A Common Fixed Point Theorem for Contraction mapping in Generalized Cone b – Metric spaces

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Abstract: In this Manuscript, we prove a common fixed point theorem for Reich type contractions in generalized cone b- metric spaces without assuming normality of cone. Our results improve, extend and generalize the results of Rangamma and Reddy [5].

Keywords: Cone, Solid cone, Fixed point, Common fixed point,Generalized cone b- metric space, Reich contractions.

1. Introduction

The concept of cone metric spaces introduced by Huang and Zhang [1] in the year 2007. The well known generalization of cone metric spaces are cone rectangular metric space [2], cone b- metric space [3]. Recently, George, et al. [4] introduced the concept of generalized cone b-metric space, which generalizes the cone metric space, cone rectangular metric space and cone b -metric spaces. They have also proved the basic version of the fixed point theorem and Kannan type fixed point theorem in generalized cone b- metric space without the assumption that the cone is normal.

Very recently, Rangamma and Reddy [5] proved a fixed point theorem for Reich contractions in generalized cone b- metric spaces, which extend and generalize the results of George, R. et al. [4]. In this paper, we improve, extend and generalize the results of Rangamma and Reddy [5] in generalized cone b- metric spaces by proving a common fixed-point theorem for Reich type contraction mappings in generalized cone b- metric spaces.

2. Preliminaries

Definition 2.1([1]). A subset P of a real Banach space E is called a cone, if it has following properties:

1. P is non empty, closed and $P \neq \{\theta\}$;

2. $a, b \in R, a, b \geq 0, x, y \in P \Rightarrow ax + by \ \in P$;
3. $P \cap (-P) = \{\theta\}$.

For a given cone $P \subset E$, we can define a partial ordering $\leq$ on E with respect to P by $x \leq y$ if and only if $y - x \in P$. We shall write $x < y$, if $x \leq y$ and $x \neq y$, while $x \ll y$ will stands for $y - x \in int(P)$, where $int(P)$denotes the interior of P and θ is zero vector in E. A cone P is called solid if $int(P) \neq \phi$.A cone P is called normal if there is a number $k > 1$ such that for all $x, y \in X, \theta \leq x \leq y \Rightarrow \|x\| \ \leq k\|y\|$. The least positive number k satisfying this condition is called the normal constant of P.

Remark 2.2 (see [7]). Let E be an ordered Banach space with a positive cone P and $a, b, c \in P$. The following properties hold:

1. If $a \leq b$ and $b \leq c$, then $a \leq c$;
2. If $\theta \leq u \ll c$ for each $c \in int(P)$, then $u = \theta$;
3. If $a \leq b + c$ for each $c \in int(P)$, then $a \leq b$;
4. If $\theta \leq x \leq y$, and $a \geq 0$, then $\theta \leq ax \leq ay$;
5. If $\theta \leq x_n \leq y_n$ for each $n \in N$, and $\lim_{n\to\infty} x_n = x, \lim_{n\to\infty} y_n = y$, then $\theta \leq x \leq y$;
6. If $\theta \leq d(x_n, x) \leq b_n$ and $b_n \to \theta$, then $d(x_n, x) \ll c$, where x_n and x are a sequence and a give point in X, respectively;
7. If $a \leq \lambda a$, where $a \in P$ and $0 < \lambda < 1$, then $a = \theta$;
8. If $c \in int(P), \theta \leq a_n$ and $a_n \to \theta$, then there exists $n_0 \in N$ such that for all $n > n_0$, we have $a_n \ll c$.

Definition 2.3 (see[1]).Let X be a non empty set, E be a real Banach space, P be a solid cone in E and $\leq$ be a partial ordering with respect to P. Suppose that the mapping $d: X \times X \to E$ satisfies:

(1) $\theta < d(x, y)$, for all $x, y \in X$ with $x \neq y$ and $d(x, y) = \theta$, if and only if $x = y$;

(2) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(3) $d(x, y) \leq d(x, z) + d(z, y)$, for all $x, y, z \in X$.

Then d is called a cone metric on X and the pair (X, d) is called a cone metric space.

