On Some Fixed Point Theorems in b-complete b-Metric Spaces

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Abstract:

Some new Results on fixed point and common fixed point in b - complete b - metric spaces using Geraghty [12] type Contractive mapping and two examples are represented to show suitable of our results.

Keyword: b - complete, b - metric spaces, Geraghty type, Contractive mapping.

1. Introduction:

The field of extension is a growing the field of the b-complete b-metric space in this articles in 1973, Geraghty [11] introduce a class of function to generalize the Banach contraction principle and Bakhtin [3] introduce b-metric spaces as generalize of metric spaces. Since then, several papers have been published on the fixed point theory in such spaces. For further works and results in b-metric spaces, we refer readers to references [1, 2, 4, 5, 6, 8, 9, 10, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30] and we introduce some type of Geraghty contractive mapping and comparison contractive mapping in extended b-metric space and we establish some fixed point result is b-complete b-metric space ours result generalize several comparable result in the literature.

2. Preliminaries :

Theorem : 2.1 Let (X, d) be a complete metric space. Let $f:X \to X$ be given mapping satisfying:

$$d(fx, fy) \leq \alpha(d(x, y))\, d(x,y),\ x, y \in X.$$

where $\alpha \in S$. Then f has a unique fixed point.

In 2011, Dukic et al. [7] reconsidered Theorem 2.1 in the framework of b-metric spaces (see also Reference [26]).

Let (X, d) be a b - metric space with parameter $s \geq 1$ and denote A the set of all functions $\alpha. : [0, \infty) \rightarrow [0, 1)$, satisfying the following condition:

$$\lim_{n \to \infty} \alpha(t_n) = 1 \Rightarrow \lim_{n \to \infty} t_n = 0.$$

Theorem 2.2: Let (X, d) be a complete b-metric space with parameter $s \geq 1$ and let $f : X \rightarrow X$ be a self map. Suppose that there exists $\beta \in S$ such that:

$$d(fx, fy) \leq \beta(d(x, y))\, d(x,y),$$

hold for all $x, y \in X$. Then f has a unique fixed point $x^* \in X$.

In recent years, many researchers have extended the result of Geraghty in the context of various metric spaces (e.g., see References [13-14]. In the present paper, we extended some fixed point theorems for Geraghty contractive mappings in b-metric spaces.

Results: [26]

Let B denote the set of all functions $\beta : [0, \infty) \rightarrow [0, 1]$ which satisfies the condition $\limsup_{n \to \infty} \beta(t_n) = 1$ implies that $t_n \rightarrow 0$ as $n \rightarrow \infty$.

Definition 1.

Lat X be a (nonempty) set and $p \geq 1$ be a given real number. A function $d : X \times X \rightarrow [0, \infty)$ is called a b-metric on X if the following conditions hold for all $x, y, z \in X$:

(i) $d(x, y) = 0$ if and only if $x = y$,

(ii) $d(x, y) = d(y, x)$,

(iii) $d(x,y) \leq p[d(x, z) + d(z, y)]$ (b-triangular inequality).

Then, the pair (X,d) is called a b-metric space with parameter p.

Definition 2. Let (X, d) be a b - metric space.

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(a) A sequence $\{x_n\}$in X is called b - convergent if and only if there exists $x \in X$ such that $d(x_n, x) \to 0$ as $n \to \infty$.

(b) $\{x_n\}$ in X is said to be b-Cauchy if and only if $d(x_n, x_m) \to 0$, as $n, m \to \infty$.

(c) The b - metric space (X, d) is called .complete if every b-Cauchy sequence in X is b-convergent.

3. Main Results:

Theorem 3.1 : Let (X, d) be a b - complete b - metric space with parameter $p \geq 1$. Let $f : X \to X$ be a self - mapping satisfying :

$$d(fx, fy) \leq \beta(M(x,y))\, M(x, y), \quad x, y \in X \tag{1}$$

where

$$M(x,y) = \max\{d(x,y), d(x, fx), d(y, fy), \tfrac{1}{3}(d(fx, y) + d(fy, x))\}.$$

and $\beta \in A$ then f has a unique fixed point.

