided quintessence and phantom models of dark energy. The quintessence model ranges between $-1 \leq \omega < 0$ and phantom model $\omega \leq -1$. Various like Super novae legacy survey, gold sample of Hubble space telescope [27-28], CMB [29-30] and large-scale structure data [31], found that the value of equation of state parameter $\omega < 1$ but might be slightly less than -1. In the our is model flat at the early time as well late time . Therefore, the model is in good agreement with the recent observations.

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