Radio Harmonic Mean Number of Complete Graphs

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Abstract: *A radio harmonic mean labeling of a connected graph G is a one to one map f from the vertex set V(G) to the set of natural numbers N such that for two distinct vertices u and v of G,* $d(u,v)+\left\lceil \frac{2\, f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G)$ *.The radio harmonic mean number of f, rhmn(f) is the maximum number assigned to any vertex of G. The radio harmonic mean number of G,* $rhmn(G)$ *is the minimum value of* $rhmn(f)$ *taken over all radio harmonic mean labeling f of G. In this paper we have determined the radio harmonic mean number of complete graph , complete bipartite graph and complete tripartite graph.*

AMS Subject classification : 05C78

Keywords: *Radio harmonic mean labeling, Radio harmonic mean graph,Complete graph,Complete Bipartite graph ,Complete Tripartite graph.*

1. Introduction

The channel assignment problems were introduced in 1980 by Hale[7]. The goal is to assign radio channels in a way so as to avoid interference between radio transmitters. Motivated by this Chartrand defined the concept of radio labeling of graphs in 2001[4]. Radio labeling, labels the vertices of a graph with non negative integers such that for any two vertices, the smaller the distance between the vertices, the greater the required difference in label. Radio labeling of graphs is applied in channel assignment problem, Sensor networks, TV and wireless networks etc..S.Ponraj et al. [9,10] defined the concept of radio mean labeling of graphs. Motivated by the notion radio mean labeling we have introduced the radio harmonic mean labeling [2,3].In this paper, we have investigated the radio harmonic mean labeling of complete graphs and determined the radio harmonic mean number of complete graphs.

In this paper, Radio Harmonic Mean Labeling and Radio Harmonic Mean Number are referred as **RHML** and **RHMN** for briefness.

2.Preliminaries

Definition 2.1

A radio harmonic mean labeling of a connected graph G is one to one map *f* from the vertex set V(G) of G to N such that for two distinct vertices *u* and *v* of G satisfies the condition $d(u,v)+\left\lceil \frac{2\, f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G)$ .A graph which admits radio harmonic mean labeling is called radio harmonic mean graph.

Definition 2.2

Radio Harmonic mean number of graph G is denoted by $rhmn(G)$. It is defined as the lowest span taken over all radio labeling of graph G.

Definition 2.3

The span of a labeling f is the maximum integer that f maps to a vertex of graph G. The radio harmonic mean number of G, $rhmn(G)$ is the lowest span taken over all radio harmonic mean labeling of the graph G.

Definition 2.4

A bipartite graph $G=(V,E)$ with vertex partition $V=\{v_1,v_2\}$ is said to be complete bipartite graph if every vertex in v_1 is connected to every vertex of v_2 and it is denoted by $K_{m,n}$ where v_1 contains m vertices and v_2 contains n vertices.

Definition 2.5

A complete tripartite graph $G=(V,E)$ with vertices can be partition into three disjoint sets such that every vertex of each set graph vertices is adjacent to every other vertex in the other two sets. It is denoted by $K_{m,n,t}$.

3.Main Results

Theorem:3.1
The radio harmonic mean number of complete graph K_n is n for $n \geq 2$.
Proof:
Let vertex set and edge set of K_n are

$V[K_n]=\{u_i : 1 \leq i \leq n\}$ and

$E[K_n]=\{u_i u_j : 1 \leq i, j \leq n, \ j > i, i \neq j\}$.

The diameter of K_n is 1 for $n \geq 2$.
The vertex labels are defined by

For $1 \leq i \leq n$

$$f(u_i)=i$$

In order to verify the definition of radio harmonic mean labeling

$$d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+diam(G) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

for every pair of vertices of G.
We need to show that

$$d(u,v)+\left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1+1 \geq 2$$

Verify the pair (u_i,u_j) for $1 \leq i, j \leq n, j > i, \ i \neq j, d(u_i,u_j)=1$

$$d(u_i,u_j)+\left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 1+\left\lceil \frac{2(i)(j)}{i+j} \right\rceil \geq 3$$

Thus, f satisfies the radio harmonic mean condition.
Therefore, the graph K_n is a radio harmonic mean graph.
Hence , $rhmn[K_n]=n$ for $n \geq 2$.

Illustration 3.2: RHML of K_6 is given below.