Proof: Let $x_0 \in X$ be any arbitrary point consider the sequence $\{x_n\}$ where

$$x_n = fx_{n-1} = f^n x_0, n \in N.$$

If there exists $n \in N$ s.t. $x_{n+1} = x_n$ then x_n is a fixed point of f and proof is completed otherwise, we have $d(x_{n+1}, x_n) > 0$ i.e. for all $n \in N$ by (i) for all $n \in N$ we have:

$$d(x_n,x_{n+1}) = d(fx_{n-1}, fx_n) \leq \beta\, M(x_{n-1}, x_n)\, M(x_{n-1}, x_n), \tag{2}$$

where:

$$M(x_{n-1}, x_n) = \max\{d(x_{n-1},x_n), d(x_{n-1}, fx_{n-1}), d(x_n, fx_n), \frac{d(fx_{n-1},x_n)+d(fx_n, x_{n-1})}{3p}\}$$

$$= \max\{d(x_{n-1},x_n), d(x_{n-1}, x_n), d(x_n, x_{n+1}), \frac{d(x_n, x_n)+d(x_{n+1}, x_{n-1})}{3p}\}$$

$$= \max\{d(x_{n-1},x_n), d(x_n, x_{n+1}), \frac{p(d(x_{n+1},x_n)+d(x_n, x_{n-1}))}{3p}\}$$

$$= \max\{d(x_{n-1},x_n), d(x_n, x_{n+1})\}$$

if $d(x_{n-1},x_n) \leq d(x_n, x_{n+1})$; then $M(x_{n-1}, x_n) = d(x_n,x_{n+1})$ from (2) we have:

$$d(x_n,x_{n+1}) \leq \beta\, (M(x_{n-1},x_n)\, M(x_{n-1},x_n)$$

$$\leq \tfrac{1}{-} d(x_n,x_{n+1})\ n \in N.$$

This is a contradiction Thus, we have:

$$M(x_{n-1}, x_n) = d(x_n, x_{n-1})$$

Then, from (2), we get:

$$d(x_n,x_{n+1}) \leq \beta\ (M(x_{n-1},x_n)\ d(x_{n-1},x_n) \qquad (3)$$

$$< d(x_{n-1},x_n) \quad n \in N$$

So $\{d\ (x_{n-1}, x_n)$ is non increasing sequence of non-negative real. Hence, there exists $r \geq 0$ such that $d(x_{n-1}, x_n) \to r$ as $n \to \infty$ we claimed that $r = 0$. Let on the contrary that $r > 0$, then from (3), we have :

$$r \leq \lim_{n\to\infty} \sup \beta\ (M(x_{n-1}, x_n))\ r$$

Then $$\frac{1}{} \leq 1 \leq \lim_{n\to\infty} \sup \beta(M\ (x_{n-1}, x_n)) \leq \frac{1}{}$$

Since $\beta \in A$, then $\lim_{n\to\infty} \sup\ (M(x_{n-1}, x_n) = 0$, so $\lim_{n\to\infty} d(x_{n-1}, x_n) = 0$, which is contradiction, that is, $r = 0$. Now we prove that $\{x_n\}$ is a b - Cauchy sequence. Suppose on the contrary that $\{x_n\}$ is not a b - Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find subsequences $\{\ x_{m_{(k)}}\ \}$ and $\{\ x_{n_{(k)}}\ \}$ of $\{x_n\}$ such that $n(k)$ is the smallest index for which $n(k) > m(k) > (k)$,

$$d(x_{m(k)}, x_{n(k)} \geq \varepsilon \qquad (4)$$

and

$$d(x_{m(k)}, x_{n(k)-1} \geq \varepsilon \qquad (5)$$

From (5) and using the b - triangular inequality, we have :

$$\varepsilon \leq d(x_{m(k)}, x_{n(k)}) \leq p(d(x_{m(k)}, x_{m(k)+1}) + d(x_{m(k)+1}, x_{n(k)})).$$

Then, we get :

$$\frac{\varepsilon}{p} \leq \lim_{k\to\infty} \sup d(x_{m(k)+1}, x_{n(k)}\ ), \qquad (6)$$

