Intuitionistic Fuzzy Semi γ* Generalized Closed Sets

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ABSTRACT

The purpose of this paper is to introduce and study the concepts of intuitionistic fuzzy semi γ* generalized closed sets and intuitionistic fuzzy semi γ* generalized open sets in intuitionistic fuzzy topological spaces. We investigate some of their properties.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy semi γ* generalized closed set and intuitionistic fuzzy semi γ* generalized open set.

1 Introduction

The concept of fuzzy sets was introduced by Zadeh [13] in 1965 and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets. On the other hand Coker [2] has introduced intuitionistic fuzzy topological space using the notion of intuitionistic fuzzy sets. Intuitionistic fuzzy generalized γ closed set is introduced by Kanimozhi and Jayanthi [6] in 2016. After, Riya and Jayanthi [7] have introduced intuitionistic fuzzy γ* generalized closed sets in 2017. In this paper we have introduced a new type of intuitionistic fuzzy closed set called intuitionistic fuzzy semi γ* generalized closed sets. We have investigated some of their properties. Additionally we have obtained some interesting theorems.

2. Preliminaries

Definition 2.1: [1] An **intuitionistic fuzzy set** (IFS) A is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$$

where the functions $\mu_A : X \rightarrow [0,1]$ and $\nu_A : X \rightarrow [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$.

Definition 2.2: [1] Let A and B be two IFSs of the form

$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle : x \in X\}$.

Then,

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,

(b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,

(c) $A^c = \{\langle x, \nu_A(x), \mu_A(x)\rangle : x \in X\}$,

(d) $A \cup B = \{\langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x)\rangle : x \in X\}$,

(e) $A \cap B = \{\langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x)\rangle : x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \langle x, 0, 1\rangle$ and $1_\sim = \langle x, 1, 0\rangle$ are respectively the empty set and the whole set of X.

Definition 2.3: [2] An **intuitionistic fuzzy topology** (IFT) on X is a family τ of IFSs in X satisfying the following axioms:

(i) $0_\sim, 1_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(iii) $\cup G_i \in \tau$ for any family $\{G_i : i \in I\} \in \tau$.

In this case the pair (X, τ) is called an **intuitionistic fuzzy topological space** (IFTS) and any IFS in τ is known as an **intuitionistic fuzzy open set** (IFOS) in X. The complement A^c of an IFOS A in an IFTS (X, τ) is called an **intuitionistic fuzzy closed set** (IFCS) in X.

Definition 2.4: [10] Two IFSs A and B are said to be **q-coincident** (A_qB) if and only if there exits an element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.5: [10] Two IFSs A and B are said to be **not q-coincident** ($A_{\bar{q}}B$) if and only if $A \subseteq B^c$.

Definition 2.6: [3] An **intuitionistic fuzzy point** (IFP), written as $p_{(\alpha,\beta)}$ is defined to be an IFS of X given by

$$p_{(\alpha,\beta)}(x) = \begin{cases} (\alpha, \beta) & \text{if } x = p, \\ (0,1) & \text{otherwise.} \end{cases}$$

An intuitionistic fuzzy point $p_{(\alpha,\beta)}$ is said to belong to a set A if $\alpha \leq \mu_A$ and $\beta \geq \nu_A$.

Definition 2.7: [8] An IFS A in (X, τ) is an **intuitionistic fuzzy Q-set** if int(cl(A)) = cl(int(A)).

Definition 2.8: [4] An IFS $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ in an IFTS (X, τ) is said to be an

(i) **intuitionistic fuzzy regular closed set** (IFRCS) if cl(int(A)) = A,

(ii) **intuitionistic fuzzy semi closed set** (IFSCS) if int(cl(A)) ⊆ A,

(iii) **intuitionistic fuzzy pre closed set** (IFPCS) if cl(int(A)) ⊆ A,

(iv) **intuitionistic fuzzy α closed set** (IFαCS) if cl(int(cl(A))) ⊆ A,

(v) **intuitionistic fuzzy β closed set** (IFβCS) if int(cl(int(A))) ⊆ A.

