Primary Bipolar Intuitionistic M Fuzzy Group

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Abstract: *The concept of a primary bipolar intuitionistic M fuzzy group and primary bipolar intuitionistic anti M fuzzy group are a new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are defined and some related properties are investigated. The purpose of the study is to implement the fuzzy set theory and the group theory of bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group. The relation between a primary bipolar intuitionistic M fuzzy subgroup and primary bipolar intuitionistic anti M fuzzy subgroup are established.*

Keywords: *M fuzzy group, anti M fuzzy group, bipolar intuitionistic fuzzy set , bipolar intuitionistic M fuzzy group, bipolar intuitionistic anti M fuzzy group, primary bipolar intuitionistic M fuzzy group , primary bipolar intuitionistic anti M fuzzy group, primary bipolar intuitionistic M fuzzy subgroup and primary bipolar intuitionistic anti M fuzzy subgroup.*

1. Introduction

The concept of fuzzy sets was initiated by I.A.Zadeh [1] then it has become a vigorous area of research in engineering, medical science, graph theory. Rosenfeld [6] gave the idea of fuzzy subgroup. Bipolar valued fuzzy sets was introduced by K.M.Lee [8] are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0,1] to [-1,0]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the membership degree (0,1] indicates that elements somewhat satisfy the property and the membership degree[-1,0) indicates that elements somewhat satisfy the implicit counter property. The author W.R.Zhang [3] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. The author Mourad Oqla [9] commenced the concept of an intuitionistic anti fuzzy M subgroups. Chakrabarthy and R.Nanda [5] investigated note on union and intersection of intuitionistic fuzzy sets, Notes on intuitionistic fuzzy sets. P.S.Das, A.Rajeshkumar [7,10] were analyzed fuzzy groups and level subgroups. R.Muthuraj [11, 12] introduced the concept of bipolar fuzzy subgroup of a M fuzzy group and bipolar anti M fuzzy group.

2. Preliminaries

Definition:1

A bipolar-valued fuzzy set (BVFS) A in X is defined as an object of the form A = $\{ \langle x, A^{+}(x), A^{-}(x)\rangle / x \in X \}$ whereA^{+}: X → [0,1] and A^{-}: X → [-1,0] denotes the positive degree of membership function and the negative degree of membership function.

Example:

A=$\{\langle x, 0.4, -0.5\rangle, \langle y, 0.3, -0.6\rangle, \langle z, 0.4, -0.6\rangle\}$ is a bipolar-valued fuzzy subset of X=$\{x, y, z\}$.

Definition:2

Let G be a group. A bipolar-valued fuzzy subset A of G is said to be a bipolar-valued fuzzy subgroup of G (BVFSG).if the following conditions are satisfied

i. $A^{+}(xy) \geq \min(A^{+}(x), A^{+}(y))$
ii. $A^{+}(x^{-1}) \geq A^{+}(x)$
iii. $A^{-}(xy) \leq \max(A^{-}(x), A^{-}(y))$
iv. $A^{-}(x^{-1}) \leq A^{-}(x)$ for all x and y in G

Example:

Let G=$\{1, -1, i, -i\}$ be a group with respect to the ordinary multiplication, then A=$\{\langle 1, 0.6, -0.6\rangle, \langle -1, 0.5, -0.4\rangle, \langle i, 0.3, -0.2\rangle, \langle -i, 0.3, -0.2\rangle\}$ is a bipolar-valued fuzzy subgroup of G.

Definition:3

Let G be a non-empty set ,A bipolar intuitionistic fuzzy set (BIFS) A in G is an object of the form A =$\{ \langle x, \mu_A^{+}(x), \upsilon_A^{+}(x), \mu_A^{-}(x), \upsilon_A^{-}(x)\rangle / x \in G\}$ where μ_A^{+}: G→ [0,1], μ_A^{-}: G→ [-1,0] and υ_A^{+}: G→ [0,1], υ_A^{-}: G→ [-1,0] is called the degree of positive membership, degree of negative membership and the degree of positive non-membership ,degree of negative non-membership respectively.

Definition:4

Let G be a group. A bipolar-valued intuitionistic fuzzy subset A of G is called a bipolar intuitionistic fuzzy subgroup of G (BIFSG).if for all $x, y \in G$

i. $\mu_A^{+}(xy) \geq \min(\mu_A^{+}(x), \mu_A^{+}(y))$ and $\upsilon_A^{+}(xy) \leq \max(\upsilon_A^{+}(x), \upsilon_A^{+}(y))$
ii. $\mu_A^{-}(xy) \leq \max(\mu_A^{-}(x), \mu_A^{-}(y))$ and $\upsilon_A^{-}(xy) \geq \min(\upsilon_A^{-}(x), \upsilon_A^{-}(y))$
iii. $\mu_A^{+}(x^{-1}) = \mu_A^{+}(x)$ and $\upsilon_A^{+}(x^{-1}) = \upsilon_A^{+}(x)$
iv. $\mu_A^{-}(x^{-1}) = \mu_A^{-}(x)$ and $\upsilon_A^{-}(x^{-1}) = \upsilon_A^{-}(x)$