Therefore

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$$\lim_{k\to\infty} \sup M(x_{m(k)}, x_{n(k)-1}) = \lim_{k\to\infty} \sup \max \{d(x_{m(k)}, x_{n(k)-1}), d(x_{m(k)}, fx_{m(k)}),$$

$$d(x_{n(k)-1}, fx_{n(k)-1}), \frac{(d(fx_{m(k)}, x_{n(k)-1}) + d(fx_{n(k)-1}, x_{m(k)}))}{3p}\}$$

$$= \lim_{k\to\infty} \sup \max\{d(x_{m(k)}, x_{n(k)-1}), d(x_{m(k)}, x_{m(k)+1},$$

$$d(x_{n(k)-1}, x_{n(k)}), \frac{d(x_{m(k)+1}, x_{n(k)-1}) + d(x_{n(k)}, x_{m(k)})}{3p}\}$$

$$\leq \lim_{k\to\infty} \sup \max\{d(x_{m(k)}, x_{n(k)-1}), d(x_{m(k)}, x_{m(k)+1,})$$

$$d(x_{n(k)-1)}, x_{n(k)}), \frac{pd(x_{m(k)+1}, x_{m(k)}) + pd(x_{m(k)}, x_{n(k)-1})}{3p}$$

$$+ \frac{pd(x_{n(k)}, x_{n(k)-1}) + pd(x_{n(k)-1}, x_{m(k)})}{3p}$$

$$\leq \varepsilon$$

From (6) and (1), we have

$$\frac{\varepsilon}{p} \leq \lim_{k\to\infty} \sup d(x_{m(k)+1}, x_{n(k)})$$

$$\leq \lim_{k\to\infty} \sup \beta(M(x_{m(k)}, x_{n(k)-1}))\, M(x_{m(k)}, x_{n(k)-1})$$

$$\leq \varepsilon \lim_{k\to\infty} \sup \beta(M(x_{m(k)}, x_{n(k)-1}).$$

Then $\frac{1}{p} \leq \lim_{k\to\infty} \sup \beta(M(x_{m(k)}, x_{n(k)-1}) \leq \frac{1}{p}$. Since $\beta \in A$, so $M(x_{m(k)}, x_{n(k)-1}) \to 0$, as a result, $d(x_{m(k)}, x_{n(k)-1}) \to 0$. From (4) and using the b - triangular inequality, we have :

$$\varepsilon \leq d(x_{m(k)}, x_{n(k)}) \leq p(d(x_{m(k)}, x_{n(k)-1}) + d(x_{m(k)-1)}, x_{n(k)})).$$

Therefore, $\lim_{k\to\infty} d(x_{m(k)}, x_{n(k)}) = 0$. This is contradicts with (4). Hence, $\{x_n\}$ is b-Cauchy sequence. The completeness of X implies that there exists $u \in X$ such that $x_n \to u$. We showed that u is a fixed point of f. By b - triangular inequality and Condition (1), we have :

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$$d(u, fu) \leq p(d(u, fx_n) + d(fx_n, fu))$$

$$\leq p\, d(u, fx_n) + p\, \beta(M(x_n, u))\, M(x_n, u).$$

Letting $n\to\infty$ in the above inequality, we obtain :

$$d(u, fu) \leq p \lim_{n\to\infty} \sup d(u, x_{n+1}) + p \lim_{n\to\infty} \sup \beta(M(x_n,u)) \lim_{n\to\infty} \sup M(x_n, u), \quad (7)$$

where :

$$\lim_{n\to\infty} \sup M(x_n,u) = \lim_{n\to\infty} \sup \max(d(x_n, u), d(x_n, fx_n), d(u, fu), \tfrac{1}{3p}(d(fx_n,u) + d(fu, x_n))$$

$$= \lim_{n\to\infty} \sup \max\{d(x_n, u), d(x_n, x_{n+1}), d(u, fu), \tfrac{1}{3}(d(x_{n+1},u) + d(fu, x_n))\}$$

$$\leq \lim_{n\to\infty} \sup \max\{d(x_n, u), d(x_n, x_{n+1}), d(u, fu), \tfrac{1}{3}(pd(x_n,u) + pd(u, fu)$$

$$+ d(x_{n+1}, u)\}$$

$$\leq d(u, fu)$$

Hence, from (7), we have :

$$d(u, fu) \leq p \lim_{n\to\infty} \sup \beta(M(x_n, u))\, d(u, fu).$$

Consequently $\frac{1}{\_}$, $\lim_{n\to\infty} \sup \beta(M(x_n, u)) \leq \frac{1}{\_}$ Since $\beta \in A$, we conclude $\lim_{n\to\infty} \sup M(x_n, u) = 0$.