Definition 2.9: [5] An IFS $A = \langle x, \mu_A, \nu_A\rangle$ in an IFTS (X, τ) is said to be an

(i) **intuitionistic fuzzy γ closed set** (IFγCS) if cl(int(A)) ∩ int(cl(A)) ⊆ A,

(ii) **intuitionistic fuzzy γ open set** (IFγOS) if A ⊆ int(cl(A)) ∪ cl(int(A)).

Result 2.10: [6] Let A be an IFS in (X, τ), then

γcl(A) ⊇ A ∪ cl(int(A)) ∩ int(cl(A)),

γint(A) ⊆ A ∩ cl(int(A)) ∩ int(cl(A)).

Definition 2.11: [9] An IFS A in (X, τ) is an **intuitionistic fuzzy nowhere dense** set if there exists no IFOS U such that U ⊆ cl(A). That is int(cl(A)) = $0_\sim$.

Proposition 2.12: [9] Let A be an IFS in X. If A is an **intuitionistic fuzzy nowhere dense** set in X, then int(A) = $0_\sim$.

Definition 2.13: [7] An IFS A of an IFTS (X, τ) is said to be **an intuitionistic fuzzy γ* generalized closed set** (IFγ*GCS) if cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ).

3. Intuitionistic fuzzy semi γ* generalized closed sets

In this section we have introduced intuitionistic fuzzy semi γ* generalized closed set and investigated some of its properties.

Definition 3.1: An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy semi γ* generalized closed set (IF semi γ*GCS) if int(cl(A)) ∩ cl(int(A)) ⊆ U whenever A ⊆ U and U is an IFSOS in (X, τ).

Example 3.2: Let X = {a, b} and τ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨ x , (0.5_a, 0.6_b) , (0.5_a, 0.4_b) ⟩.Then (X, τ) is an IFTS .

Here IFSO(X) = { $0_\sim$,$1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a \geq 0.5$, $\mu_b \geq 0.6$, $\nu_a \leq 0.5$, $\nu_b \leq 0.4$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Here the IFS A= ⟨ x, (0.2_a, 0.3_b), (0.8_a, 0.7_b) ⟩ is an IF semi γ*GCS in X.

Theorem 3.3: Every IFCS is an IF semi γ*GCS in (X, τ) but not conversely in general.

Proof: Let A be an IFCS in (X, τ). Let A ⊆ U where U is an IFSOS in (X, τ). Since A is an IFCS, cl(A) = A . Now int(cl(A)) ∩ cl(int(A)) = int(A) ∩ cl(int(A)) ⊆ A ∩ cl(A) = A ∩ A = A ⊆ U. Hence A is an IF semi γ*GCS in (X, τ).

Example 3.4: Let X={a,b} and τ = {$0_\sim$, G_1, G_2 ,$1_\sim$} be an IFT on X, where G_1= ⟨ x , (0.5_a, 0.4_b) , (0.5_a, 0.6_b) ⟩, G_2= ⟨ x , (0.4_a, 0.3_b) , (0.6_a, 0.7_b) ⟩. Then (X, τ) is an IFTS.

Here IFSO(X) = { $0_\sim$,$1_\sim$,$\mu_a \in [0,1]$,$\mu_b \in [0,1]$,$\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0.4 \leq \mu_a \leq 0.5$, $0.4 \leq \mu_b \leq 0.6$, $0.5 \leq \nu_a \leq 0.6$, $0.4 \leq \nu_b \leq 0.7$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Here the IFS A = ⟨x, (0.3_a, 0.4_b), (0.5_a, 0.6_b)⟩ is an IF semi γ*GCS but not an IFCS in (X, τ), as cl(A) = $G_1^{\,c} \neq$ A .

Theorem 3.5: Every IFSCS is an IF semi γ*GCS in (X, τ) but not conversely in general.

Proof: Let A be an IFSCS in (X, τ). Let A ⊆ U where U is an IFSOS in (X, τ). Since A is an IFSCS, int(cl(A)) ⊆ A . Now int(cl(A)) ∩ cl(int(A)) ⊆ A ∩ cl(int(A)) ⊆ A ∩ cl(A) = A ⊆ U. Hence A is an IF semi γ*GCS in (X, τ).

Example 3.6: In example 3.4, the IFS A= ⟨x, (0.3_a, 0.4_b), (0.5_a, 0.6_b) ⟩ is an IF semi γ*GCS but not an IFSCS in (X, τ), as int(cl(A)) = G_1 ⊈ A.

