# SOME PROPERTIES OF BIPOLAR VALUED I-FUZZY IDEAL OF A RING

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**ABSTRACT:**
In this paper, bipolar valued I-fuzzy ideal of a ring is introduced and some properties are discussed. These properties are useful to further research. Bipolar valued I-fuzzy ideal of a ring is a generalized form of bipolar valued fuzzy ideal of a ring.

**KEY WORDS:**
Interval valued fuzzy subset, bipolar valued fuzzy subset, bipolar valued I-fuzzy subset, bipolar valued I-fuzzy ideal, product, strongest, intersection, pseudo bipolar valued I-fuzzy coset.

## 1. INTRODUCTION

In 1965, Zadeh [12] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc. Lee [5] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0 ) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [5, 6]. Fuzzy subgroup was introduced by Azriel Rosenfeld [2]. Anitha.M.S., et.al.[1] defined the bipolar valued fuzzy subgroups of a group and Balasubramanian.A et.al.[3] introduced about bipolar interval valued fuzzy subgroups of a group. Grattan-Guiness had introduced about fuzzy membership mapped onto interval and many valued quantities. Palaniappan. N & K. Arjunan[8] defined the operation on fuzzy and anti fuzzy ideals. Santhi.V.K and K. Anbarasi,[9] have introduced about bipolar valued multi fuzzy subhemirings of a hemiring. A study on interval valued fuzzy, anti fuzzy, intuitionistic fuzzy subrings of a ring by Somasundra Moorthy.M.G[10], the thesis was useful to write the paper.

After that K.Murugalingam and K.Arjunan[7] have discussed about interval valued fuzzy subsemiring of a semiring and then bipolar valued multi fuzzy subsemirings of a semiring have been introduced by Yasodara.B and KE.Sathappan[11]. Here, the concept of bipolar valued I-fuzzy ideal of a ring is introduced and estaiblished some results.

## 2.PRELIMINARIES.

**Definition 2.1.([12])** Let X be any nonempty set. A mapping $[M] : X \to D[0, 1]$ is called an interval valued fuzzy subset ( I-fuzzy subset) of X, where D[0, 1] denotes the family of all closed subintervals of [0, 1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where $M^-$ and $M^+$ are fuzzy subsets of X such that $M^-(x) \le M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0, 1] ) and not a number from the interval [0, 1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

**Definition 2.2.([5])** A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) >/ x \in X\}$, where $A^+ : X \to [0, 1]$ and $A^- : X \to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

**Example 1:** $A = \{ < a, 0.4, -0.2 >, < b, 0.6, -0.8 >, < c, 0.3, -0.9 >\}$ is a bipolar valued fuzzy subset of X = {a, b, c }.

**Definition 2.3.** A bipolar interval valued fuzzy subset ( bipolar valued I-fuzzy subset ) [A] in X is defined as an object of the form $[A] = \{ < x, [A]^+(x), [A]^-(x) > / x \in X \}$, where $[A]^+ : X \to D[0, 1]$ and $[A]^- : X \to D[-1, 0]$, where D[0, 1] denotes the family of all closed subintervals of [0, 1] and D[−1, 0] denotes the family of all closed subintervals of [−1, 0]. The positive interval membership degree $[A]^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued I-fuzzy subset [A] and the negative interval membership degree $[A]^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued I-fuzzy subset [A]. Note that [0] = [0, 0], [1] = [1, 1] and [−1] = [−1, −1].

**Example 2:** $[A] = \{ < a, [0.5, 0,6], [-0.3, -0.2] >, < b, [0.1, 0.4], [-0.7, -0.3] >, < c, [0.5, 0.6], [ -0.4, -0.2] > \}$ is a bipolar valued I-fuzzy subset of X = {a, b, c }.

**Definition 2.4.** Let R be a ring. A bipolar valued I-fuzzy subset [A] of R is said to be a bipolar valued I-fuzzy ideal of R (BVI-FI) if the following conditions are satisfied,

(i) $[A]^+(x-y) \ge \text{rmin}\{ [A]^+(x), [A]^+(y) \}$

(ii) $[A]^+(xy) \ge \text{rmax}\{ [A]^+ (x), [A]^+ (y) \}$

(iii) $[A]^- (x-y) \le \text{rmax}\{ [A]^- (x), [A]^- (y) \}$

(iv) $[A]^- (xy) \le \text{rmin}\{ [A]^- (x), [A]^- (y) \}$ for all x and y in R.