Definition:5

Let G be a group. A bipolar-valued intuitionistic fuzzy subset A of G is called a bipolar intuitionistic anti- fuzzy subgroup of G.if for all $x, y \in G$

i. $\mu_A^{+}(xy) \leq \max(\mu_A^{+}(x), \mu_A^{+}(y))$ and $\upsilon_A^{+}(xy) \geq \min(\upsilon_A^{+}(x), \upsilon_A^{+}(y))$
ii. $\mu_A^{-}(xy) \geq \min(\mu_A^{-}(x), \mu_A^{-}(y))$ and $\upsilon_A^{-}(xy) \leq \max(\upsilon_A^{-}(x), \upsilon_A^{-}(y))$
iii. $\mu_A^{+}(x^{-1}) = \mu_A^{+}(x)$ and $\upsilon_A^{+}(x^{-1}) = \upsilon_A^{+}(x)$
iv. $\mu_A^{-}(x^{-1}) = \mu_A^{-}(x)$ and $\upsilon_A^{-}(x^{-1}) = \upsilon_A^{-}(x)$

Definition:6

Let G be a M group and A be a bipolar intuitionistic fuzzy subgroup of G, then A is called bipolar intuitionistic M fuzzy group of G. if for all $x \in$ G and $m \in$ M

i. $\mu_A^{+}(mx) \geq \mu_A^{+}(x)$ and $\upsilon_A^{+}(mx) \leq \upsilon_A^{+}(x)$
ii. $\mu_A^{-}(mx) \leq \mu_A^{-}(x)$ and $\upsilon_A^{-}(mx) \geq \upsilon_A^{-}(x)$

Definition:7

Let G be a M group and A be a bipolar intuitionistic anti fuzzy subgroup of G, then A is called bipolar intuitionistic anti- M fuzzy group of G. if for all $x \in$ G and $m \in$ M

i. $\mu_A^{+}(xy) \leq \mu_A^{+}(x)$ and $\upsilon_A^{+}(xy) \geq \upsilon_A^{+}(x)$
ii. $\mu_A^{-}(xy) \geq \mu_A^{-}(x)$ and $\upsilon_A^{-}(xy) \leq \upsilon_A^{-}(x)$

Definition:8

Let G be a group and A be a bipolar fuzzy subgroup of G, then A is called a primary bipolar M fuzzy group of G. if for all $x, y \in$ G and $m \in$ M, then either $\mu_A^{+}(mxy) \leq \mu_A^{+}(x^{p})$, for some $p \in Z_{+}$ or else $\mu_A^{+}(mxy) \leq \mu_A^{+}(y^{q})$, for some $q \in Z_{+}$ and either $\mu_A^{-}(mxy) \geq \mu_A^{-}(x^{p})$, for some $p \in Z_{+}$ or else $\mu_A^{-}(mxy) \geq \mu_A^{-}(y^{q})$, for some $q \in Z_{+}$

Definition:9

Let G be a group and A be a bipolar fuzzy subgroup of G, then A is called a primary bipolar anti M fuzzy group of G. if for all $x, y \in$ G and $m \in$ M,then either $\mu_A^+(mxy) \geq \mu_A^+(x^p)$, for some $p \in Z_+$or else $\mu_A^+(mxy) \geq \mu_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \leq \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) \geq \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \leq \mu_A^-(y^q)$, for some $q \in Z_+$

Definition:10

Let G be an M group and A be a bipolar intuitionistic fuzzy subgroup of G, then A is called a primary bipolar intuitionistic M fuzzy group of G. if for all $x, y \in$ G and $m \in$ M, then either $\mu_A^+(mxy) \leq \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) \geq \upsilon_A^+(x^p)$, for some $p \in Z_+$ or else $\mu_A^+(mxy) \leq \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) \geq \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \geq \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) \leq \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \geq \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) \leq \upsilon_A^-(y^q)$, for some $q \in Z_+$

Example:

$$\mu_A^+(x) = \begin{cases} 0.7 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0.2 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} -0.8 & if\ x = 1 \\ -0.5 & if x = -1 \\ -0.3 & if\ x = i, -i \end{cases} \qquad \upsilon_A^-(x) = \begin{cases} -0.1 & if\ x = 1 \\ -0.4 & if x = -1 \\ -0.6 & if\ x = i, -i \end{cases}$$

Definition:11

Let G be an M group and A be a bipolar intuitionistic anti fuzzy subgroup of G, then A is called a primary bipolar intuitionistic anti M fuzzy group of G. if for all $x, y \in$ G and $m \in$ M, then either $\mu_A^+(mxy) \geq \mu_A^+(x^p)$ and $\upsilon_A^+(mxy) \leq \upsilon_A^+(x^p)$, for some $p \in Z_+$or else $\mu_A^+(mxy) \geq \mu_A^+(y^q)$ and $\upsilon_A^+(mxy) \leq \upsilon_A^+(y^q)$, for some $q \in Z_+$ and either $\mu_A^-(mxy) \leq \mu_A^-(x^p)$ and $\upsilon_A^-(mxy) \geq \upsilon_A^-(x^p)$, for some $p \in Z_+$ or else $\mu_A^-(mxy) \leq \mu_A^-(y^q)$ and $\upsilon_A^-(mxy) \geq \upsilon_A^-(y^q)$, for some $q \in Z_+$