Theorem 3.7: Every IFPCS is an IF semi γ*GCS in (X, τ) but not conversely in general.

Proof: Let A be an IFPCS in (X, τ). Let A ⊆ U where U is an IFSOS in (X, τ). Now int(cl(A)) ∩ cl(int(A)) ⊆ int(cl(A)) ∩ A ⊆ cl(A) ∩ A = A ⊆ U, by hypothesis. Hence A is an IF semi γ* GCS in (X, τ).

Example 3.8: In example 3.4, the IFS A= ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩ is an IF semi γ*GCS but not an IFPCS in (X, τ), as cl(int(A)) = cl(G_1) = G_1^c ⊈ A.

Theorem 3.9: Every IFRCS is an IF semi γ*GCS in (X, τ) but not conversely in general.

Proof: Let A be an IFRCS in (X, τ). Since every IFRCS is an IFCS, by theorem 3.3, A is an IF semi γ*GCS in (X, τ).

Example 3.10: In example 3.4, the IFS A= ⟨x, (0.3_a, 0.4_b), (0.5_a, 0.6_b) ⟩ is an IF semi γ*GCS but not an IFRCS in (X, τ), as cl(int(A)) = $0_\sim$ ≠ A.

Theorem 3.11: Every IFαCS is an IF semi γ*GCS in (X, τ) but not conversely in general.

Proof: Let A be an IFαCS in (X, τ). Let A ⊆ U where U be an IFSOS in (X, τ). Now int(cl(A)) ∩ cl(int(A)) ⊆ cl(int(cl(A))) ∩ cl(int(cl(A))) ⊆ A ∩ A = A ⊆ U, by hypothesis. Hence A is an IF semi γ*GCS in (X, τ).

Example 3.12: : In example 3.4, the IFS A= ⟨x , (0.3_a, 0.4_b) , (0.5_a, 0.6_b)⟩ is an IF semi γ*GCS but not an IFαCS in (X, τ), as cl(int(cl(A))) = G_1^c ⊈ A .

Theorem 3.13: Every IFROS is an IF semi γ*GCS in (X, τ) but not conversely in general.

Proof: Let A be an IFROS in (X, τ). Then int(cl(A)) = A and int(A) = A. Let A ⊆ U where U is an IFSOS in (X, τ). Now int(cl(A)) ∩ cl(int(A)) = A ∩ cl(A) = A ⊆ U. Hence A is an IF semi γ*GCS in (X, τ).

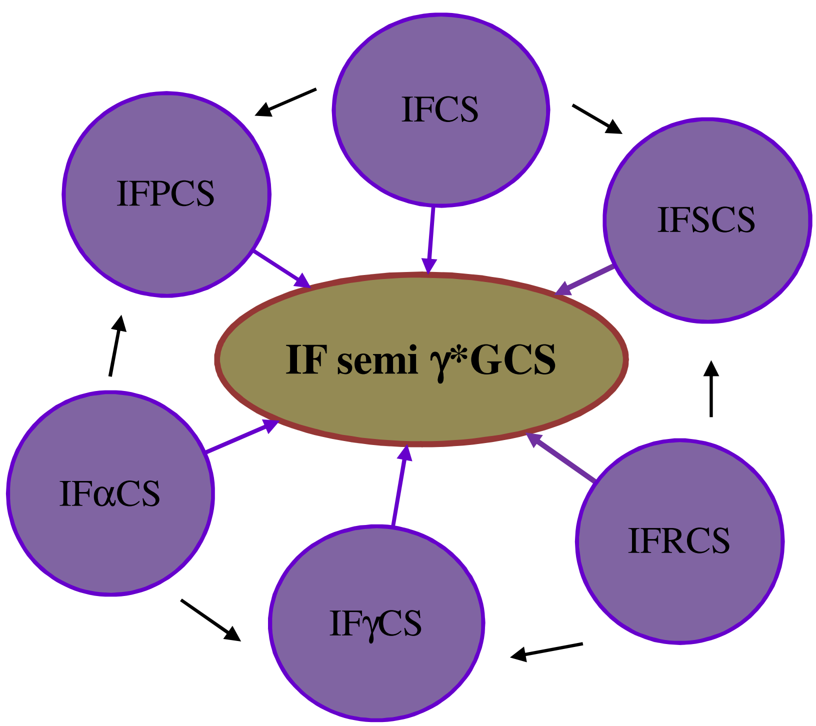
Example 3.14: In example 3.4, the IFS A = ⟨x, (0.3_a, 0.4_b) , (0.5_a, 0.6_b) ⟩ is an IF semi γ*GCS but not an IFROS in (X, τ), as int(cl(A)) = G_1 ≠A.