**Example 3:** Let $R = Z_3 = \{ 0, 1, 2 \}$ be a ring with respect to the ordinary addition and multiplication. Then $[A] = \{ < 0, [0.6, 0.7], [-0.7, -0.6] >, < 1, [0.5, 0.6], [-0.6, -0.5] >, < 2, [0.5, 0.6], [-0.6, -0.5] > \}$ is a bipolar valued I-fuzzy ideal of R.

**Definition 2.5.** Let $[A] = \langle [A]^+, [A]^- \rangle$ and $[B] = \langle [B]^+, [B]^- \rangle$ be any two bipolar valued I-fuzzy subsets of sets G and H, respectively. The product of [A] and [B], denoted by $[A]\times[B]$, is defined as $[A]\times[B] = \{ \langle (x, y), ([A]\times[B])^+(x, y), ([A]\times[B])^-(x, y) \rangle / \text{ for all } x \text{ in } G$ and y in H } where $([A]\times[B])^+(x, y) = \text{rmin} \{ [A]^+(x), [B]^+(y) \}$ and $([A]\times[B])^-(x, y) = \text{rmax} \{ [A]^-(x), [B]^-(y) \}$ for all x in G and y in H.

**Definition 2.6.** Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar valued I-fuzzy subset in a set S, the strongest bipolar valued I-fuzzy relation on S, that is a bipolar valued I-fuzzy relation on [A] is $[V] = \{ \langle (x, y), [V]^+(x, y), [V]^-(x, y) \rangle / x \text{ and } y \text{ in } S\}$ given by $[V]^+(x, y) = \text{rmin} \{[A]^+(x), [A]^+(y) \}$ and $[V]^-(x, y) = \text{rmax} \{ [A]^-(x), [A]^-(y) \}$ for all x and y in S.

**Definition 2.7.** Let $[A] = \langle [A]^+, [A]^- \rangle$ and $[B] = \langle [B]^+, [B]^- \rangle$ be two bipolar valued I-fuzzy subsets of a set X. We define the following relations and operations:
(i) $[A] \subset [B]$ if and only if $[A]^+(u) \leq [B]^+(u)$ and $[A]^-(u) \geq [B]^-(u)$, $\forall\, u\in X$.
(ii) $[A] = [B]$ if and only if $[A]^+(u) = [B]^+(u)$ and $[A]^-(u) = [B]^-(u)$, $\forall\, u\in X$.
(iii) $[A]\cap[B] = \{ \langle u, \text{rmin} ( [A]^+(u), [B]^+(u) ), \text{rmax} ( [A]^-(u), [B]^-(u) ) \rangle / u\in X \}$.
(iv) $[A]\cup[B] = \{ \langle u, \text{rmax} ([A]^+(u), [B]^+(u) ), \text{rmin} ( [A]^-(u), [B]^-(u) ) \rangle / u\in X \}$.

**Definition 2.8.** Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar valued I-fuzzy ideal of a ring R and a in R. Then the **pseudo bipolar valued I-fuzzy coset** $(a[A])^p = \langle (a[A]^+)^p, (a[A]^-)^p \rangle$ is defined by $(a[A]^+)^p(x) = p(a)\, [A]^+(x)$ and $(a[A]^-)^p(x) = -p(a)[A]^-(x)$, for every x in R and for some p in P, where P is a collection of all bipolar valued fuzzy subsets of R.

## 3.SOME PROPERTIES OF BIPOLAR VALUED I-FUZZY IDEAL OF A RING

**Theorem 3.1.** Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar valued I-fuzzy ideal of a ring R. Then $[A]^+ (-x) = [A]^+ (x)$, $[A]^- (-x) = [A]^- (x)$, $[A]^+ (x) \leq [A]^+ (0)$, $[A]^- (x) \geq [A]^- (0)$, for all x in R, where 0 is the identity element in R.