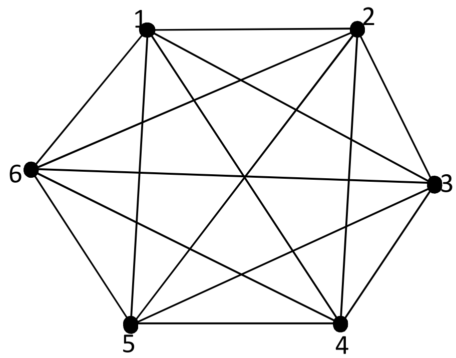

Figure. 1. RHML of K_6

Theorem:3.3

The radio harmonic mean number of complete bipartite graph $K_{m,n}$ is $m+n$ for $n \geq 2$.

Proof:

Let $V[K_{m,n}] = \{u_i, v_j : 1 \leq i \leq m, 1 \leq j \leq n\}$ and

$E[K_{m,n}] = \{u_i v_j : 1 \leq i \leq m \text{ and } 1 \leq j \leq n\}$.

The diameter of $K_{m,n}$ is 2 for $m,n \geq 2$.

The vertex labels are defined by

For $1 \leq i \leq m$

$f(u_i) = i$

For $1 \leq j \leq n$

$f(v_j) = m + j$

In order to verify the definition of radio harmonic mean labeling (1),we need to show that

$$d(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1 + 2 \geq 3$$

for every pair vertices of (u,v) *where* $u \neq v$.

Case (i): Verify the pair (u_i, v_j) for $1 \leq i \leq m, 1 \leq j \leq n$, $d(u_i, v_j) = 1$

$$d(u_i, v_j) + \left\lceil \frac{2f(u_i)f(v_j)}{f(u_i)+f(v_j)} \right\rceil = 1 + \left\lceil \frac{2(i)(m+j)}{m+i+j} \right\rceil \geq 3$$

Case (ii): Verify the pair (u_i, u_j) for $1 \leq i, j \leq m$, $j > i$, $i \neq j$, $d(u_i, u_j) = 2$

$$d(u_i, u_j) + \left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 2 + \left\lceil \frac{2(i)(j)}{i+j} \right\rceil \geq 4$$

Case (iii): Verify the pair (v_i, v_j) for $1 \leq i, j \leq n$, $j > i$, $i \neq j$, $d(v_i, v_j) = 2$

$$d(v_i, v_j) + \left\lceil \frac{2f(v_i)f(v_j)}{f(v_i)+f(v_j)} \right\rceil = 2 + \left\lceil \frac{2(m+i)(m+j)}{2m+i+j} \right\rceil \geq 6$$

In all the above cases f satisfies the radio harmonic mean condition (1).

Therefore, the graph $K_{m,n}$ is a radio harmonic mean graph.

Hence, $rhmn[K_{m,n}] = m+n$ for $m,n \geq 2$.

Illustration 3.4: RHML of $K_{4,5}$ is given below

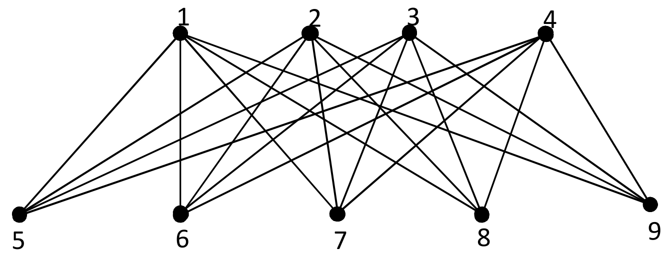

Figure 2. RHML of $K_{4,5}$

Theorem 3.5

The radio harmonic mean number of complete tripartite graph $K_{m,n,t}$ is $m+n+t$ for $m,n,t \geq 2$.

Proof:

Let vertex and edge set of the complete tripartite graph are

$$V[K_{m,n,t}] = \{u_i, v_j, w_k : 1 \leq i \leq m,\ 1 \leq j \leq n \text{ and } 1 \leq k \leq t\} \text{ and}$$

$$E[K_{m,n,t}] = \{u_i v_j, u_i w_k, v_j w_k : 1 \leq i \leq m,\ 1 \leq j \leq n \text{ and } 1 \leq k \leq t\}.$$

The diameter of $K_{m,n,t}$ is 2 for $m,n,t \geq 2$.

The vertex labels are defined by

For $1 \leq i \leq m$

$$f(u_i) = i$$

For $1 \leq j \leq n$

$$f(v_j) = m + j$$

For $1 \leq k \leq t$

$$f(w_k) = m + n + k$$

In order to verify the definition of radio harmonic mean labeling (1),We need to show that

$$d(u,v) + \left\lceil \frac{2f(u)f(v)}{f(u)+f(v)} \right\rceil \geq 1 + 2 \geq 3$$

for every pair of vertices of (u,v) *where* $u \neq v$.