Example:

$$\mu_A^+(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.7 & if\ x = i, -i \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0.5 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.2 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} -0.3 & if\ x = 1 \\ -0.5 & if x = -1 \\ -0.8 & if\ x = i, -i \end{cases} \qquad \upsilon_A^-(x) = \begin{cases} -0.6 & if\ x = 1 \\ -0.4 & if x = -1 \\ -0.1 & if\ x = i, -i \end{cases}$$

Definition:12

Let G be a group. A bipolar-valued intuitionistic fuzzy set A of G is called a primary bipolar intuitionistic fuzzy subgroup of G .if for all $x, y \in G$ and $m \in M$

i. $\mu_A^+(mxy) \leq \min(\mu_A^+(x^p), \mu_A^+(y^p))$ and
$\upsilon_A^+(mxy) \geq \max\ (\upsilon_A^+(x^p), \upsilon_A^+(y^p))$, for some $p \in Z_+$

ii. $\mu_A^-(mxy) \geq \max\left(\mu_A^-(x^p), \mu_A^-(y^p)\right)$and
$\upsilon_A^-(mxy) \leq \min\left(\upsilon_A^-(x^p), \upsilon_A^-(y^p)\right)$, for some $p \in Z_+$

iii. $\mu_A^+(mx^{-1}) = \mu_A^+(x^p), \mu_A^-(mx^{-1}) = \mu_A^-(x^p)$ and
$\upsilon_A^+(mx^{-1}) = \upsilon_A^+(x^p), \upsilon_A^-(mx^{-1}) = \upsilon_A^-(x^p)$, for some $p \in Z_+$

Example:

$$\mu_A^+(x) = \begin{cases} 0.7 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.4 & if\ x = i, -i \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0.2 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.5 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} -0.8 & if\ x = 1 \\ -0.5 & if x = -1 \\ -0.3 & if\ x = i, -i \end{cases} \qquad \upsilon_A^-(x) = \begin{cases} -0.1 & if\ x = 1 \\ -0.4 & if x = -1 \\ -0.6 & if\ x = i, -i \end{cases}$$

Definition:13

Let G be a group. A bipolar-valued intuitionistic fuzzy set A of G is called a primary bipolar intuitionistic anti- fuzzy subgroup of G .if for all $x, y \in G$ and $m \in M$

i. $\mu_A^+(mxy) \geq \max(\mu_A^+(x^p), \mu_A^+(y^p))$ and
$\upsilon_A^+(mxy) \leq \min(\upsilon_A^+(x^p), \upsilon_A^+(y^p))$, for some $p \in Z_+$

ii. $\mu_A^-(mxy) \leq \min(\mu_A^-(x^p), \mu_A^-(y^p))$ and
$\upsilon_A^-(mxy) \geq \max(\upsilon_A^-(x^p), \upsilon_A^-(y^p))$, for some $p \in Z_+$

iii. $\mu_A^+(mx^{-1}) = \mu_A^+(x^p), \mu_A^-(mx^{-1}) = \mu_A^-(x^p)$ and
$\upsilon_A^+(mx^{-1}) = \upsilon_A^+(x^p), \upsilon_A^-(mx^{-1}) = \upsilon_A^-(x^p)$, for some $p \in Z_+$

Example:

$$\mu_A^+(x) = \begin{cases} 0.4 & if\ x = 1 \\ 0.6 & if x = -1 \\ 0.7 & if\ x = i, -i \end{cases} \qquad \upsilon_A^+(x) = \begin{cases} 0.5 & if\ x = 1 \\ 0.3 & if x = -1 \\ 0.2 & if\ x = i, -i \end{cases}$$

$$\mu_A^-(x) = \begin{cases} -0.3 & if\ x = 1 \\ -0.5 & if x = -1 \\ -0.8 & if\ x = i, -i \end{cases} \qquad \upsilon_A^-(x) = \begin{cases} -0.6 & if\ x = 1 \\ -0.4 & if x = -1 \\ -0.1 & if\ x = i, -i \end{cases}$$

3. Some properties of primary bipolar intuitionistic M fuzzy group and primary bipolar intuitionistic anti M fuzzy group

Theorem:1

If A is any primary bipolar M fuzzy group of G,A $\neq$G then the bipolar fuzzy subset B of G defined by,

$$\mu_B^+(x) = \begin{cases} \alpha, & if\ x \in A \\ \beta, & if\ x \in R{\sim}A\ where\ \alpha, \beta \in [0,1], \alpha > \beta \end{cases}$$

$$\upsilon_B^+(x) = \begin{cases} \alpha', & if\ x \in A \\ \beta', & if\ x \in R{\sim}A\ where\ \alpha', \beta' \in [0,1], \alpha' < \beta' \end{cases}$$

$$\mu_B^-(x) = \begin{cases} \alpha, & if\ x \in A \\ \beta, & if x \in R{\sim}A\ where\ \alpha, \beta \in [-1,0], \alpha < \beta \end{cases}$$

$$\upsilon_B^-(x) = \begin{cases} \alpha', & if\ x \in A \\ \beta', & if\ x \in R{\sim}A\ where\ \alpha', \beta' \in [-1,0], \alpha' > \beta' \end{cases}$$

is a primary bipolar intuitionistic M fuzzy group of G.