**Proof.** Let x be in R. Now, $[A]^+(x) = [A]^+( -(-x)) \geq [A]^+(-x) \geq [A]^+ (x)$ for all x in R. And $[A]^-(x) = [A]^-( -(-x) ) \leq [A]^-(-x) \leq [A]^-(x)$ for all x in R. And $[A]^+(0) = [A]^+(x-x) \geq \text{rmin} \{ [A]^+(x), [A]^+(-x) \} = [A]^+(x)$ for all x in R. And $[A]^-(0) = [A]^-(x-x) \leq \text{rmax}\{ [A]^-(x), [A]^-(-x) \} = [A]^-(x)$ for all x in R.

**Theorem 3.2.** Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar valued I-fuzzy ideal of a ring R. Then
(i) $[A]^+ (x-y) = [A]^+ (0)$ implies that $[A]^+ (x) = [A]^+ (y)$ for all x, y in R.
(ii) $[A]^-(x-y) = [A]^- (0)$ implies that $[A]^- (x) = [A]^- (y)$ for all x, y in R.

**Proof.** (i) Let x, y in R. Now, $[A]^+(x) = [A]^+( x-y+y ) \geq \text{rmin} \{ [A]^+(x-y), [A]^+(y) \} = \text{rmin} \{ [A]^+(0), [A]^+(y) \} = [A]^+(y) = [A]^+( y-x+x) \geq \text{rmin}\{ [A]^+(y-x), [A]^+ (x) \} = \text{rmin}\{ [A]^+(0), [A]^+(x) \} = [A]^+(x)$ for all x, y in R. (ii) And $[A]^-(x) = [A]^-(x-y+y) \leq \text{rmax} \{ [A]^-(x-y), [A]^-(y) \} = \text{rmax}\{ [A]^-(0), [A]^-(y) \} = [A]^-(y) = [A]^-(y-x+x) \leq \text{rmax} \{ [A]^-(y-x), [A]^-(x) \} = \text{rmax} \{ [A]^-(0), [A]^-(x) \} = [A]^-(x)$ for all x, y in R.

**Theorem 3.3.** Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar valued I-fuzzy ideal of a ring R.
(i) If $[A]^+(x-y) = [1]$, then $[A]^+ (x) = [A]^+ (y)$ for all x, y in R.

(ii) If $[A]^-(x-y) = [-1]$, then $[A]^-(x) = [A]^-(y)$ for all x, y in R.

**Proof.** (i) Let x, y in R. Now, $[A]^+(x) = [A]^+(x-y+y) \geq \text{rmin}\{ [A]^+(x-y), [A]^+(y) \} = \text{rmin}\{ [1], [A]^+(y) \} = [A]^+(y) = [A]^+(-y) = [A]^+(-x+x-y) \geq \text{rmin}\{ [A]^+(-x), [A]^+(x-y) \} = \text{rmin}\{ [A]^+(-x), [1] \} = [A]^+(-x) = [A]^+(x)$ for all x, y in R. (ii) And $[A]^-(x) = [A]^-(x-y+y) \leq \text{rmax}\{ [A]^-(x-y), [A]^-(y) \} = \text{rmax}\{ [-1], [A]^-(y) \} = [A]^-(y) = [A]^-(-y) = [A]^-(-x+x-y) \leq \text{rmax}\{ [A]^-(-x), [A]^-(x-y) \} = \text{rmax}\{ [A]^-(-x), [-1] \} = [A]^-(-x) = [A]^-(x)$ for all x, y in R.

**Theorem 3.4.** Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar valued I-fuzzy ideal of a ring R. (i) If $[A]^+(x-y) = [0]$ then either $[A]^+(x) = [0]$ or $[A]^+(y) = [0]$ for x, y in R. (ii) If $[A]^+(xy) = [0]$ then either $[A]^+(x) = [0]$ or $[A]^+(y) = [0]$ for x, y in R. (iii) If $[A]^-(x-y) = [0]$ then either $[A]^-(x) = [0]$ or $[A]^-(y) = [0]$ for x, y in R. (iv) If $[A]^-(xy) = [0]$ then either $[A]^-(x) = [0]$ or $[A]^-(y) = [0]$ for x, y in R.