Theorem 3.15: Every IFγCS is an IF semi γ*GCS in (X, τ) but not conversely in general.

Proof: Let A be an IFγCS in (X, τ). Let A ⊆ U and U be an IFSOS in (X, τ). Now int(cl(A)) ∩ cl(int(A)) ⊆ A ⊆ U by hypothesis. Hence A is an IF semi γ*GCS in (X, τ).

Example 3.16 : In example 3.4, the IFS A = ⟨x, (0.5_a, 0.3_b) , (0.5_a, 0.6_b) ⟩ is an IF semi γ*GCS but not an IFγCS in (X, τ), as cl(int(A)) ∩ int(cl(A)) = $G_1 \cap G_1^c = G_1 \nsubseteq$ A.

In the following diagram we have provided the relation between various types of intuitionistic fuzzy closedness. The reverse implications are not true in this diagram in general.



Remark 3.17: The union of any two IF semi γ*GCSs need not be an IF semi γ*GCS in (X, τ) in general.

Example 3.18: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.5, 0.6_b), (0.5_a, 0.4_b)\rangle$. Then (X, τ) is an IFTS. Here

Here $IFSO(X) = \{ 0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \mu_a \geq 0.5, \mu_b \geq 0.6, \nu_a \leq 0.5, \nu_b \leq 0.4$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

Let $A = \langle x, (0.5_a, 0.4_b), (0.5_a, 0.5_b)\rangle$ and $B = \langle x, (0.3_a, 0.6_b), (0.7_a, 0.4_b)\rangle$ be two IFSs in X. Then, A and B are IF semi γ*GCSs. But their union $A \cup B = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$ is not an IF semi γ*GCS in X. Since, $A \cup B \subseteq U = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)\rangle$, but $int(cl(A \cup B)) \cap cl(int(A \cup B)) = int(1_\sim) \cap cl(G) = 1_\sim \cap 1_\sim = 1_\sim \nsubseteq U$.

Remark 3.19: The intersection of any two IF semi γ*GCSs need not be an IF semi γ*GCS in (X, τ) in general.

Example 3.20: Let $X = \{a, b\}$ and $\tau = \{0_\sim, G, 1_\sim\}$ be an IFT on X, where $G = \langle x, (0.5, 0.4_b), (0.5_a, 0.6_b)\rangle$. Then (X, τ) is an IFTS. Here

$IFSO(X) = \{ 0_\sim, 1_\sim, \mu_a \in [0,1], \mu_b \in [0,1], \nu_a \in [0,1], \nu_b \in [0,1] \,/\, \mu_a = 0.5, 0.4 \leq \mu_b \leq 0.6, \nu_a = 0.5, 0.4 \leq \nu_b \leq 0.6$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1\}$.

Let $A = \langle x, (0.5_a, 0.7_b), (0.4_a, 0.3_b)\rangle$ and $B = \langle x, (0.6_a, 0.5_b), (0.3_a, 0.4_b)\rangle$ be two IFSs in X. Then A and B are IF semi γ*GCSs but their intersection $A \cap B = \langle x, (0.5_a, 0.5_b), (0.4_a, 0.4_b)\rangle$ is not an IF semi γ*GCS in X, as $A \cap B \subseteq U = \langle x, (0.5_a, 0.5_b), (0.4_a, 0.4_b)\rangle$ where as $int(cl(A \cap B)) \cap cl(int(A \cap B)) = int(1_\sim) \cap cl(G) = 1_\sim \cap G^c = G^c \nsubseteq U$.

Theorem 3.21: Let (X, τ) be an IFTS. Then for every $A \in$ IF semi γ*GC(X) and for every $B \in$ IFS(X), $A \subseteq B \subseteq cl(int(A)) \Rightarrow B \in$ IF semi γ*GC(X).