Proof:

Let $a, b \in G$ and $m \in M$ and let $\mu_B^+(mab) > \mu_B^+(a^p)$ for some $p \in Z_+$ then $\mu_B^+(a^p) \neq \alpha$ so that $\mu_B^+(a^p) = \beta$ and$\mu_B^+(mab) = \alpha$, Hence $a^p \notin A$, for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar M fuzzy group it follows that $b^q \in A$, for some$q \in Z_+$. Consequently $\mu_B^+(b^q) = \alpha = \mu_B^+(mab)$. Let $a, b \in G$ and $m \in M$ and let $\upsilon_B^+(mab) < \upsilon_B^+(a^p)$, for some $p \in Z_+$ then $\upsilon_B^+(a^p) \neq \alpha'$ so that $\upsilon_B^+(a^p) = \beta'$ and$\upsilon_B^+(mab) = \alpha'$, , Hence $a^p \notin A$, for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar M fuzzy group it follows that $b^q \in A$, for some $q \in Z_+$.Consequently $\upsilon_B^+(b^q) = \alpha' = \upsilon_B^+(mab)$. Let $a, b \in G$ and $m \in M$ and let $\mu_B^-(mab) < \mu_B^-(a^p)$, for some $p \in Z_+$ then $\mu_B^-(a^p) \neq \alpha$ so that $\mu_B^-(a^p) = \beta$ and $\mu_B^-(mab) = \alpha$, Hence $a^p \notin A$, for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar M fuzzy group it follows that $b^q \in A$, for some$q \in Z_+$.Consequently $\mu_B^-(b^q) = \alpha = \mu_B^-(mab)$. Let $a, b \in G$ and $m \in M$ and let $\upsilon_B^-(mab) > \upsilon_B^-(a^p)$, for some $p \in Z_+$ then $\upsilon_B^-(a^p) \neq \alpha'$ so that $\upsilon_B^-(a^p) = \beta'$ and $\upsilon_B^-(mab) = \alpha'$, Hence $a^p \notin A$, for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar M fuzzy group it follows that $b^q \in A$, for some $q \in Z_+$. Consequently $\upsilon_B^-(b^q) = \alpha' = \upsilon_B^-(mab)$.Hence B is a primary bipolar intuitionistic M fuzzy group of G.

Theorem:2

If A is any primary bipolar anti M fuzzy group of G, A ≠G then the bipolar fuzzy subset B of G defined by,

$$\mu_B^+(x) = \begin{cases} \alpha, & if\ x \in A \\ \beta, & if\ x \in R{\sim}A\ where\ \alpha, \beta \in [0,1], \alpha < \beta \end{cases}$$

$$\upsilon_B^+(x) = \begin{cases} \alpha', & if\ x \in A \\ \beta', & if\ x \in R{\sim}A\ where\ \alpha', \beta' \in [0,1], \alpha' > \beta' \end{cases}$$

$$\mu_B^-(x) = \begin{cases} \alpha, & if\ x \in A \\ \beta, & if x \in R{\sim}A\ where\ \alpha, \beta \in [-1,0], \alpha > \beta \end{cases}$$

$$\upsilon_B^-(x) = \begin{cases} \alpha', & if\ x \in A \\ \beta', & if\ x \in R{\sim}A\ where\ \alpha', \beta' \in [-1,0], \alpha' < \beta' \end{cases}$$

is a primary bipolar intuitionistic anti M fuzzy group of G.

Proof:

Let $a, b \in G$ and $m \in M$ and let $\mu_B^+(mab) < \mu_B^+(a^p)$, for some $p \in Z_+$ then $\mu_B^+(a^p) \neq \alpha$ so that $\mu_B^+(a^p) = \beta$ and$\mu_B^+(mab) = \alpha$, Hence $a^p \notin A$, for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar anti M fuzzy group it follows that $b^q \in A$ for some$q \in Z_+$.Consequently $\mu_B^+(b^q) = \alpha = \mu_B^+(mab)$. Let $a, b \in G$ and $m \in M$ and let $\upsilon_B^+(mab) > \upsilon_B^+(a^p)$, for some $p \in Z_+$ then $\upsilon_B^+(a^p) \neq \alpha'$ so that $\upsilon_B^+(a^p) = \beta'$ and $\upsilon_B^+(mab) = \alpha'$, Hence $a^p \notin A$ for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar anti M fuzzy group it follows that $b^q \in A$, for some $q \in Z_+$.Consequently $\upsilon_B^+(b^q) = \alpha' = \upsilon_B^+(mab)$. Let $a, b \in G$ and $m \in M$ and let $\mu_B^-(mab) > \mu_B^+(a^p)$, for some $p \in Z_+$ then $\mu_B^-(a^p) \neq \alpha$ so that $\mu_B^-(a^p) = \beta$ and $\mu_B^-(mab) = \alpha$, Hence $a^p \notin A$, for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar anti M fuzzy group it follows that $b^q \in A$, for some $q \in Z_+$.Consequently $\mu_B^-(b^q) = \alpha = \mu_B^-(mab)$. Let $a, b \in G$ and $m \in M$ and let $\upsilon_B^-(mab) < \upsilon_B^-(a^p)$, for some $p \in Z_+$ then $\upsilon_B^-(a^p) \neq \alpha'$ so that $\upsilon_B^-(a^p) = \beta'$ and $\upsilon_B^-(mab) = \alpha'$, Hence $a^p \notin A$, for some $p \in Z_+$. Since $ab \in A$ and A is a primary bipolar anti M fuzzy group it follows that $b^q \in A$, for some $q \in Z_+$.Consequently $\upsilon_B^-(b^q) = \alpha' = \upsilon_B^-(mab)$.Hence B is a primary bipolar intuitionistic anti M fuzzy group of G.