Therefore $fu = u$. To see that the fixed point $u \in X$ is unique, suppose there is $v \neq u$ in X $Tv = v$ from (1) we get:

$$d(u, v) = d(fu, fv) \leq \beta(M(u,v))\, M(u,v),$$

where

$$M(u,v) = \max\{d(u,v), d(u, fu), d(v, fv), \tfrac{(d(fu,v) + d(fv,u))}{3p}\}$$

$$\leq d(u,v)$$

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Therefore we have $d(u,v) < \frac{1}{}d(u,v)$. Then u = v which is a contradiction.

$$\Rightarrow u = v$$

which is contradiction.

Example 1 Let $X = \{2,3,4\}$ and $d : X \times X \to [0,\infty)$ be defined as follows:

(i) $d(2, 3) = d(3,2) = 1$

(ii) $d(2,4) = d(4,2) = \frac{1}{9}$

(iii) $d(3,4) = d(4,3) = \frac{6}{9}$

(iv) $d(2,2) = d(3,3) = d(4,4) = 0$

It is easy to check that (X,d) is a b-metric space with constant $p = \frac{5}{2}$. Set $f_1 = f_3 = 2$ and $f_2 = 4$ and $\beta(t) = (\frac{2}{5})e^{-t}$. $t > 0$ and $\beta(0) \in [0, \frac{2}{5})$. then we have:

$$d(f_1, f_2) = d(2, 4) = \frac{1}{9} \leq \frac{2}{5}e^{-1} = \beta(M(1,2))\, M(1,2).$$

$$d(f_1, f_3) = d(2, 2) = 0 \leq \beta(M(1,3))\, M(1,3).$$

$$d(f_2, f_3) = d(4, 2) = \frac{1}{9} \leq \frac{2}{5} e^{-6/9/(6/9)} = \beta(M(2,3))\, M(2,3).$$

Theorem 3.2 Let (X, d) be a b-complete b-metric space with parameter $p \geq 1$. Let f_1, f_2 be self mapping on X which satisfy:

$$pd(f_1x, f_2y) \leq \beta M(x, y))\, M(x,y),\ x, y \in X. \tag{8}$$

where

$M(x, y) = \max \{d(x, y), d(x, f_1x), d(y, f_2y), d(f_1x, f_2y)\}$and $\beta \in A$. If f_1 or f_2 are continuous, then f_1 and f_2 have a unique common fixed point.

Proof: Let x_0 be arbitrary. Define the sequence (x_n) in X by $x_{2n+1} = f_1x_{2n}$ and $x_{2n+2} = f_2x_{2n+1}$ for all n = 0,1 , ... From (8), for all n = 0,1,2, ..., we have

$$pd(x_{2n+1}, (x_{2n+2}) = pd\,(f_1x_{2n}, f_2x_{2n+1}) \tag{9}$$

$$\leq \beta\,(M(x_{2n}, x_{2n+1}))\, M(x_{2n}, x_{2n+1})$$

where

$$M(x_{2n}, x_{2n+1}) = \max\{d(x_{2n}, x_{2n+1}), d(x_{2n}, f_1x_{2n}), d(x_{2n+1}, f_2x_{2n+1}), d(f_1x_{2n}, f_2x_{2n+1})$$

$$= \max\{d(x_{2n}, x_{2n+1}), d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2}), d(x_{2n+1}, x_{2n+2})$$

$$= \max\{d(x_{2n}, x_{2n+1}), d(x_{2n+1}, x_{2n+2})\}\ n = 0, 1, 2, ...$$

If $M(x_{2n}, x_{2n+1}) = d(x_{2n+1}, x_{2n+2})$ then:

$$pd(x_{2n+1}, (x_{2n+2}) \leq \beta\,(M(x_{2n}, x_{2n+1}))\, d(x_{2n+1}, x_{2n+2}) < \frac{1}{\ }(d(x_{2n+1}, x_{2n+2}))$$

which is contradiction. Hence we have $M(x_{2n}, x_{2n+1}) = d(x_{2n}, x_{2n+1})$

from (9) we have

$$d(x_{2n+1}, x_{2n+2}) \leq \beta\,(M(x_{2n}, x_{2n+1}))\, d(x_{2n}, x_{2n+1})$$

$$< \frac{1}{\ }(d(x_{2n}, x_{2n+2}) \tag{10}$$

Then, we get $d(x_{2n+1}, x_{2n+2}) \leq d(x_{2n}, x_{2n+1})$. Similarly, $d(x_{2n+3}, x_{2n+2}) \leq d(x_{2n+2}, x_{2n+1})$.