**Proof.** Let x, y in R. (i) By the definition $[A]^+(x-y) \geq \text{rmin}\{ [A]^+(x), [A]^+(y) \}$ which implies that $[0] \geq \text{rmin}\{ [A]^+(x), [A]^+(y) \}$. Therefore either $[A]^+(x) = [0]$ or $[A]^+(y) = [0]$. (ii) By the definition $[A]^+(xy) \geq \text{rmax}\{ [A]^+(x), [A]^+(y) \}$ which implies that $[0] \geq \text{rmax}\{ [A]^+(x), [A]^+(y) \}$. Therefore either $[A]^+(x) = [0]$ or $[A]^+(y) = [0]$. (iii) By the definition $[A]^-(x-y) \leq \text{rmax}\{ [A]^-(x), [A]^-(y) \}$ which implies that $[0] \leq \text{rmax}\{[A]^-(x), [A]^-(y) \}$. Therefore either $[A]^-(x) = [0]$ or $[A]^-(y) = [0]$. (iv) By the definition $[A]^-(xy) \leq \text{rmin}\{ [A]^-(x), [A]^-(y) \}$ which implies that $[0] \leq \text{rmin}\{ [A]^-(x), [A]^-(y) \}$. Therefore either $[A]^-(x) = [0]$ or $[A]^-(y) = [0]$.

**Theorem 3.5.** If $[A] = \langle [A]^+, [A]^- \rangle$ is a bipolar valued I-fuzzy ideal of a ring R, then $H = \{ x \in R \mid [A]^+(x) = [1], [A]^-(x) = [-1] \}$ is either empty or a ideal of R.

**Proof.** If no element satisfies this condition then H is empty. If x and y in H then $[A]^+(x-y) \geq \text{rmin}\{ [A]^+(x), [A]^+(y) \} = \text{rmin}\{ [1], [1] \} = [1]$. Therefore $[A]^+(x-y) = [1]$. And $[A]^+(xy) \geq \text{rmax}\{ [A]^+(x), [A]^+(y) \} = \text{rmax}\{ [1], [1] \} = [1]$. Therefore $[A]^+(xy) = [1]$. Also $[A]^-(x-y) \leq \text{rmax}\{ [A]^-(x), [A]^-(y) \} = \text{rmax}\{[-1], [-1] \} = [-1]$. Therefore $[A]^-(x-y) = [-1]$. And $[A]^-(xy) \leq \text{rmin}\{ [A]^-(x), [A]^-(y) \} = \text{rmin}\{[-1], [-1]\} = [-1]$. Therefore $[A]^-(xy) = [-1]$. That is $x-y \in H$ and $xy \in H$. Hence H is a ideal of R. Hence H is either empty or a ideal of R.

**Theorem 3.6.** If $[A] = \langle [A]^+, [A]^- \rangle$ and $[B] = \langle [B]^+, [B]^- \rangle$ are two bipolar valued I-fuzzy ideal of a ring R, then their intersection $[A] \cap [B]$ is a bipolar valued I-fuzzy ideal of R.

**Proof.** Let $[A] = \{ < x, [A]^+(x), [A]^-(x) > / x \in R \}$, $[B] = \{ < x, [B]^+(x), [B]^-(x) > / x \in R \}$. Let $[C] = [A] \cap [B]$ and $[C] = \{ < x, [C]^+(x), [C]^-(x) > / x \in R \}$. Let x, y in R. Now $[C]^+(x-y) = \text{rmin}\{ [A]^+(x-y), [B]^+(x-y) \} \geq \text{rmin}\{ \text{rmin}\{ [A]^+(x), [A]^+(y) \}, \text{rmin}\{ [B]^+(x), [B]^+(y) \} \} \geq \text{rmin}\{ \text{rmin}\{ [A]^+(x), [B]^+(x) \}, \text{rmin}\{[A]^+(y), [B]^+(y)\} \} = \text{rmin}\{ [C]^+(x), [C]^+(y) \}$. Therefore $[C]^+(x-y) \geq \text{rmin}\{ [C]^+(x), [C]^+(y) \}$, for all x, y in R. And $[C]^+(xy) = \text{rmin}\{ [A]^+(xy), [B]^+(xy) \} \geq \text{rmin}\{ \text{rmax}\{[A]^+(x), [A]^+(y) \}, \text{rmax}\{ [B]^+(x), [B]^+(y) \}\} \geq \text{rmax}\{ \text{rmin}\{ [A]^+(x), [B]^+(x) \}, \text{rmin}\{[A]^+(y), [B]^+(y)\}\} = \text{rmax}\{ [C]^+(x), [C]^+(y) \}$. Therefore $[C]^+(xy) \geq \text{rmax}\{ [C]^+(x), [C]^+(y) \}$, for all x, y in R. Also $[C]^-(x-y) = \text{rmax}\{ [A]^-(x-y), [B]^-(x-y) \} \leq \text{rmax}\{ \text{rmax}\{[A]^-(x), [A]^-(y) \}, \text{rmax}\{ [B]^-(x), [B]^-(y) \}\} \leq \text{rmax}\{ \text{rmax}$