Theorem:3

If A is a primary bipolar intuitionistic M-fuzzy group of G. then $\bar{\bar{A}} = A$ is a primary bipolar intuitionistic M-fuzzy group of G.

Proof:

Consider $x, y \in A$ and $m \in$ M

Now $\mu_{\bar{\bar{A}}}^+(mxy) = \upsilon_{\bar{A}}^+(mxy)$

$= \mu_A^+(mxy)$

$\leq \mu_A^+(x^p)$

Therefore $\mu_{\bar{\bar{A}}}^+(mxy) \leq \mu_A^+(x^p)$, for some $p \in Z_+$

Consider $\upsilon_{\bar{\bar{A}}}^+(mxy) = \mu_{\bar{A}}^+(mxy)$

$= \upsilon_A^+(mxy)$

$\geq \upsilon_A^+(x^p)$

Therefore $\upsilon_{\bar{\bar{A}}}^+(mxy) \geq \upsilon_A^+(x^p)$, for some $p \in Z_+$

Consider $\mu_{\bar{\bar{A}}}^-(mxy) = \upsilon_{\bar{A}}^-(mxy)$

$= \mu_A^-(mxy)$

$\geq \mu_A^-(x^p)$

Therefore $\mu^{-}_{\bar{\bar{A}}}(mxy) \geq \mu^{-}_{A}(x^{p})$, for some $p \in Z_{+}$

Consider $\upsilon^{-}_{\bar{\bar{A}}}(mxy) = \mu^{-}_{\bar{A}}(mxy)$

$= \upsilon^{-}_{A}(mxy)$

$\leq \upsilon^{-}_{A}(x^{p})$

Therefore $\upsilon^{-}_{\bar{\bar{A}}}(mxy) \leq \upsilon^{-}_{A}(x^{p})$, for some $p \in Z_{+}$

Therefore $\bar{\bar{A}} = A$ is a primary bipolar intuitionistic M-fuzzy group of G

Theorem:4

Intersection of any two primary bipolar intuitionistic M fuzzy group is again a primary bipolar intuitionistic M fuzzy group of G.

Proof:

Let A and B be two primary bipolar intuitionistic M fuzzy group of G.

Consider $x, y \in A \cap B$ and $m \in$ M

Now $\mu^{+}_{A\cap B}(mxy) = \min\,(\mu^{+}_{A}(mxy), \mu^{+}_{B}(mxy))$

$\leq \min\,(\mu^{+}_{A}(x^{p}), \mu^{+}_{B}(x^{p}))$

$= \mu^{+}_{A\cap B}(x^{p})$

Therefore $\mu^{+}_{A\cap B}(mxy) \leq \mu^{+}_{A\cap B}(x^{p})$, for some $p \in Z_{+}$

Consider $\upsilon^{+}_{A\cap B}(mxy) = \max\,(\upsilon^{+}_{A}(mxy), \upsilon^{+}_{B}(mxy))$

$\geq \max\,(\upsilon^{+}_{A}(x^{p}), \upsilon^{+}_{B}(x^{p}))$

$= \upsilon^{+}_{A\cap B}(x^{p})$

Therefore $\upsilon^{+}_{A\cap B}(mxy) \geq \upsilon^{+}_{A\cap B}(x^{p})$, for some $p \in Z_{+}$

Consider $\mu^{-}_{A\cap B}(mxy) = \max\,(\mu^{-}_{A}(mxy), \mu^{-}_{B}(mxy))$

$\geq \max\,(\mu^{-}_{A}(x^{p}), \mu^{-}_{B}(x^{p}))$

$= \mu^{-}_{A\cap B}(x^{p})$

Therefore $\mu^{-}_{A\cap B}(mxy) \geq \mu^{-}_{A\cap B}(x^{p})$, for some $p \in Z_{+}$

Consider $\upsilon^{-}_{A\cap B}(mxy) = \min\,(\upsilon^{-}_{A}(mxy), \upsilon^{-}_{B}(mxy))$

$\leq \min\,(\upsilon^{-}_{A}(x^{p}), \upsilon^{-}_{B}(x^{p}))$

$= \upsilon^{-}_{A\cap B}(x^{p})$

Therefore $\upsilon^{-}_{A\cap B}(mxy) \leq \upsilon^{-}_{A\cap B}(x^{p})$, for some $p \in Z_{+}$

Therefore $A \cap B$ is a primary bipolar intuitionistic M-fuzzy group of G.

Theorem:5

Union of any two primary bipolar intuitionistic M fuzzy group is also a primary bipolar intuitionistic M fuzzy group if either is contained in the other.