So, we have $d(x_n, x_{n+1}) \leq d(x_{n-1}, x_n)$. Thus $\{d(x_n, x_{n+1})\}$ is a decreasing sequence, hence there exist $r \geq 0$ s.t. $d(x_n, x_{n+1}) \to r$ as $n \to \infty$ we showed that $r = 0$. Suppose $r > 0$ Letting $n \to \infty$ in (10) we obtain

$$r \leq \lim_{n\to\infty} \sup \beta\,(M(x, y))\, r.$$

Then, we have:

$$\frac{1}{\ } \leq 1 \leq \lim_{n\to\infty} \sup \beta\,(M(x_{2n}, x_{2n+1}) \leq \frac{1}{\ }.$$

Since $\beta \in A$, we have

$$\lim_{n\to\infty} M(x_{2n}, x_{2n+1}) = 0.$$

Hence, $$r = \lim_{n\to\infty} d(x_{2n}, x_{2n+1}) = 0,$$

which is a contradiction. Now, we show that $\{x_{2n}\}$ is a b-Cauchy sequence. Suppose that $\{x_{2n}\}$ is not a b-Cauchy sequence. Then there exists $\varepsilon > 0$ for which we can find subsequences $\{x_{2m(k)}\}$ and $\{x_{2n(k)}\}$ of $\{\{x_{2n}\}$ s.t. n(k) is the smallest index for which $n(k) > m(k) > k$,

$$d(x_{2n(k)}, x_{2m(k)}) \geq \varepsilon, \quad (11)$$

and

$$d(x_{2n(k)}, x_{2m(k)-2}) \leq \varepsilon \quad (12)$$

From (8) and (11) and using b-triangular inequality, we have

$$\varepsilon \leq d(x_{2n(k)}, x_{2m(k)})$$

$$\leq pd(x_{2n(k)}, x_{2n(k)+1}) + pd(x_{2n(k)+1}, x_{2m(k)})$$

$$= pd(x_{2n(k)}, x_{2n(k)+1}) + pd(f_1x_{2n(k)}, f_2x_{2m(k)-1})$$

$$\leq pd(x_{2n(k)}, x_{2n(k)+1}) + \beta(M(x_{2n(k)}, x_{2m(k)-1}))\ M(x_{2n(k)}, x_{2m(k)-1}), \quad (13)$$

where

$$M(x_{2n(k)}, x_{2m(k)-1}) = \max\{d(x_{2n(k)}, x_{2m(k)-1}), d(x_{2n(k)}, f_1x_{2n(k)}), d(x_{2m(k)-1}, f_2x_{2m(k)-1}), d(f_1x_{2n(k)}, f_2\, x_{2m(k)-1})\}$$

Letting $k\to\infty$ we have:

$$\lim_{k\to\infty} \sup M(x_{2n(k)}, x_{2m(k)-1}) = \lim_{k\to\infty} \sup d(x_{2n(k)}, x_{2m(k)-1}).$$

From b-triangular inequality, we have:

$$d(x_{2n(k)}, x_{2m(k)-1}) \leq p(d(x_{2n(k)}, x_{2m(k)-2}) + d(x_{2m(k)-2}, x_{2m(k)-1})).$$

Letting again $k\to\infty$ in the above inequality, we get:

$$\lim_{k\to\infty} \sup d(x_{2n(k)}, x_{2m(k)-1}) \leq p\varepsilon \quad (14)$$