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{ $[A]^-(x)$, $[B]^-(x)$ }, rmax{ $[A]^-(y)$, $[B]^-(y)$ }}= rmax { $[C]^-(x)$, $[C]^-(y)$ }. Therefore $[C]^-(x-y) \leq$ rmax{ $[C]^-(x)$, $[C]^-(y)$ }, for all x, y in R. And $[C]^-(xy)$ = rmax{$[A]^-(xy)$, $[B]^-(xy)$ }$\leq$ rmax{ rmin{ $[A]^-(x)$, $[A]^-(y)$ }, rmin { $[B]^-(x)$, $[B]^-(y)$ }} $\leq$ rmin { rmax {$[A]^-(x)$, $[B]^-(x)$}, rmax {$[A]^-(y)$, $[B]^-(y)$}}= rmin { $[C]^-(x)$, $[C]^-(y)$ }. Therefore $[C]^-(xy) \leq$ rmin {$[C]^-(x)$, $[C]^-(y)$ }, for all x, y in R. Hence $[A]\cap[B]$ is a bipolar valued I-fuzzy ideal of R.

**Theorem 3.7.** The intersection of a family of bipolar valued I-fuzzy ideals of a ring R is a bipolar valued I-fuzzy ideal of R.

**Proof.** The proof follows from the Theorem 3.6.

**Theorem 3.8.** If $[A] = \langle [A]^+, [A]^- \rangle$ and $[B] = \langle [B]^+, [B]^- \rangle$ are any two bipolar valued I-fuzzy ideals of the rings $R_1$ and $R_2$ respectively, then $[A]\times[B] = \langle ([A]\times[B])^+, ([A]\times[B])^- \rangle$ is a bipolar valued I-fuzzy ideal of $R_1\times R_2$.