Proof:

Let A and B be two primary bipolar intuitionistic M fuzzy group of G. To prove that $A \cup B$ is a primary bipolar intuitionistic M fuzzy group of G, if $A \subseteq B$ *or* $B \subseteq A$

If $A \subseteq B \Longrightarrow A \cup B = B$

$B \subseteq A \Longrightarrow A \cup B = A$

Consider $x, y \in A \cup B$ and $m \in$ M

Now $\mu^{+}_{A\cup B}(mxy) = \max(\mu^{+}_{A}(mxy), \mu^{+}_{B}(mxy))$

$\leq \max(\mu^{+}_{A}(x^{p}), \mu^{+}_{B}(x^{p}))$

$= \mu^{+}_{A\cup B}(x^{p})$

Therefore $\mu^{+}_{A\cup B}(mxy) \leq \mu^{+}_{A\cup B}(x^{p})$, for some $p \in Z_{+}$

Consider $\upsilon^{+}_{A\cup B}(mxy) = \min(\upsilon^{+}_{A}(mxy), \upsilon^{+}_{B}(mxy))$

$\geq \min(\upsilon^{+}_{A}(x^{p}), \upsilon^{+}_{B}(x^{p}))$

$= \upsilon^{+}_{A\cup B}(x^{p})$

Therefore $\upsilon^{+}_{A\cup B}(mxy) \geq \upsilon^{+}_{A\cup B}(x^{p})$, for some $p \in Z_{+}$

Consider $\mu^{-}_{A\cup B}(mxy) = \min(\mu^{-}_{A}(mxy), \mu^{-}_{B}(mxy))$

$\geq \min(\mu^{-}_{A}(x^{p}), \mu^{-}_{B}(x^{p}))$

$= \mu^{-}_{A\cup B}(x^{p})$

Therefore$\mu^{-}_{A\cup B}(mxy) \geq \mu^{-}_{A\cup B}(x^{p})$, for some $p \in Z_{+}$

Consider $\upsilon^{-}_{A\cup B}(mxy) = \max(\upsilon^{-}_{A}(mxy), \upsilon^{-}_{B}(mxy))$

$\leq \max(\upsilon^{-}_{A}(x^{p}), \upsilon^{-}_{B}(x^{p}))$

$= \upsilon^{-}_{A\cup B}(x^{p})$

Therefore $\upsilon^{-}_{A\cup B}(mxy) \leq \upsilon^{-}_{A\cup B}(x^{p})$, for some $p \in Z_{+}$

Hence Union of any two primary bipolar intuitionistic M fuzzy group is also a primary bipolar intuitionistic M fuzzy group if either is contained in the other.

Theorem:6

If A is a primary bipolar intuitionistic anti M-fuzzy group of G. then $\bar{\bar{A}} = A$ is a primary bipolar intuitionistic anti M-fuzzy group of G.

Proof:

Consider $x, y \in A$ and $m \in$ M

Now $\mu^{+}_{\bar{\bar{A}}}(mxy) = \upsilon^{+}_{\bar{A}}(mxy)$

$= \mu^{+}_{A}(mxy)$

$\geq \mu^{+}_{A}(x^{p})$

Therefore$\mu^{+}_{\bar{\bar{A}}}(mxy) \geq \mu^{+}_{A}(x^{p})$, for some $p \in Z_{+}$

Consider $\upsilon^{+}_{\bar{\bar{A}}}(mxy) = \mu^{+}_{\bar{A}}(mxy)$

$= \upsilon^{+}_{A}(mxy)$

$\leq \upsilon^{+}_{A}(x^{p})$

Therefore $\upsilon^{+}_{\bar{\bar{A}}}(mxy) \leq \upsilon^{+}_{A}(x^{p})$, for some $p \in Z_{+}$

Consider $\mu^{-}_{\bar{\bar{A}}}(mxy) = \upsilon^{-}_{\bar{A}}(mxy)$

$= \mu^{-}_{A}(mxy)$

$\leq \mu^{-}_{A}(x^{p})$

Therefore $\mu^{-}_{\bar{\bar{A}}}(mxy) \leq \mu^{-}_{A}(x^{p})$, for some $p \in Z_{+}$

Consider $\upsilon^{-}_{\bar{\bar{A}}}(mxy) = \mu^{-}_{\bar{A}}(mxy)$

$= \upsilon^{-}_{A}(mxy)$

$\geq \upsilon_A^-(x^p)$

Therefore $\upsilon_{\bar{\bar{A}}}^-(mxy) \geq \upsilon_A^-(x^p)$, for some $p \in Z_+$

Therefore $\bar{\bar{A}} = A$ is a primary bipolar intuitionistic anti M-fuzzy group of G

Theorem:7

Intersection of any two primary bipolar intuitionistic anti M fuzzy group is again a primary bipolar intuitionistic anti M fuzzy group of G.

Proof:

Let A and B be two primary bipolar intuitionistic anti M fuzzy group of G.