Proof: Let $B \subseteq U$ and U be an IFSOS in X. Since $A \subseteq B$, $A \subseteq U$, by hypothesis. Since $B \subseteq cl(int(A))$, $clint(B) \subseteq cl(int(A))$. Also $int(cl(B)) \subseteq int(cl(cl(int(A)))) \subseteq int(cl(int(A))) \subseteq int(cl(A))$. Therefore $cl(int(B)) \cap int(cl(B)) \subseteq cl(int(A)) \cap int(cl(A)) \subseteq U$, by hypothesis. Hence $B \in$ IF semi γ*GC(X).

Theorem 3.22: If A is both an IFSOS and an IF semi γ*GCS in (X, τ), then A is an IFγCS in (X, τ).

Proof: Since A is an IFSOS and A ⊆ A, we have, by hypothesis int(cl(A)) ∩ cl(int(A)) ⊆ A. Hence A is an IFγCS in (X, τ).

Theorem 3.23: If A is both an IFSOS and an IF semi γ*GCS in (X, τ), then A is an IFβCS in (X, τ).

Proof: Let A be an IFSOS and an IF semi γ*GCS in X. Then int(cl(A)) ∩ cl(int(A)) ⊆ A, as A ⊆ A. Now int(cl(int(A))) ⊆ int(cl(int(A))) ∩ cl(int(A)) ⊆ int(cl(A) ∩ cl(int(A)) ⊆ A, by hypothesis. Therefore int(cl(int(A))) ⊆ A and hence A is an IFβCS in (X, τ).

Theorem 3.24: If A is both an IFOS and an IF semi γ*GCS in (X, τ), then A is an IFSCS in (X, τ).

Proof: Let A be an IFOS and an IF semi γ*GCS in X. Then int(cl(A)) ∩ cl(int(A)) ⊆ A, as A ⊆ A and every IFOS is an IFSOS in X. Now int(cl(A)) = int(cl(A)) ∩ cl(A) = int(cl(A)) ∩ cl(int(A)) ⊆ A. Therefore int(cl(A)) ⊆ A and A is an IFSCS in (X, τ).

Theorem 3.25: If A is both an IFOS and an IF semi γ*GCS in (X, τ), then A is an IFROS in (X, τ).

Proof: Let A be an IFOS and an IF semi γ*GCS in X. Then int(cl(A)) ∩ cl(int(A)) ⊆ A, as A ⊆ A, and every IFOS is an IFSOS in Y.

Now int(cl(A)) = int(cl(A)) ∩ cl(A) = int(cl(A)) ∩ cl(int(A)) ⊆ A, by hypothesis. Hence int(cl(A)) ⊆ A. Since A is an IFOS, it is IFPOS. Hence A ⊆ int(cl(A)). Therefore A = int(cl(A)) and hence A is an IFROS in (X, τ).

Theorem 3.26: An IFS A of an IFTS (X, τ) is an IF semi γ*GCS if and only if $A_{\bar{q}}$ F ⇒ (int(cl(A)) ∩ cl(int(A)) $)_{\bar{q}}$F for every IFSCS F of X.

Proof: Necessity: Let F be an IFSCS in X and $A_{\bar{q}}$ F, then A $\subseteq F^c$, by Definition 2.5, where F^c is an IFSOS. Then int(cl(A)) ∩ cl(int(A)) $\subseteq F^c$, by hypothesis. Therefore again by Definition 2.5, (int(cl(A)) ∩ cl(int(A)))$_{\bar{q}}$ F.

Sufficiency: Let U be an IFSOS such that A $\subseteq$ U. Then U^c is an IFSCS and A $\subseteq (U^c)^c$. By hypothesis, $A_{\bar{q}}$ $U^c \Rightarrow$ (int(cl(A)) ∩ cl(int(A)))$_{\bar{q}}$ U^c. Hence int(cl(A)) ∩ cl(int(A)) $\subseteq (U^c)^c$ = U. Therefore int(cl(A)) ∩ cl(int(A)) $\subseteq$ U and A is an IF semi γ*GCS in X.

Theorem 3.27: For an IFOS A in (X, τ), the following conditions are equivalent:

(i) A is an IFCS,

(ii) A is an IF semi γ*GCS and an IFQ-set.