**Proof:** Let [A] and [B] be two bipolar valued I-fuzzy ideals of the rings $R_1$ and $R_2$ respectively. Let $x_1$, $x_2$ be in $R_1$, $y_1$ and $y_2$ be in $R_2$. Then ( $x_1, y_1$ ) and ( $x_2, y_2$ ) are in $R_1\times R_2$. Now, $([A]\times[B])^+[ (x_1, y_1) - (x_2, y_2) ] = ([A]\times[B])^+( x_1-x_2, y_1-y_2 )$ = rmin { $[A]^+(x_1-x_2)$, $[B]^+(y_1-y_2)$ }$\geq$ rmin { rmin{ $[A]^+(x_1)$, $[A]^+(x_2)$ }, rmin { $[B]^+(y_1)$, $[B]^+(y_2)$ }} = rmin { rmin { $[A]^+(x_1)$, $[B]^+(y_1)$ }, rmin { $[A]^+(x_2)$, $[B]^+(y_2)$ } } = rmin { $([A]\times[B])^+(x_1, y_1)$, $([A]\times[B])^+(x_2, y_2)$ }. Therefore $([A]\times[B])^+[ (x_1, y_1) - (x_2, y_2) ] \geq$ rmin { $([A]\times[B])^+(x_1, y_1)$, $([A]\times[B])^+(x_2, y_2)$ }. And $([A]\times[B])^+[(x_1, y_1) (x_2, y_2)] = ([A]\times[B])^+(x_1x_2, y_1y_2)$ = rmin { $[A]^+(x_1x_2)$, $[B]^+(y_1y_2)$ } $\geq$ rmin { rmax { $[A]^+(x_1)$, $[A]^+(x_2)$ }, rmax { $[B]^+(y_1)$, $[B]^+(y_2)$ } } = rmax { rmin { $[A]^+(x_1)$, $[B]^+(y_1)$ }, rmin { $[A]^+(x_2)$, $[B]^+(y_2)$ } } = rmax{ $([A]\times[B])^+(x_1, y_1)$, $([A]\times[B])^+(x_2, y_2)$ }. Therefore $([A]\times[B])^+[ (x_1, y_1)(x_2, y_2) ] \geq$ rmax{ $([A]\times[B])^+(x_1, y_1)$, $([A]\times[B])^+(x_2, y_2)$ }. Also $([A]\times[B])^-[ (x_1, y_1) - (x_2, y_2) ] = ([A]\times[B])^-( x_1-x_2, y_1-y_2)$ = rmax {$[A]^-(x_1-x_2)$, $[B]^-(y_1-y_2)$ } $\leq$ rmax { rmax { $[A]^-(x_1)$, $[A]^-(x_2)$ }, rmax { $[B]^-(y_1)$, $[B]^-(y_2)$ }} = rmax { rmax { $[A]^-(x_1)$, $[B]^-(y_1)$ }, rmax {$[A]^-(x_2)$, $[B]^-(y_2)$ }} = rmax {$([A]\times[B])^-(x_1, y_1)$, $([A]\times[B])^-(x_2, y_2)$ }. Therefore $([A]\times[B])^-[ (x_1, y_1) -(x_2, y_2) ] \leq$ rmax { $([A]\times[B])^-(x_1, y_1)$, $([A]\times[B])^-(x_2, y_2)$ }. And $([A]\times[B])^-[ (x_1, y_1)(x_2, y_2) ] = ([A]\times[B])^-(x_1x_2, y_1y_2)$ = rmax { $[A]^-(x_1x_2)$, $[B]^-(y_1y_2)$ } $\leq$ rmax { rmin { $[A]^-(x_1)$, $[A]^-(x_2)$ }, rmin { $[B]^-(y_1)$, $[B]^-(y_2)$ } } = rmin { rmax {$[A]^-(x_1)$, $[B]^-(y_1)$ }, rmax { $[A]^-(x_2)$, $[B]^-(y_2)$ } } = rmin{ $([A]\times[B])^-(x_1, y_1)$, $([A]\times[B])^-(x_2, y_2)$ }. Therefore $([A]\times[B])^-[(x_1, y_1)(x_2, y_2) ] \leq$ rmin{ $([A]\times[B])^-(x_1, y_1)$, $([A]\times[B])^-(x_2, y_2)$ }. Hence $[A]\times[B]$ is a bipolar valued I-fuzzy ideal of $R_1\times R_2$.

**Theorem 3.9.** Let $[A] = \langle [A]^+, [A]^- \rangle$ be a bipolar valued I-fuzzy subset of a ring R and $[V] = \langle [V]^+, [V]^- \rangle$ be the strongest bipolar valued I-fuzzy relation of R. Then [A] is a bipolar valued I-fuzzy ideal of R if and only if [V] is a bipolar valued I-fuzzy ideal of R×R.

**Proof.** Suppose that [A] is a bipolar valued I-fuzzy ideal of R. Then for any $x = (x_1, x_2)$, $y = (y_1, y_2)$ are in R×R. Now $[V]^+(x-y) = [V]^+[ (x_1, x_2)-(y_1, y_2) ] = [V]^+( x_1-y_1, x_2-y_2 )$ = rmin { $[A]^+(x_1-y_1)$, $[A]^+(x_2-y_2)$ } $\geq$ rmin { rmin { $[A]^+(x_1)$, $[A]^+(y_1)$ }, rmin {$[A]^+(x_2)$, $[A]^+(y_2)$ } } = rmin { rmin { $[A]^+(x_1)$, $[A]^+(x_2)$ }, rmin { $[A]^+(y_1)$, $[A]^+(y_2)$ } } = rmin {$[V]^+(x_1, x_2)$, $[V]^+(y_1, y_2)$ } = rmin { $[V]^+(x)$, $[V]^+(y)$ }. Therefore $[V]^+(x-y) \geq$ rmin { $[V]^+(x)$, $[V]^+(y)$ } for all x, y in R×R. And $[V]^+(xy) = [V]^+[ (x_1, x_2)(y_1, y_2) ] = [V]^+(x_1y_1,$