Definition 2.4 (see [3]).Let X be a non empty set, E be a real Banach space, P be a solid cone in E and $\leq$ be a partial ordering with respect to $P$and $s \geq 1$ be a real number. Suppose that the mapping $d: X \times X \to E$ satisfies:

1. $\theta < d(x, y)$, for all $x, y \in X$ with $x \neq y$ and $d(x, y) = \theta$, if and only if $x = y$;

(4) $d(x, y) = d(y, x)$, for all $x, y \in X$;

(5) $d(x, y) \leq s[d(x, z) + d(z, y)]$, for all $x, y, z \in X$.

Then d is called a cone b- metric on X and the pair (X, d) is called a cone b-metric space.

Definition 2.5 (see [2]). Let X be a non empty set, E be a real Banach space, P be a solid cone in E, $\leq$ be a partial ordering with respect to $P$and $d: X \times X \to E$be a mapping such that for all $x, y \in X$ and for all distinct point $w, z \in X$, each distinct from x and y:

1. $\theta < d(x, y)$, for all $x, y \in X$ with $x \neq y$ and $d(x, y) = \theta$, if and only if $x = y$;
2. $d(x, y) = d(y, x)$, for all $x, y \in X$;
3. $d(x, y) \leq d(x, w) + d(w, z) + d(z, y)$, for all $x, y, z, w \in X$.

Then d is called a cone rectangular metric on$X$ and the pair (X, d) is called a cone rectangular metric space.

Definition 2.6 (see [4]). Let X be a non empty set, E be a real Banach space, P be a solid cone in E , $\leq$ be a partial ordering with respect to $P$and$s \geq 1$be a real number. Let the map$d: X \times X \to E$ be such that for all $x, y \in X$ and for all distinct point $w, z \in X$,(each distinct from x and y):

1. $\theta < d(x, y)$, for all $x, y \in X$ with $x \neq y$ and $d(x, y) = \theta$, if and only if $x = y$;
2. $d(x, y) = d(y, x)$, for all $x, y \in X$;
3. $d(x, y) \leq s[d(x, w) + d(w, z) + d(z, y)]$, for all $x, y, z, w \in X$.

Then d is called a generalized cone b-metric on$X$ and the pair (X, d) is called a generalized cone b- metric space.

Definition 2.7 (see[8]). Let (X, d) be a generalized cone b- metric space with coefficients $s \geq 1$. The sequence $\{x_n\}$ in X is said to be:

1. A convergent sequence if for every $c \in E$ with $\theta \ll c$, there exists $n_0 \in N$ such that for

   all $n > n_0, d(x_n, x) \ll c$ for some $x \in X$. We say thatthe sequence $\{x_n\}$ converges to x and we denote this by $\lim_{n\to\infty} x_n = x$ or $x_n \to x$, as $n \to \infty$.
2. a Cauchy sequence if for every $c \in E$ with $\theta \ll c$, there exists $n_0 \in N$ such that for all

$$m, n > n_0, d(x_n, x_m) \ll c.$$

3. The generalized cone b- metric space (X, d) is said to be complete if every Cauchy sequence is convergent in X.

Definition 2.8 (see[9],[10]). Let (X, d) be a generalized cone b- metric space with coefficient $s \geq 1$. A self map $T: X \to X$ is called Reich contraction if there exists $a, b, c \geq 0$ with $a + b + c < 1/s$ such that
$d(Tx, Ty) \leq ad(x, y) + bd(x, Tx) + cd(y, Ty)$, for all $x, y \in X$.

3. MAIN RESULTS

Theorem 3.1: Let (X, d) be a complete generalized cone b- metric space with coefficient $s \geq 1, P$ be a solid cone in real Banach space E and let $T_1, T_2: X \to X$ be a mapping satisfying

$$d(T_1x, T_2y) \leq ad(x, y) + bd(x, T_1x) + cd(y, T_2y) \quad \dots \qquad (3.1.1)$$

for all $x, y \in X$, and $a, b, c \geq 0$ with $a + b + c < \frac{1}{s}$, then T_1 and T_2 have a unique common fixed point in X.

