Edge Trimagic Graceful Labeling of Degree Splitting Graphs

J. A. Jose Ezhil[1*], M. Regees[2] & T. Shyla Isac Mary[3]
[1*]*Research Scholar, Reg. No – 18113112092011, Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, Kanyakumari District, Tamil Nadu, India.*
[2]*Assistant Professor, Department of Mathematics, Malankara Catholic College, Mariagiri, Kaliakavilai – 629153, Tamilnadu, India.*
[3]*Assistant Professor, Department of Mathematics, Nesamony Memorial Christian College, Marthandam – 629165, Kanyakumari District, Tamil Nadu, India.*
[*]*Email:ezhilja@gmail.com*
Affiliated to Manonmaniam Sundaranar University, Abishekapatti, Tirunelveli - 627 012, Tamil Nadu, India.

Abstract *- A (p, q) graph G is called an edge trimagic graceful if there exists a bijection $\varphi : V(G) \cup E(G) \rightarrow \{1, 2, 3, \ldots, p + q\}$ such that for each edge xy in E(G), $|\varphi(x) - \varphi(xy) + \varphi(y)| = c_1$ or c_2 or c_3, where c_1, c_2 and c_3 are constants. In this paper, we proved some of the degree splitting graphs are edge trimagic graceful graphs.*

Keywords - Graph, splitting graph, degree splitting graph trimagic, graceful, trimagic graceful.

1. INTRODUCTION

Splitting Graph was introduced by E. Sambathkumar and Walikar. H. B [4], for each point v of a graph G, takes a new point v′, join v′ to all points of G adjacent to v. The graph S(G) thus obtained is called the splitting graph of G. For a graph G(V,E), if V_i denote the set of all vertices of degree i, the degree splitting graph DS(G) is the graph obtained from G by adding new vertices w_i for each V_i with $|V_i| > 2$, and joining w_i with every vertex in V_i . Degree splitting graph DS(G) was introduced by R. Ponraj and S. Somasundaram [3]. In this paper, we have discussed about the super edge trimagic graceful labeling for some Degree Splitting Graphs such as $DS(K_{1,n})$, $DS(P_{,n})$ and $DS[K_2 + nK_1]$.

Definition [5]**.** Let G = (V, E) be a graph with $V = S_1 \cup S_2 \cup \ldots, \cup S_t \cup T$, where each S_i is a set of vertices having at least two vertices and $T = V - \cup S_i$. The ***degree splitting graph*** of G is denoted by DS(G) and is obtained from G by adding vertices $u_1, u_2, \ldots, u_t$ and joining u_i to each vertex of S_i $(1 \le i \le t)$.

2. MAIN RESULTS

Theorem 2.1.

The degree splitting graph of the graph $[K_2 + nK_1]$, admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for even $n \ge 3$.

Proof:

Let the vertex set of the degree splitting of Star graph $K_2 + n K_1$ be, $V[DS(K_2 + nK_1)] = \{a, b, u, v, w_i / 1 \le i \le n\}$ and the edge set be $E[DS(K_2 + nK_1)] = \{uv, ua, va, uw_i, vw_i, bw_i / 1 \le i \le n\}$. Then the degree splitting graph of the graph $(K_2 + nK_1)$ has n + 4 vertices and 3n + 3 edges.

Define a bijection $\varphi: V \cup E \rightarrow \{1, 2, 3, \ldots, 4n + 7\}$ such that

$\varphi(u) = 1; \varphi(v) = 2$

$\varphi(a) = 3 ; \varphi(b) = 4$

$\varphi(w_i) = i + 4, 1 \le i \le n ; \varphi(uv) = n + 5$

$\varphi(ua) = n + 6 ; \varphi(va) = n + 7$

$\varphi(uw_i) = n + 7 + i, 1 \le i \le n$

$\varphi(\mathrm{bw_i}) = 3n + 7 + i,\ 1 \le i \le n$

and $\varphi(\mathrm{vw_i}) = 2n + 7 + i,\ 1 \le i \le n$.

Hence for each edge uv $\in E[DS(K_2 + nK_1)]$, $\left|\varphi(u) - \varphi(uv) + \varphi(v)\right|$ will get any one of the constants $c_1 = \left|-n-2\right|$, $c_2 = \left|-2n-1\right|$ and $c_3 = \left|1-3n\right|$. Therefore the degree splitting graph of the graph $(K_2 + nK_1)$ admits an edge trimagic graceful labeling for even $n \ge 3$. Since the graph $DS(K_2 + nK_1)$ has n + 4 vertices and these n + 4 vertices have labels 1, 2, ..., n + 4 for all $n \ge 3$, hence $DS(K_2 + nK_1)$ is a super edge trimagic graceful.