$x_2y_2$) = rmin {$[A]^+(x_1y_1)$, $[A]^+(x_2y_2)$ } ≥ rmin{rmax {$[A]^+(x_1)$, $[A]^+(y_1)$}, rmax {$[A]^+(x_2)$, $[A]^+(y_2)$}} = rmax { rmin {$[A]^+(x_1)$, $[A]^+(x_2)$ }, rmin{$[A]^+(y_1)$, $[A]^+(y_2)$}} = rmax{ $[V]^+(x_1, x_2)$, $[V]^+(y_1, y_2)$ }= rmax {$[V]^+(x)$, $[V]^+(y)$ }. Therefore $[V]^+(xy)$ ≥ rmax {$[V]^+(x)$, $[V]^+(y)$ } for all x and y in R×R. Also we have $[V]^-(x-y) = [V]^-[ (x_1, x_2)-(y_1, y_2) ] = [V]^-(x_1-y_1, x_2-y_2)$ = rmax { $[A]^-(x_1-y_1)$, $[A]^-(x_2-y_2)$ } ≤ rmax { rmax { $[A]^-(x_1)$, $[A]^-(y_1)$ }, rmax {$[A]^-(x_2)$, $[A]^-(y_2)$ } } = rmax { rmax {$[A]^-(x_1)$, $[A]^-(x_2)$ }, rmax { $[A]^-(y_1)$, $[A]^-(y_2)$ }}= rmax { $[V]^-(x_1, x_2)$, $[V]^-(y_1, y_2)$ } = rmax { $[V]^-(x)$, $[V]^-(y)$ }. Therefore $[V]^-(x-y)$ ≤ rmax { $[V]^-(x)$, $[V]^-(y)$ } for all x, y in R×R. And $[V]^-(xy) = [V]^-[ (x_1, x_2)(y_1, y_2) ] = [V]^-(x_1y_1, x_2y_2)$ = rmax { $[A]^-(x_1y_1)$, $[A]^-(x_2y_2)$ } ≤ rmax { rmin { $[A]^-(x_1)$, $[A]^-(y_1)$ }, rmin {$[A]^-(x_2)$, $[A]^-(y_2)$ } } = rmin { rmax {$[A]^-(x_1)$, $[A]^-(x_2)$ }, rmax { $[A]^-(y_1)$, $[A]^-(y_2)$ } } = rmin { $[V]^-(x_1, x_2)$, $[V]^-(y_1, y_2)$ } = rmin{ $[V]^-(x)$, $[V]^-(y)$ }. Therefore $[V]^-(xy)$ ≤ rmin{ $[V]^-(x)$, $[V]^-(y)$ } for all x, y in R×R. This proves that [V] is a bipolar valued I-fuzzy ideal of R×R. Conversely assume that [V] is a bipolar valued I-fuzzy ideal of R×R, then for any x = $(x_1, x_2)$ and y = $(y_1, y_2)$ are in R×R, we have rmin { $[A]^+(x_1-y_1)$, $[A]^+(x_2-y_2)$ } = $[V]^+(x_1-y_1, x_2-y_2)$ = $[V]^+[( x_1, x_2)-(y_1, y_2)] = [V]^+(x-y)$ ≥ rmin{ $[V]^+(x)$, $[V]^+(y)$ }= rmin { $[V]^+(x_1, x_2)$, $[V]^+(y_1, y_2)$ } = rmin { rmin{ $[A]^+(x_1)$, $[A]^+(x_2)$ }, rmin { $[A]^+(y_1)$, $[A]^+(y_2)$ } }. If $x_2 = y_2 = 0$, we get, $[A]^+(x_1-y_1)$ ≥ rmin { $[A]^+(x_1)$, $[A]^+(y_1)$ } for all $x_1$ and $y_1$ in R. And rmin { $[A]^+(x_1y_1)$, $[A]^+(x_2y_2)$ }= $[V]^+(x_1y_1, x_2y_2) = [V]^+[( x_1, x_2)(y_1, y_2)] = [V]^+(xy)$ ≥ rmax { $[V]^+(x)$, $[V]^+(y)$ } = rmax { $[V]^+(x_1, x_2)$, $[V]^+(y_1, y_2)$ } = rmax {rmin { $[A]^+(x_1)$, $[A]^+(x_2)$ }, rmin { $[A]^+(y_1)$, $[A]^+(y_2)$ } }. If $x_2 = y_2 = 0$, we get $[A]^+(x_1y_1)$ ≥ rmax { $[A]^+(x_1)$, $[A]^+(y_1)$ } for all $x_1$ and $y_1$ in R. Also rmax { $[A]^-(x_1-y_1)$, $[A]^-(x_2-y_2)$ } = $[V]^-(x_1-y_1, x_2-y_2) = [V]^-[(x_1, x_2)-(y_1, y_2)]$ = $[V]^-(x-y)$ ≤ rmax {$[V]^-(x)$, $[V]^-(y)$ } = rmax { $[V]^-(x_1, x_2)$, $[V]^-(y_1, y_2)$ }= rmax { rmax { $[A]^-(x_1)$, $[A]^-(x_2)$ }, rmax { $[A]^-(y_1)$, $[A]^-(y_2)$ }}. If $x_2 = y_2 = 0$, we get $[A]^-(x_1-y_1)$ ≤ rmax {$[A]^-(x_1)$, $[A]^-(y_1)$ } for all $x_1$ and $y_1$ in R. And rmax {$[A]^-(x_1y_1)$, $[A]^-(x_2y_2)$ } = $[V]^-(x_1y_1, x_2y_2) = [V]^-[ (x_1, x_2)(y_1, y_2) ] = [V]^-(xy)$ ≤ rmin { $[V]^-(x)$, $[V]^-(y)$ } = rmin { $[V]^-(x_1, x_2)$, $[V]^-(y_1, y_2)$ } = rmin { rmax { $[A]^-(x_1)$, $[A]^-(x_2)$ }, rmax { $[A]^-(y_1)$, $[A]^-(y_2)$ } }. If $x_2 = y_2 = 0$, we get $[A]^-(x_1y_1)$ ≤ rmin { $[A]^-(x_1)$, $[A]^-(y_1)$ } for all $x_1$ and $y_1$ in R. Hence [A] is a bipolar valued I-fuzzy ideal of R.