Proof: Let x_0 be any arbitrary point of X. We define the sequence $\{x_{2n}\}$ in X such that

$$x_{2n+1} = T_1x_{2n}$$

and

$$x_{2n+2} = T_2x_{2n+1}, \text{ for all } n = 0,1,2,\dots \qquad (3.1.2)$$

If $x_{2n} = x_{2n+1}$, then $x_{2n} = T_1x_{2n}$, that is x_{2n} is a fixed point of T_1 and if $x_{2n+1} = x_{2n+2}$, then $x_{2n+1} = T_2x_{2n+1}$, that is x_{2n+1} is a fixed point of T_2. Assume that, $x_{2n} \neq x_{2n+1}$, for all $n \in N$. Then from (3.1.1) it follow that,

$$d(x_{2n}, x_{2n+1}) = d(T_1x_{2n-1}, T_2x_{2n})$$

$$\leq ad(x_{2n-1}, x_{2n}) + bd(x_{2n-1}, \quad T_1x_{2n+1}) + cd(x_{2n}, T_2x_{2n})$$

$$\leq ad(x_{2n-1}, x_{2n}) + bd(x_{2n-1}, \quad x_{2n}) + cd(x_{2n}, x_{2n+1})$$

$$= (a + b)d(x_{2n-1}, x_{2n}) + cd(x_{2n}, x_{2n+1})$$

Which implies that

$d(x_{2n}, x_{2n+1}) \leq h\, d(x_{2n-1}, x_{2n})$, for all $n \in N$, where $h = \frac{a+b}{1-c}$.

It is easy to see that $0 < h < \frac{1}{s}$.By repeating this process, we obtain

$$d(x_{2n}, x_{2n+1}) \leq h\, d(x_{2n-1}, x_{2n})$$

$$\leq h^2 d(x_{2n-1}, x_{2n-2})$$

$$\leq \cdots \leq$$

$\leq h^n d(x_0, x_1)\ldots$ (3.1.3)

for all $n \in N$, where $0 < h < 1/s < 1$. Using b- rectangular inequality, (3.1.1), (3.1.2) (3.1.3), and the fact that $a + b + c < \frac{1}{s}, i.e., a < \frac{1}{s}$, we get,

$$d(x_{2n}, x_{2n+2}) = d(T_1 x_{2n-1}, T_2 x_{2n+1})$$

$$\leq ad(x_{2n-1}, x_{2n+1}) + bd(x_{2n-1}, \quad T_1 x_{2n-1}) + cd(x_{2n+1}, T_2 x_{2n+1})$$

$$= ad(x_{2n-1}, x_{2n+1}) + bd(x_{2n-1}, \quad x_{2n}) + cd(x_{2n+1}, x_{2n+2})$$

$\leq as[d(x_{2n-1}, x_{2n}) + d(x_{2n}, x_{2n+2}) + d(x_{2n+2,} x_{2n+1})] + bd(x_{2n-1}, \ x_{2n})$

$+ cd(x_{2n+1}, x_{2n+2})$

Which implies that

$$d(x_{2n}, x_{2n+2}) \leq \left[\frac{sa + b}{1 - sa}\right] d(x_{2n-1}, x_{2n}) + \left[\frac{sa + c}{1 - sa}\right] d(x_{2n+1}, x_{2n+2})$$

$$\leq \left[\frac{sa + b}{1 - sa}\right] h^{n-1} d(x_0, x_1) + \left[\frac{sa + c}{1 - sa}\right] h^n d(x_0, x_1)$$

$$\leq \left[\frac{sa + b}{1 - sa} + \frac{sa + c}{1 - sa} . h\right] h^{n-1} d(x_0, x_1)$$

$$\leq \left[\frac{sa + b}{1 - sa} + \frac{sa + c}{1 - sa}\right] h^{n-1} d(x_0, x_1)$$

$=\left[\frac{2sa+b+c}{1-sa}\right] h^{n-1} d(x_0, x_1)$

Hence,

$d(x_{2n}, x_{2n+2}) \leq \mu h^{n-1} d(x_0, x_1)\ldots$ (3.1.4)

for all $n \in N$, where $\mu = \frac{2sa+b+c}{1-sa} \geq 0$. For the sequence $\{x_{2n}\}$, we consider $d(x_{2n}, x_{2n+2p})$ in to two cases.