Proof: (i) ⇒ (ii) Since A is an IFCS, it is an IF semi γ*GCS, by Theorem 3.3, Now int(cl(A)) = int(A) = A = cl(A) = cl(int(A)), by hypothesis. Hence A is an IFQ-set.

(ii) ⇒ (i) Since A is an IFOS and an IF semi γ*GCS, by Theorem 3.25, A is an IFROS. Therefore A = int(cl(A)) = cl(int(A)) = cl(A), by hypothesis. Hence A is an IFCS in X.

Theorem 3.28: For an IF semi γ*GCS A in an IFTS (X,τ), the following conditions hold:

(i) If A is an IFROS, then scl(A) is an IF semi γ*GCS,

(ii) If A is an IFRCS, then sint(A) is an IF semi γ*GCS.

Proof: (i) Let A be an IFROS in (X, τ). Then int(cl(A)) = A. By definition, we have scl(A) = A ∪ int(cl(A)) = A. Since A is an IF semi γ*GCS in X, scl(A) is an IF semi γ*GCS in X.

(ii) Let A be an IFRCS in (X, τ). Then cl(int(A)) = A. By definition, we have sint(A) = A ∩ cl(int(A)) = A. Since A is an IF semi γ*GCS in X, sint(A) is an IF semi γ*GCS in X.

Theorem 3.29: If an IFS A of an IFTS (X, τ) is an intuitionistic fuzzy nowhere dense, then A is an IF semi γ*GCS in X.

Proof: If A is an intuitionistic fuzzy nowhere dense subset, then int(cl(A)) = $0_\sim$. Let A $\subseteq$ U where U is an IFSOS in X. Then int(cl(A)) ∩ cl(int(A)) = $0_\sim$ ∩ cl(int(A)) = $0_\sim \subseteq$ U and hence A is an IF semi γ*GCS in X.

4. Intuitionistic fuzzy semi γ* generalized open sets

In this section we have discussed and analyzed some of the properties of intuitionistic fuzzy semi γ* generalized open set and produced many interesting characterization theorems.

Definition 4.1: The complement A^c of an IF semi γ*GCS A in an IFTS (X, τ) is called an **intuitionistic fuzzy semi γ* generalized open set** (IF semi γ*GOS) in X.

Example 4.2: Let X = {a, b} and τ= {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨ x , (0.5_a, 0.6_b) , (0.5_a, 0.4_b) ⟩.Then (X, τ) is an IFTS .

Here IFSC(X) = { $0_\sim$,$1_\sim$,$\mu_a \in [0,1]$,$\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\nu_a \geq 0.5$, $\nu_b \geq 0.6$, $\mu_a \leq 0.5$, $\mu_b \leq 0.4$ and $0 \leq \mu_a + \nu_a \leq 1$ and $0 \leq \mu_b + \nu_b \leq 1$}.

Here the IFS A= ⟨ x, (0.8_a, 0.7_b), (0.2_a, 0.3_b) ⟩ is an IF semi γ*GOS in X.

Theorem 4.3: Every IFOS, IFSOS, IFPOS, IFαOS and IFγOS is an IF semi γ*GOS in (X, τ) but not conversely in general.

Proof: Straightforward.

Example 4.4: In example 3.4, the IFS A = ⟨x, (0.5_a, 0.6_b), (0.3_a, 0.4_b)⟩ is an IF semi γ*GOS but not an IFOS, IFSOS and IFαOS in (X, τ).

Example 4.5: In example 3.4, the IFS A = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.4_b)⟩ is an IF semi γ*GOS but not an IFPOS in (X, τ).

Example 4.6: In example 3.4, the IFS A = ⟨x, (0.5_a, 0.6_b), (0.5_a, 0.3_b)⟩ is an IF semi γ*GOS but not an IFγOS in (X, τ).

Remark 4.7: The union of two IF semi γ*GOSs need not be an IF semi γ*GOS in (X, τ) in general.

Example 4.8: Obvious from Example 3.20, by taking complement of A and B in the respective example.

Remark 4.9: The intersection of two IF semi γ*GOSs need not be an IF semi γ*GOS in (X, τ) in general.

Example 4.10: Obvious from Example 3.18, by taking complement of A and B in the respective example.

Theorem 4.11: Let (X, τ) be an IFTS. Then for every A ∈ IF semi γ*GO(X) and for every B ∈ IFS(X), int(cl(A)) ⊆ B ⊆ A ⇒ B ∈ IF semi γ*GO(X).