Example 2.2. An edge trimagic graceful labeling of the degree splitting graph $DS(K_2 + 6K_1)$ is given in figure 1.

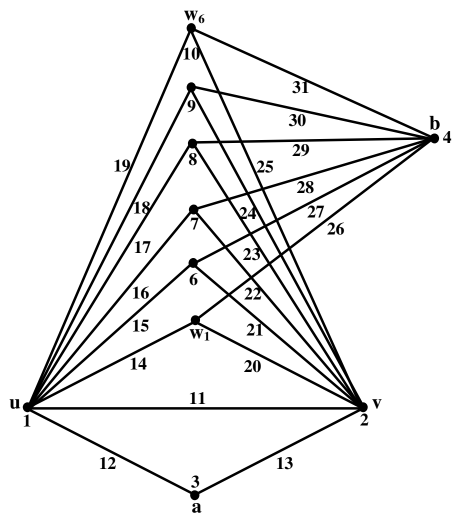

Figure 1. $DS(K_2 + 6K_1)$ with constants $c_1 = 8$, $c_2 = 13$ and $c_3 = 17$.

Theorem 2.3.
The degree splitting graph of the path graph $DS(P_n)$ admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for all $n \ge 3$.

Proof:

Let the vertex set of the degree splitting of the path graph P_n be, $V[DS(P_n)] = \{u, v, u_i / 1 \le i \le n\}$ and the edge set be $E[DS(P_n)] = \{uu_i, vu_1, vu_n / 2 \le i \le n-1\}$. Then the graph $DS(P_n)$ has n + 2 vertices and 2n – 1 edges.

Case 1: n is odd

Define a bijection φ: $V \cup E \rightarrow \{1, 2, 3, ..., 3n+1\}$ such that

$\varphi(u) = 1$; $\varphi(v) = n + 2$

$$\varphi(u_i) = \begin{cases} \frac{i+3}{2}, & 1 \le i \le n, \text{ i is odd} \\ \frac{n+i+3}{2}, & 1 \le i \le n, \text{ i is even} \end{cases}$$

$\varphi(vu_1) = n + 3$; $\varphi(vu_n) = \frac{3n+5}{2}$

$$\varphi(uu_i) = \begin{cases} n + 1 + \frac{i+3}{2}, & 2 \le i \le n-1, \text{ i is odd} \\ \frac{3n+5+i}{2}, & 2 \le i \le n-1, \text{ i is even} \end{cases}$$

and $\varphi(u_iu_{i+1}) = 2n + i + 2,\ 1 \le i \le n$.

Hence for each edge uv $\in DS(P_n)$, $\left|\varphi(u) - \varphi(uv) + \varphi(v)\right|$ will assign to any one of the constants $c_1 = \left|\frac{3-3n}{2}\right|$, $c_2 = 1$ and $c_3 = \left|-n\right|$. Therefore the degree splitting graph of the path graph $DS(P_n)$ admits an edge trimagic graceful labeling for odd $n \ge 3$.

Case 2: n is even

Define a bijection φ: V ∪ E → {1, 2, 3, ..., 3n+1} such that

φ(u) = 1; φ(v) = n + 2

$$\varphi(u_i) = \begin{cases} \frac{i+3}{2}, & 1 \le i \le n, \text{i is odd} \\ \frac{n+i+2}{2}, & 1 \le i \le n, \text{i is even} \end{cases}$$

$\varphi(vu_1)$ = n + 3 ; $\varphi(vu_n)$ = 2n + 2

$$\varphi(uu_i) = \begin{cases} n + 2 + \frac{i+1}{2}, 2 \le i \le n-1, \text{i is odd} \\ n + 2 + \frac{n+i}{2}, 2 \le i \le n-1, \text{i is even} \end{cases}$$

and $\varphi(u_iu_{i+1})$ = 2n + i + 2, 1 ≤ i ≤ n.

Hence for each edge uv ∈ DS(P_n), $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will assign to any one of the constants $c_1 = \left|\frac{2-3n}{2}\right|$, c_2 = 1, and $c_3 = |-n|$. Therefore the degree splitting graph of the path graph DS(P_n) admits an edge trimagic graceful labeling for even n ≥ 3.

Since the degree splitting graph of the path graph DS(P_n) has n + 2 vertices and these n + 2 vertices have labels 1, 2, ..., n + 2 for all n ≥ 3, hence DS(P_n) is a super edge trimagic graceful. Hence, the theorem follows from the case 1 and case 2.