Case1. Suppose $2p$ is odd, say $2m+1$, for $m \geq 1$, then by using b- rectangular inequality (3.1.3) and the fact $sh^2 < 1$, we get

$$d(x_{2n}, x_{2n+2m+1}) \leq s\left[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + d(x_{2n+2,}x_{2n+2m+1})\right]$$

$$\leq s[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})]$$

$$+s^2[(x_{2n+2}, x_{2n+3}) + d(x_{2n+3}, x_{2n+4}) + d(x_{2n+4,}x_{2n+2m-1})]$$

$$\leq s[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})]$$

$$+s^2[(x_{2n+2}, x_{2n+3}) + d(x_{2n+3}, x_{2n+4}) + \cdots + s^{2m}d(x_{2n+2m}, x_{2n+2m+1})$$

$$\leq s[h^n d(x_0, x_1) + h^{n+1}d(x_0, x_1)] + s^2[h^{2n+2}d(x_0, x_1) + h^{2n+3}d(x_0, x_1]$$

$$+\ldots+ s^{2m}h^{2n+2m}d(x_0, x_1)$$

$$\leq sh^{2n}[1 + sh^2 + \cdots +]d(x_0, x_1) + sh^{2n+1}[1 + sh^2 + \cdots]d(x_0, x_1)$$

$$= (1 + h)sh^{2n}[1 + sh^2 + \cdots]d(x_0, x_1).$$

Hence,

$d(x_{2n}, x_{2n+2m+1}) \leq \left(\frac{1+h}{1-sh^2}\right)sh^2 d(x_0, x_1)$, for all $n \in N$. Let $\theta \ll c$ be given.

Since $sh^2 < 1$, we notice that $\left(\frac{1+h}{1-sh^2}\right)sh^2 d(x_0, x_1) \to \theta$, as $n \to \infty$. By remark 2.2, for any $c \in int\ P$, we find $N_1 \in N$ such that for each $n > N_1$, we have

$$\left(\frac{1+h}{1-sh^2}\right)sh^2 d(x_0, x_1) \ll c.$$

Thus, $d(x_{2n}, x_{2n+2m+1}) \leq \left(\frac{1+h}{1-sh^2}\right)sh^2 d(x_0, x_1) \ll c$, for all $n > N_1$and $m \geq 1$.

Case 2. Suppose $2p$ is even, say $2m$, for $m \geq 1$, then by using b- rectangular inequality,(3.1.3), (3.1.4) and the fact that $sh^2 < 1$, we get,

$$d(x_{2n}, x_{2n+2m}) \leq s\left[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2}) + d(x_{2n+2,}x_{2n+2m})\right]$$

$$\leq s[d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x_{2n+2})]$$

$$+s^2[(x_{2n+2}, x_{2n+3}) + d(x_{2n+3}, x_{2n+4}) + d(x_{2n+4,}x_{2n+2m})]$$

$$\leq s[d(x_{2n},x_{2n+1})+d(x_{2n+1},x_{2n+2})]$$

$$+s^2[(x_{2n+2},x_{2n+3})+d(x_{2n+3},x_{2n+4})$$

$$+\cdots+s^{2m-1}[d(x_{2n+2m-4},x_{2n+2m-3})+d(x_{2n+2m-3},x_{2n+2m-2})$$

$$+d(x_{2n+2m-2,}x_{2n+2m})]$$

$$\leq s[h^n d(x_0,x_1)+h^{n+1}d(x_0,x_1)]+s^2[h^{2n+2}d(x_0,x_1)+h^{2n+3}d(x_0,x_1]$$

$$+\ldots+s^{2m-1}[h^{2n+2m-4}d(x_0,x_1)+h^{2n+2m-3}d(x_0,x_1)]$$

$$+s^{2m-1}\mu h^{2n+2m-3}\,d(x_{2n+2m-2},x_{2n+2m})$$

$$\leq sh^{2n}[1+sh^2+\cdots+]d(x_0,x_1)+sh^{2n+1}[1+sh^2+\cdots]d(x_0,x_1)$$

$$+s^{2m-1}\mu h^{2n+2m-3}d(x_0,x_1)$$

$$=(1+h)sh^{2n}[1+sh^2+\cdots]d(x_0,x_1)+s^{2m-1}\mu h^{2n+2m-3}d(x_0,x_1)$$

That is, $d(x_{2n},x_{2n+2m})\leq\left(\frac{1+h}{1-sh^2}\right)sh^2d(x_0,x_1)+s^{2m-1}\,\mu h^{2n+2m-3}d(x_0,x_1)$,