Proof: Let A be an IF semi γ*GOS of X and let B ⊆ U where U is an IFSOS. Since A^c is an IF semi γ*GCS and $A^c \subseteq B^c \subseteq cl(int(A^c))$, B^c is an IF semi γ*GCS, by Theorem 3.21. This implies B is an IF semi γ*GOS in X and B ∈ IF semi γ*GO(X).

Theorem 4.12: An IFS A of an IFTS (X, τ) is an IF semi γ*GOS if and only if F ⊆ cl(int(A)) ∪ int(cl(A)) whenever F is an IFSCS and F ⊆ A.

Proof: Necessity: Suppose A is an IF semi γ*GOS in X. Let F be an IFSCS, such that F ⊆ A. Then F^c is an IFSOS and $A^c \subseteq F^c$, by hypothesis. Since A^c is an IF semi γ*GCS, we have, $int(cl(A^c)) \cap cl(int(A^c)) \subseteq F^c$. Therefore F ⊆ cl(int(A)) ∪ int(cl(A)).

Sufficiency: Let U be an IFSOS, such that $A^c \subseteq U$ and $U^c \subseteq A$, then $U^c \subseteq cl(int(A)) \cup int(cl(A))$, by hypothesis. Therefore $int(cl(A^c)) \cap cl(int(A^c)) \subseteq U$ and A^c is an IF semi γ*GCS. Hence A is an IF semi γ*GOS in X.

Theorem 4.13: Let (X, τ) be an IFTS. Then for every A∈ IFS(X) and for every B ∈ IFRC(X), B ⊆ A ⊆ cl(int(B)) ∩ int(cl(B)) implies A ∈ IF semi γ*GO(X).

Proof: Let B be an IFRCS in X. Then B = cl(int(B)). By hypothesis, A ⊆ int(cl(B)) ∩ cl(int(B)) = int(cl(B)) ∩ B ⊆ cl(B) ∩ B ⊆ B = cl(int(B)) ⊆ cl(int(A)) as B ⊆ A. Therefore A is an IFSOS and by Theorem 4.3, A is an IF semi γ*GOS. Hence A ∈ IF semi γ*GO(X).

Theorem 4.14: An IFS A of an IFTS (X, τ) is an IF semi γ*GOS then F ⊆ cl(int(cl(A))) whenever F is an IFSCS and F ⊆ A.

Proof: Suppose A is an IF semi γ*GOS in X. Let F be an IFSCS such that F ⊆ A. Then F^c is an IFSOS and $A^c \subseteq F^c$. By hypothesis, A^c is an IF semi γ*GCS, we have $int(cl(A^c)) \cap cl(int(A^c)) \subseteq F^c$. Now $int(cl(int(A^c))) = int(cl(int(A^c))) \cap cl(int(A^c)) \subseteq int(cl(A^c)) \cap cl(int(A^c)) \subseteq F^c$. Therefore F ⊆ cl(int(cl(A))) .

5. Theoretical applications of intuitionistic fuzzy semi γ* generalized closed sets

In this section we have investigated many theoretical applications of intuitionistic fuzzy semi γ* generalized closed set by defining new spaces and obtained many interesting propositions.

Definition 5.1: An IFTS (X, τ) is an **intuitionistic fuzzy semi γ_c^* $T_{1/2}$** (IF semi γ_c^* $T_{1/2}$) **space** if every IF semi γ*GCS is an IFCS in X.

Definition 5.2: An IFTS (X, τ) is an **intuitionistic fuzzy semi γ_γ^* $T_{1/2}$** (IF semi γ_γ^* $T_{/1/2}$) **space** if every IF semi γ*GCS is an IFγCS in X.