**Theorem 3.10.** Let [A] = ⟨ $[A]^+$, $[A]^-$ ⟩ be a bipolar valued I-fuzzy ideal of a ring R. Then the pseudo bipolar valued I-fuzzy coset $(a[A])^p$ is a bipolar valued I-fuzzy ideal of the ring R, for every a in R and p in P.

**Proof.** For every x and y in R, $(a[A]^+)^p(x-y) = p(a)[A]^+(x-y)$ ≥ p(a) rmin { $[A]^+(x)$, $[A]^+(y)$ } = rmin { $p(a)[A]^+(x)$, $p(a)[A]^+(y)$ } = rmin { $(a[A]^+)^p(x)$, $(a[A]^+)^p(y)$ }. Therefore $(a[A]^+)^p(x-y)$ ≥ rmin { $(a[A]^+)^p(x)$, $(a[A]^+)^p(y)$ } for all x and y in R. And $(a[A]^+)^p(xy)$ = $p(a)[A]^+(xy)$ ≥ p(a) rmax { $[A]^+(x)$, $[A]^+(y)$ } = rmax { $p(a)[A]^+(x)$, $p(a)[A]^+(y)$ } = rmax { $(a[A]^+)^p(x)$, $(a[A]^+)^p(y)$ }. Thus $(a[A]^+)^p(xy)$ ≥ rmax {$(a[A]^+)^p(x)$, $(a[A]^+)^p(y)$ } for all x and y in R. Also $(a[A]^-)^p(x-y) = p(a)[A]^-(x-y)$ ≤ p(a) rmax { $[A]^-(x)$, $[A]^-(y)$ } = rmax { $p(a)[A]^-(x)$, $p(a)[A]^-(y)$ } = rmax { $(a[A]^-)^p(x)$, $(a[A]^-)^p(y)$ }. Therefore $(a[A]^-)^p(x-y)$ ≤ rmax { $(a[A]^-)^p(x)$, $(a[A]^-)^p(y)$ } for all x and y in R. And $(a[A]^-)^p(xy) = p(a)[A]^-(xy)$ ≤ p(a) rmin { $[A]^-(x)$, $[A]^-(y)$ } = rmin { $p(a)[A]^-(x)$, $p(a)[A]^-(y)$ } = rmin { $(a[A]^-)^p(x)$, $(a[A]^-)^p(y)$ }. Therefore $(a[A]^-)^p(xy)$ ≤ rmin { $(a[A]^-)^p(x)$, $(a[A]^-)^p(y)$ } for all x and y in R. Hence $(a[A])^p$ is a bipolar valued I-fuzzy ideal of the ring R.

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