$$\leq\left(s\frac{1+h}{1-sh^2}+s^{2m-1}\mu h^{2m-3}\right)h^{2n}d(x_0,x_1),$$

for all $n\in N,m\in N$,where $\mu\geq 0$. Let $\theta\ll c$ be given. Since, $sh^2<1$, we notice that

$\left(s\frac{1+h}{1-sh^2}+s^{2m-1}\mu h^{2m-3}\right)h^n d(x_0,x_1)\to\theta$, as $n\to\infty$. By remark 2.2 for any $c\in int(P)$, we find $N_2\in N$ such that

$\left(s\frac{1+h}{1-sh^2}+s^{2m-1}\mu h^{2m-3}\right)h^{2n}d(x_0,x_1)\ll c$, for all $n>N_1$ and $m\geq 1$.

Thus, $d(x_{2n},x_{2n+2m})\leq\left(s\frac{1+h}{1-sh^2}+s^{2m-1}\mu h^{2m-3}\right)h^{2n}d(x_0,x_1)\ll c$, ,for all $n>N_1$ and $m\geq 1$.

Let $N_0=\max\{N_1,N_2\}$. Thus for each $c\in int(P)$, we have $d(x_{2n},x_{2n+2p})\ll c$,for all $n>N_0$ and $p\geq 1$. Therefore, $\{x_{2n}\}$ is Cauchy sequence in X. Since, X is a complete generalized cone b- metric space, then there exists $x^*\in X$ such that

$\lim_{n\to\infty}x_{2n}=\lim_{n\to\infty}T_1x_{2n-1}=x^*$,…... (3.1.5)

We shall show that $T_1x^* = x^*$.

Using b- rectangular inequality, (1.3.1),(1.3.2), $b < 1/s$ and for any $n \in N$ we have,

$$d(T_1x^*, x^*) \leq s[d(T_1x^*, x_{2n}) + d(x_{2n}, x_{2n+1}) + d(x_{2n+1}, x^*)]$$

$$\leq s[ad(x^*, x_{2n-1}) + d(x^*, T_1x^*) + cd(x_{2n-1}, T_1x_{2n-1})]$$

$$+ sd(x_{2n}, x_{2n+1}) + sd(x_{2n+1}, x^*)$$

Which implies that

$$d(T_1x^*, x^*) \leq \frac{1}{1 - sb}[sad(x^*, x_{2n-1}) + sc\, d(x_{2n-1}, x_{2n}) + sd(x_{2n}, x_{2n+1}) + sd(x_{2n+1,}x^*)]$$

$$\leq \frac{1}{1 - sb}\left[sad(x^*, x_{2n-1}) + sh^{2n-1}c\, d(x_0, x_1) + sh^{2n}d(x_0, x_1) + sd(x_{2n+1,}x^*)\right]$$

$$\leq \frac{1}{1 - sb}\left[sad(x^*, x_{2n-1}) + sh^{2n-1}(c + \mu)d(x_0, x_1) + sd\left(x_{2n+1,}x^*\right)\right].$$

Let $\theta \ll c$ be given. We choose natural numbers N_3, N_4 and N_5 such that

$d\left(x_{2n-1\,,}x^*\right) \ll \frac{c(1-sb)}{3sa}$, for all $n > N_3$,

$d\left(x_{2n-1\,,}x^*\right) \ll \frac{c(1-sb)}{3s}$, for all $n > N_4$ and

$h^{2n-1}d(x_0, x_1) \ll \frac{c(1-sb)}{3s(c+\mu)}$, for all $n > N_{5.}$

Let $N = \max\{N_3, N_4, N_5\}$. Thus for each $c \in int(P)$, we have

$d(T_1x^*, x^*) \ll c$, for all $n > N$. It follows that, $d(T_1x^*, x^*) = \theta$, that is

$T_1x^* = x^*$. Thus x^* is a fixed point of T_1 in X.