From (13) and (14)

$$\varepsilon \leq \limsup_{k\to\infty} (\beta(M(x_{2n(k)}, x_{2m(k)-1}))M(x_{2n(k)}, x_{2m(k)-1}))$$

$$\varepsilon \leq \limsup_{k\to\infty} \beta(M(x_{2n(k)}, x_{2m(k)-1})) \limsup_{k\to\infty} d(x_{2n(k)}, x_{2m(k)-1})$$

$$\leq p\varepsilon \limsup_{k\to\infty} \beta(M(x_{2n(k)}, x_{2m(k)-1})).$$

Therefore,

$$\underline{1} \leq \limsup_{k\to\infty} \beta(M(x_{2n(k)}, x_{2m(k)-1}) \leq \underline{1}.$$

Since $\beta \in A$ it follows that

$$\lim_{k\to\infty} M(x_{2n(k)}, x_{2m(k)-1}) = 0.$$

Consequently,

$$\lim_{k\to\infty} d(x_{2n(k)}, x_{2m(k)-1}) = 0. \qquad (15)$$

From (11) and using the b-triangular inequality, we get:

$$\varepsilon \leq d(x_{2n(k)}, x_{2m(k)}) \leq p(d(x_{2n(k)}, x_{2m(k)-1}) + d(x_{2m(k)-1}, x_{2m(k)})).$$

Letting $k\to\infty$ in the above inequality and using form (15), we obtain:

$$\limsup_{k\to\infty} d(x_{2n(k)}, x_{2m(k)}) = 0.$$

This is contradicts (11). This implies that $\{x_{2n}\}$ is a b-Cauchy sequence and so is $\{x_n\}$.

There exist $x^* \in X$ s.t. $= \lim_{n\to\infty} x_n = x^*$ if f_1 is continuous we have :

$$f_1 x^* \lim_{n\to\infty} f_1 x_{2n} = \lim_{n\to\infty} f_1 x_{2n+1} = x^*$$

From (8), we have:

$$pd(x^*, f_2x^*) = pd(f_1x^*, f_2x^*) \leq \beta(M(x^*, x^*))\, M(x^*, x^*),$$

where

$$M(x^*, x^*) = \max\{d(x^*, x^*), d(x^*, f_1x^*), d(x^*, f_2x^*), d(f_1x^*, f_2x^*)\}$$

$$= d(x^*, f_2x^*)$$

Since $\beta \in A$ we have

$$pd\,(x^*, f_2x^*) \leq \beta(M(x^*, x^*))\, d(x^*, f_2x^*) \leq \frac{1}{}d(x^*, f_2x^*)$$

Hence $f_2x^* = x^*$. If f_2 is continuous, then by similar argument to that of above, one can show that f_1, f_2 have common fixed point. Now, we prove the uniqueness of the common fixed point. Let $y = f_1y = f_2y$, is another common fixed point for f_1 and f_2. From (8), we obtain:

$$pd\,(x^*, y) = pd\,(f_1x^*, f_2y) \leq \beta\,(M(x^*,y))\, M(x^*, y)$$

where

$$M(x^*, y) = \max\{d(x^*, y), d(x^*, f_1x^*)\ d(y, f_2y), d(f_1x^*, f_2y)\}$$

$$= d(x^*, y)$$

Therefore $x^* = y$ and the common fixed point f_1 and f_2 is unique

In theorem (2) $f_1 = f_2$, we get the following result.

Corollary 1:

Let (X, d) be a b-complete b-metric space with $p \geq 1$ and f_1 be self mapping on which satisfy:

$$pd\,(f_1x, f_1y) \leq \beta\,(M(x, y))\,(M(x, y), \quad x, y \in X \tag{16}$$

where $M(x, y) = \max\,\{d(x, y), d(x, f_1x), d(y, f_1y), d(f_1x, f_1y)\}$ and f_1 is continuous. Then f_1 has a unique fixed point.

Example : Let $X = [0, 1]$ and $d: X \times X \rightarrow [0, \infty)$ be defined by $d(x, y) = |x - y|^2$, for all $x, y \in [0, 1]$, It is easy to check that (X, d) is a b-metric space with parameter $p = 3$. Set $f_1x = \frac{}{3}$ for all $x \in X$ and $\beta(t) = \frac{1}{3}$ for all $t > 0$. Then,

$$3d(f_1x, f_1y) = 3\,\left|\frac{x}{3} - \frac{y}{3}\right|^2$$

$= \frac{1}{3} |x - y|^2$

$3d(f_1x, f_1y) \leq \beta\ (M(x, y))\ M(x,y)$.

Then, the conditions of corollary 1 are satisfied.

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