Consider $x, y \in A \cap B$ and $m \in$ M

Now $\mu_{A\cap B}^+(mxy) = \min\left(\mu_A^+(mxy), \mu_B^+(mxy)\right)$

$\geq \min\left(\mu_A^+(x^p), \mu_B^+(x^p)\right)$

$= \mu_{A\cap B}^+(x^p)$

Therefore $\mu_{A\cap B}^+(mxy) \geq \mu_{A\cap B}^+(x^p)$, for some $p \in Z_+$

Consider $\upsilon_{A\cap B}^+(mxy) = \max\left(\upsilon_A^+(mxy), \upsilon_B^+(mxy)\right)$

$\leq \max\left(\upsilon_A^+(x^p), \upsilon_B^+(x^p)\right)$

$= \upsilon_{A\cap B}^+(x^p)$

Therefore $\upsilon_{A\cap B}^+(mxy) \leq \upsilon_{A\cap B}^+(x^p)$, for some $p \in Z_+$

Consider $\mu_{A\cap B}^-(mxy) = \max\left(\mu_A^-(mxy), \mu_B^-(mxy)\right)$

$\leq \max\left(\mu_A^-(x^p), \mu_B^-(x^p)\right)$

$= \mu_{A\cap B}^-(x^p)$

Therefore $\mu_{A\cap B}^-(mxy) \leq \mu_{A\cap B}^-(x^p)$, for some $p \in Z_+$

Consider $\upsilon_{A\cap B}^-(mxy) = \min\left(\upsilon_A^-(mxy), \upsilon_B^-(mxy)\right)$

$\geq \min\left(\upsilon_A^-(x^p), \upsilon_B^-(x^p)\right)$

$= \upsilon_{A\cap B}^-(x^p)$

Therefore $\upsilon_{A\cap B}^-(mxy) \geq \upsilon_{A\cap B}^-(x^p)$, for some $p \in Z_+$

Therefore $A \cap B$ is a primary bipolar intuitionistic anti M-fuzzy group of G.

Theorem:8

Union of any two primary bipolar intuitionistic anti M fuzzy group is also a primary bipolar intuitionistic anti M fuzzy group if either is contained in the other.

Proof:

Let A and B be two primary bipolar intuitionistic anti M fuzzy group of G.

To prove that $A \cup B$ is a primary bipolar intuitionistic anti M fuzzy group of G, if $A \subseteq B$ *or* $B \subseteq A$

If $A \subseteq B \Longrightarrow A \cup B = B$

$B \subseteq A \Longrightarrow A \cup B = A$

Consider $x, y \in A \cup B$ and $m \in$ M

Now $\mu_{A\cup B}^+(mxy) = \max\left(\mu_A^+(mxy), \mu_B^+(mxy)\right)$

$\geq \max\left(\mu_A^+(x^p), \mu_B^+(x^p)\right)$

$= \mu_{A\cup B}^+(x^p)$

Therefore $\mu^+_{A\cup B}(mxy) \geq \mu^+_{A\cup B}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^+_{A\cup B}(mxy) = \min(\upsilon^+_A(mxy), \upsilon^+_B(mxy))$

$\leq \min(\upsilon^+_A(x^p), \upsilon^+_B(x^p))$

$= \upsilon^+_{A\cup B}(x^p)$

Therefore $\upsilon^+_{A\cup B}(mxy) \leq \upsilon^+_{A\cup B}(x^p)$, for some $p \in Z_+$

Consider $\mu^-_{A\cup B}(mxy) = \min(\mu^-_A(mxy), \mu^-_B(mxy))$

$\leq \min(\mu^-_A(x^p), \mu^-_B(x^p))$

$= \mu^-_{A\cup B}(x^p)$

Therefore $\mu^-_{A\cup B}(mxy) \leq \mu^-_{A\cup B}(x^p)$, for some $p \in Z_+$

Consider $\upsilon^-_{A\cup B}(mxy) = \max(\upsilon^-_A(mxy), \upsilon^-_B(mxy))$

$\geq \max(\upsilon^-_A(x^p), \upsilon^-_B(x^p))$

$= \upsilon^-_{A\cup B}(x^p)$

Therefore $\upsilon^-_{A\cup B}(mxy) \geq \upsilon^-_{A\cup B}(x^p)$, for some $p \in Z_+$

Hence Union of any two primary bipolar intuitionistic anti M fuzzy group is also a primary bipolar intuitionistic anti M fuzzy group if either is contained in the other.

Theorem:9

Let $\mu_A$and υ_A be a bipolar intuitionistic fuzzy subset of a M fuzzy subgroup, then $\mu_A = (\mu^+_A, \mu^-_A)$ is a primary bipolar intuitionistic M fuzzy subgroup of G if and only if $\upsilon_A = (\upsilon^+_A, \upsilon^-_A)$ is a primary bipolar intuitionistic anti M fuzzy subgroup of G.

Proof:

Given that $\mu_A = (\mu^+_A, \mu^-_A)$ is a primary bipolar intuitionistic M fuzzy subgroup of G.

To prove that $\upsilon_A = (\upsilon^+_A, \upsilon^-_A)$ is a primary bipolar intuitionistic anti M fuzzy subgroup of G.