Similarly, we can prove that $T_2x^* = x^*$. Thus, $T_1x^* = x^* = T_2x^*$. Hence, x^* is a common fixed point of $T_1$and T_2 in X.

Now to prove uniqueness: Suppose $y^*$is another common fixed point of T_1 and T_2 in X. Then it follows from (1.3.1) that

$$d(x^*, y^*) = d(T_1x^*, T_2y^*)$$

$$\leq ad(x^*, y^*) + bd(x^*, T_1x^*) + cd(y^*, T_2y^*)$$

$$= ad(x^*, y^*) + bd(x^*, x^*) + cd(y^*, y^*)$$

$$= ad(x^*, y^*)$$

$< \frac{1}{s} d(x^*, y^*)$, which is contradiction. Therefore, we must have $x^* = y^*$.

Hence, $x^*$is a unique common fixed-point of $T_1$and T_2 in X. This completes prof of theorem.

With the suitable values of a, b, and c, we obtain the following corollaries on generalized cone b- metric spaces.

Corollary 3.2:Let (X, d) be a complete generalized cone b- metric space with coefficient $s \geq 1, P$ be a solid cone in real Banach space E and let $T_1, T_2: X \to X$ be a mapping satisfying

$d(T_1x, T_2y) \leq \alpha[d(x, T_1x) + d(y, T_2y)]$ ... (3.2.1)

for all $x, y \in X$,where $\alpha \in [0, \frac{1}{s}+1]$, then T_1 and T_2 have a unique common fixed point in X.

Corollary 3.3: Let (X, d) be a complete generalized cone b- metric space with coefficient $s \geq 1, P$ be a solid cone in real Banach space E and let $T_1, T_2: X \to X$ be a mapping satisfying

$d(T_1x, T_2y) \leq ad(x, y)$ ... (3.1.1)

for all $x, y \in X$, andwhere $\alpha \in \left[0, \frac{1}{s}\right)$, then T_1 and T_2 have a unique common fixed point in X.

Example 3.4. Let $X = \{a, b, c, d\}$, where $a, b, c, d \in R, E = M_{n\times n}\ (R)$ be the space of all real matrices of order $n \geq 1$ and

$P = \{M = \left(a_{ij}\right)_{1\leq i,j\leq n}: a_{ij} \geq 0$, for all $i, j\}$ is a cone E.

Define $d: X \times X \to E$ such that

$$\begin{cases} d(x, y) = d(x, y), \\ d(x, x) = O_{n\times n}, \\ d(a, b) = 0.1\ I_n; \\ d(a, c) = d(b, c) = 0.01\ I_n; \\ d(a, d) = d(b.d) = d(c, d) = 0.02\ I_n \end{cases} \qquad \text{for all } x, y \in X;$$

Where I_n is the identity matrix. In this case, (X, d) is not a cone metric space with respect to P. Since $d(a,b) = 0.1\ I_n > d(a,c) + d(c,b)$

$= 0.01\ I_n + 0.01\ I_n$

$= 0.02\ I_n$ and (X, d) is not a cone rectangular metric space with respect to P.

Since $d(a,b) = 0.1\ I_n > d(a,c) + d(c,d) + d\ (c,b)$

$= 0.01 I_n + 0.02 I_n + 0.02 I_n$

$= 0.05\ I_n$.

However, it is easy to see that (X, d) is a complete generalized cone b- metric space with coefficient $s = 2 > 1$.

Further, let $T_1, T_2 : X \to X$ be ant two maps defined by:

$$T_1 x = \begin{cases} c, & if\ x \in \{a,b,c\}, \\ b & if\ \ x = d; \end{cases}$$

and

$$T_2 x = \begin{cases} c, if\ x \in \{a,c\} \\ b, if\ x = b \\ d, if\ x = d. \end{cases}$$

It is clear that $T_1$and $T_2$are satisfy the contraction condition (1.3.1) of theorem 3.1 with $a = \frac{1}{8}, b = \frac{1}{6}, c = 1/8$and hence $x = c$is a unique common fixed point of the mappings $T_1$and T_2.

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