Case (i): Verify the pair (u_i, u_j) for $1 \leq i, j \leq m,\ j > i,\ i \neq j,\ d(u_i, u_j) = 2$

$$d(u_i,u_j) + \left\lceil \frac{2f(u_i)f(u_j)}{f(u_i)+f(u_j)} \right\rceil = 2 + \left\lceil \frac{2(i)(j)}{i+j} \right\rceil \geq 4$$

Case (ii): Verify the pair (u_i, v_j) for $1 \leq i \leq m,\ 1 \leq j \leq n,\ d(u_i, v_j) = 1$

$$d(u_i,v_j) + \left\lceil \frac{2f(u_i)f(v_j)}{f(u_i)+f(v_j)} \right\rceil = 1 + \left\lceil \frac{2(i)(m+j)}{m+i+j} \right\rceil \geq 3$$

Case (iii): Verify the pair (v_i, v_j) for $1 \leq i, j \leq n,\ j > i,\ i \neq j,\ d(v_i, v_j) = 2$

$$d(v_i,v_j) + \left\lceil \frac{2f(v_i)f(v_j)}{f(v_i)+f(v_j)} \right\rceil = 2 + \left\lceil \frac{2(m+i)(m+j)}{2m+i+j} \right\rceil \geq 6$$

Case (iv): Verify the pair (u_i, w_k) for $1 \leq i \leq m,\ 1 \leq k \leq t,\ d(u_i, w_k) = 1$

$$d(u_i,w_k) + \left\lceil \frac{2f(u_i)f(w_k)}{f(u_i)+f(w_k)} \right\rceil = 1 + \left\lceil \frac{2(i)(m+n+k)}{m+n+i+k} \right\rceil \geq 3$$

Case (v): Verify the pair (v_j, w_k) for $1 \leq j \leq n,\ 1 \leq k \leq t,\ d(v_j, w_k) = 1$

$$d(v_j, w_k) + \left\lceil \frac{2f(v_j)f(w_k)}{f(v_j)+f(w_k)} \right\rceil = 1 + \left\lceil \frac{2(m+j)(m+n+k)}{2m+i+j+k} \right\rceil \geq 5$$

Case (vi): Verify the pair (w_i, w_j) for $1 \leq i, j \leq t, j > i, i \neq j, d(w_i, w_j) = 2$

$$d(w_i, w_j) + \left\lceil \frac{2f(w_i)f(w_j)}{f(w_i)+f(w_j)} \right\rceil = 2 + \left\lceil \frac{2(m+n+i)(m+n+j)}{2m+2n+i+j} \right\rceil \geq 8$$

In all the above cases *f* satisfies the radio harmonic mean condition (1).
Therefore, the graph $K_{m,n,t}$ is a radio harmonic mean graph.

Hence, $rhmn(K_{m,n,t}] = m+n+t$ for $m,n,t \geq 2$.

Illustration 3.6: RHML of $K_{4,5,2}$ is given below

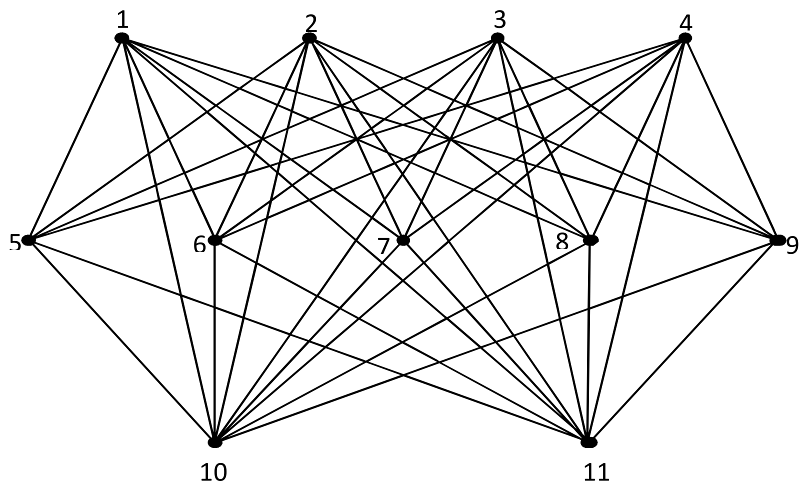

Figure 3. RHML of $K_{4,5,2}$

4. Conclusion

In this paper, the radio harmonic mean number of complete graphs was obtained. The radio labeling of graphs which would have played a very important role in the communication networks.

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