Example 5.3: Let X = {a, b} and τ = {$0_\sim$, G, $1_\sim$} be an IFT on X, where G = ⟨x, (0.5_a, 0.4_b), (0.5_a, 0.6_b)⟩. Then (X, τ) is an IFTS. Here,

IFSO(X) = { $0_\sim$, $1_\sim$, $\mu_a \in [0, 1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $\mu_a = 0.5$, $0.4 \le \mu_b \le 0.6$, $\nu_a = 0.5$, $0.4 \le \nu_b \le 0.6$ and $0 \le \mu_a + \nu_a \le 1$ and $0 \le \mu_b + \nu_b \le 1$}.Then,

IF semi γ*GC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \le \mu_a + \mu_b \le 1$, $0 \le \mu_a + \mu_b \le 1$} and

IFγC(X) = {$0_\sim$, $1_\sim$, $\mu_a \in [0,1]$, $\mu_b \in [0,1]$, $\nu_a \in [0,1]$, $\nu_b \in [0,1]$ / $0 \le \mu_a + \mu_b \le 1$, $0 \le \mu_a + \mu_b \le 1$}

Hence (X, τ) is an IF semi γ_γ^* $T_{/1/2}$ space.

Theorem 5.4: Let (X, τ) be an IF semi γ_c^* $T_{1/2}$ space. Then,

(i) any union of IF semi γ*GCS is an IF semi γ*GCS,

(ii) any intersection of IF semi γ*GOS is an IF semi γ*GOS.

Proof: (i) Let $\{A_i\}_{i \in I}$ be a collection of IF semi γ*GCSs. Since (X, τ) is an IF semi γ_c^* $T_{1/2}$ space (X, τ), every IF semi γ*GCS is an IFCS. We know that any union of IFCS is an IFCS. Since every IFCS is an IF semi γ*GCS, the union of any number IF semi γ*GCS is an IF semi γ*GCS.

(ii) can be proved by taking complement in (i).

Theorem 5.5: An IFTS (X, τ) is an IF semi γ_c^* $T_{1/2}$ space if and only if IF semi γ*GO(X) = IFO(X).

Proof: Necessity: Let A be an IF semi γ*GOS in (X, τ), then A^c is an IF semi γ*GCS (X, τ). By hypothesis, A^c is an IFCS in (X, τ). Hence A is an IFOS in X. Thus IF semi γ*GO(X) = IFO(X).

Sufficiency: Let A be an IF semi γ*GCS in (X, τ). Then A^c is an IF semi γ*GOS in (X, τ). By hypothesis, A^c is an IFOS in (X, τ). Therefore A is an IFCS in (X, τ). Hence (X, τ) is an IF semi γ_c^* $T_{1/2}$ space.

Theorem 5.6: If A is an IFPCS and if (X, τ) is an IF semi γ_c^* $T_{1/2}$ space, then A is an IFCS in X.

Proof: Let A be an IFPCS in X. Since every IFPCS is an IF semi γ*GCS, A is an IF semi γ*GCS in X. Since (X, τ) is an IF semi γ_c^* $T_{1/2}$ space, A is an IFCS in X.

Theorem 5.7: If A is an IFαCS in X where X is an IF semi γ_c^* $T_{1/2}$ space, then A is an IFCS in X.

Proof: Let A be an IFαCS in X. Since every IFαCS is an IF semi γ*GCS, A is an IF semi γ*GCS in X and A is an IFCS in X, by hypothesis.

Theorem 5.8: If A is both an IFOS and an IF semi γ*GCS in X and if X is an IF semi γ_c^* $T_{1/2}$ space, then

(i) A is an IFROS in X

(ii) A is an IFRCS in X

(iii) A is an IFQ - set in X.

Proof: Let A be an IF semi γ*GCS in X, then A is an IFCS in X, by hypothesis. Now,

(i) int (cl(A)) = int(A) = A and therefore A is an IFROS in X .

(ii) cl(int(A)) = cl(A) = A and therefore A is an IFRCS in X and

(iii) from (i) and (ii) int (cl(A)) = cl(int(A)).

Hence A is an IFQ - set in X.

Theorem 5.9: Let (X, τ) be an IF semi γ_c^* $T_{1/2}$ space and the IFS A ∈ IF semi γ*GO(X), then cl(A) ∈ IFRC(X).

Proof: Let A be an IF semi γ*GOS in X. Then, since X is an IF semi γ_c^* $T_{1/2}$ space, A is an IFOS. Since every IFOS is an IFSOS, we have A ⊆ cl(int(A)).

This implies cl(A) ⊆ cl(cl(int(A))) ⊆ cl(int(cl(A))) ⊆ cl(cl(A)) ⊆ cl(A).Therefore cl(A) = cl(int(cl(A))). Hence cl(A) ∈ IFRC(X).

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