Let $x, y \in G$ and $m \in M$

(i) $\mu^+_A(mxy) \leq \min(\mu^+_A(x^p), \mu^+_A(y^p))$, for some $p \in Z_+$

$\Leftrightarrow 1 - \upsilon^+_A(mxy) \leq \min(1 - \upsilon^+_A(x^p), 1 - \upsilon^+_A(y^p))$

$\Leftrightarrow -\upsilon^+_A(mxy) \leq -1 + \min(1 - \upsilon^+_A(x^p), 1 - \upsilon^+_A(y^p))$

$\Leftrightarrow \upsilon^+_A(mxy) \geq 1 - \min(1 - \upsilon^+_A(x^p), 1 - \upsilon^+_A(y^p))$

$\Leftrightarrow \upsilon^+_A(mxy) \geq \max(\upsilon^+_A(x^p), \upsilon^+_A(y^p))$, for some $p \in Z_+$

Therefore $\mu^+_A(mxy) \leq \min(\mu^+_A(x^p), \mu^+_A(y^p) \Leftrightarrow \upsilon^+_A(mxy) \geq \max(\upsilon^+_A(x^p), \upsilon^+_A(y^p))$, for some $p \in Z_+$

(ii) $\mu^-_A(mxy) \geq \max\left(\mu^-_A(x^p), \mu^-_A(y^p)\right)$, for some $p \in Z_+$

$\Leftrightarrow -1 - \upsilon^-_A(mxy) \geq \max(-1 - \upsilon^-_A(x^p), -1 - \upsilon^-_A(y^p))$

$\Leftrightarrow -\upsilon^-_A(mxy) \geq 1 + \max(-1 - \upsilon^-_A(x^p), -1 - \upsilon^-_A(y^p))$

$\Leftrightarrow \upsilon^-_A(mxy) \leq -1 - \max(-1 - \upsilon^-_A(x^p), -1 - \upsilon^-_A(y^p))$

$\Leftrightarrow \upsilon^-_A(mxy) \leq \min(\upsilon^-_A(x^p), \upsilon^-_A(y^p))$, for some $p \in Z_+$

Therefore $\mu^-_A(mxy) \geq \max\left(\mu^-_A(x^p), \mu^-_A(y^p)\right) \Leftrightarrow \upsilon^-_A(mxy) \leq \min\left(\upsilon^-_A(x^p), \upsilon^-_A(y^p)\right)$, for some $p \in Z_+$

(iii) $\mu^+_A(mx^{-1}) = \mu^+_A(x^p)$, for some $p \in Z_+$

$\Leftrightarrow 1 - \upsilon^+_A(mx^{-1}) = 1 - \upsilon^+_A(x^p)$

$\Leftrightarrow \upsilon^+_A(mx^{-1}) = \upsilon^+_A(x^p)$, for some $p \in Z_+$

Therefore $\mu_A^+(mx^{-1}) = \mu_A^+(x^p) \Leftrightarrow \upsilon_A^+(mx^{-1}) = \upsilon_A^+(x^p)$, for some $p \in Z_+$

(iv) $\mu_A^-(mx^{-1}) = \mu_A^-(x^p)$, for some $p \in Z_+$

$\Leftrightarrow \quad -1 - \upsilon_A^-(mx^{-1}) = -1 - \upsilon_A^-(x^p)$

$\Leftrightarrow \quad \upsilon_A^-(mx^{-1}) = \upsilon_A^-(x^p)$, for some $p \in Z_+$

Therefore $\mu_A^-(mx^{-1}) = \mu_A^-(x^p) \Leftrightarrow \upsilon_A^-(mx^{-1}) = \upsilon_A^-(x^p)$, for some $p \in Z_+$

(v) $\mu_A^+(mx) \leq \mu_A^+(x^p)$, for some $p \in Z_+$

$\Leftrightarrow \quad -\mu_A^+(mx) \geq -\mu_A^+(x^p)$

$\Leftrightarrow \quad 1 - \mu_A^+(mx) \geq 1 - \mu_A^+(x^p)$

$\Leftrightarrow \quad \upsilon_A^+(mx) \geq \upsilon_A^+(x^p)$, for some $p \in Z_+$

Therefore $\mu_A^+(mx) \leq \mu_A^+(x^p) \Leftrightarrow \upsilon_A^+(mx) \geq \upsilon_A^+(x^p)$, for some $p \in Z_+$

(vi) $\mu_A^-(mx) \geq \mu_A^-(x^p)$, for some $p \in Z_+$

$\Leftrightarrow \quad -\mu_A^-(mx) \leq -\mu_A^-(x^p)$

$\Leftrightarrow \quad -1 - \mu_A^-(mx) \leq -1 - \mu_A^-(x^p)$

$\Leftrightarrow \quad \upsilon_A^-(mx) \leq \upsilon_A^-(x^p)$, for some $p \in Z_+$

Therefore $\mu_A^-(mx) \geq \mu_A^-(x^p) \Leftrightarrow \upsilon_A^-(mx) \leq \upsilon_A^-(x^p)$, for some $p \in Z_+$

Therefore $\mu_A = (\mu_A^+, \mu_A^-)$ is a primary bipolar intuitionistic M fuzzy subgroup of G if and only if $\upsilon_A = (\upsilon_A^+, \upsilon_A^-)$ is a primary bipolar intuitionistic anti M fuzzy subgroup of G.

4. Conclusion

The concept of a primary bipolar intuitionistic M fuzzy group and primary bipolar intuitionistic anti M fuzzy group are a new algebraic structure of a bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group are defined and some related properties are investigated. The purpose of the study is to implement the fuzzy set theory and the group theory of bipolar intuitionistic M fuzzy subgroup of a M fuzzy group and anti M fuzzy group.The relation between a primary bipolar intuitionistic M fuzzy subgroup and primary bipolar intuitionistic anti M fuzzy subgroup are established. We hope that our result can also be extended to other algebraic systems.

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