Example 2.4. An edge trimagic graceful labeling of the degree splitting graphs DS(P_7) is given in figure 2.

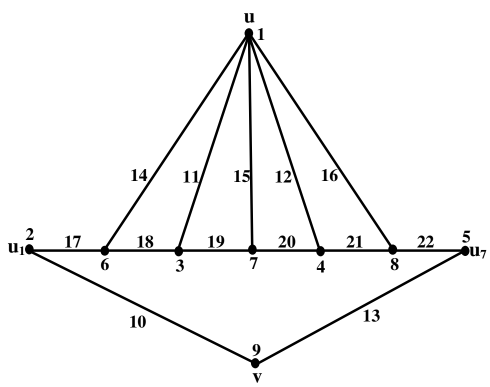

Figure 2. DS(P_7) with constants c_1 = 9, c_2 = 1 and c_3 = 7.

Theorem 2.5.

The degree splitting graph of the star graph DS($K_{1,n}$) admits an edge trimagic graceful labeling and super edge trimagic graceful labeling for all n ≥ 3.

Proof:

Let the vertex set of the degree splitting graph of the Star graph be, V[DS($K_{1,n}$)] = {v, v_i, w / 1 ≤ i ≤ n } and the edge set be E[DS($K_{1,n}$)] = {vv_i, wv_i/ 1 ≤ i ≤ n }.

Then the graph DS($K_{1,n}$) has n + 2 vertices and 2n edges.

Define a bijection φ: V ∪ E → {1, 2, 3, ..., 3n + 2} such that

φ(v) = 1; φ(w) = 2

$\varphi(v_i)$ = i + 2 , 1 ≤ i ≤ n

$\varphi(vv_1)$ = n + 3; $\varphi(wv_1)$ = n + 4

$\varphi(vv_i)$ = n + 3 + i, 2 ≤ i ≤ n

and $\varphi(wv_i)$ = 2n + 2 + i, 2 ≤ i ≤ n.

Hence for each edge uv ∈ E[DS($K_{1,n}$)], $|\varphi(u) - \varphi(uv) + \varphi(v)|$ will assign to any one of the constants $c_1 = |1 - n|$, $c_2 = |-n|$ and $c_3 = |2 - 2n|$. Therefore the graph DS($K_{1,n}$) admits an edge trimagic graceful labeling for all n ≥ 3. Since the graph DS($K_{1,n}$) has n + 2 vertices and these n + 2 vertices have labels 1, 2, ..., n + 2 for all n ≥ 3, hence DS($K_{1,n}$) is a super edge trimagic gracefulfor all n ≥ 3.

Example 2.6. An edge trimagic graceful labeling of the degree splitting graph DS($K_{1,6}$) is given in figure 3.

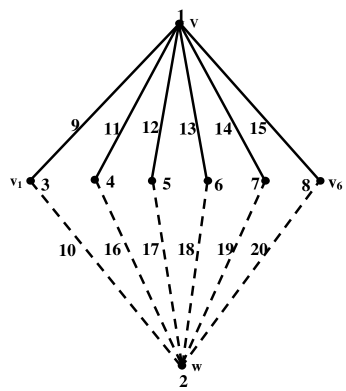

Figure 3.DS($K_{1,6}$) with constants c_1= 5, c_2= 6 and c_3= 10.

3. CONCLUSION

In this paper, we proved that some of the degree splitting graphs areadmit edge trimagic graceful labeling. Using these ideas we can find more edge trimagic graceful graphs.

REFERENCES

1. Jose Ezhil J. A, Regees. M and Shyla Isac Mary. T, "Edge Trimagic Graceful Labeling of Graphs", International Journal of Pure and Applied Mathematics, Volume 120 No. 3, 2018, pp. 487 – 496.
2. Joseph A. Gallian, "A Dynamic Survey of graph Labeling of Some Graphs", The Electronic Journal of Combinatorics (2017), #DS6.
3. Ponraj. R and Soma Sundaram. S, "On the Degree Splitting of Graphs", NATL. ACAD. SCI. LETT., Vol – 27, No.7 & 8, pp. 275 – 278, 2004.
4. Sampathkumar. E and Walikar. H. B, "On the Splitting Graph of a Graph", The Karnataka University Journal – Science, Vol. XXV &XXVI(combined) 1980 – 1981, pg. 13 – 16.
5. Saranya. D and Mary. U, "Even Graceful labeling of Some New Graphs", International Journal of Engineering and Computer Science, vol. 2, issue. 6, June, 2013, pp. 1934 – 1936.