NERVE EXCITATION INVOLVING I-FUNCTION

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ABSTRACT

The aim this paper is to study the Nerve Excitation involving I-Function of one variable.

Keywords: I-Function, Nerve Excitation.

1. INTRODUCTION:

The I-function of one variable is defined by Saxena [2, p.366-375] and we will represent here in the following manner:

$$I^{m,n}_{p_i, q_i; r}\left[x \,\middle|\, \begin{matrix} [(a_j, \alpha_j)_{1,n}], [(a_{ji}, \alpha_{ji})_{n+1, p_i}] \\ [(b_j, \beta_j)_{1,m}], [(b_{ji}, \beta_{ji})_{m+1, q_i}] \end{matrix}\right] = (1/2\pi i) \int_L \theta(s)\, x^s\, ds \qquad (1)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j s) \prod_{j=1}^{n} \Gamma(1 - a_j + \alpha_j s)}{\sum_{i=1}^{r} \left\{ \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} + \beta_{ji} s) \prod_{j=n+1}^{p_i} \Gamma(a_{ji} - \alpha_{ji} s) \right\}},$$

integral is convergent, when ($B>0$, $A \le 0$), where

$$B = \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{pi} \alpha_{ji} + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{qi} \beta_{ji}, \qquad (2)$$

$$A = \sum_{j=1}^{pi} \alpha_{ji} - \sum_{j=1}^{qi} \beta_{ji}, \qquad (3)$$

$$|\arg x| < \tfrac{1}{2} B\pi, \ \forall\, i \in (1, 2, \ldots, r). \qquad (4)$$

The cells of a nerve fibre may be conceived as an electric system. The protoplasm contains a large number of different ions, both cations (positive electric charge) and anions (negative electric charge). When an electric current is applied to a nerve fibre, the cations move to the cathode, the anions to the anode, and the electric equilibrium are disturbed. This phenomenon leads to the excitation of the nerve.

Based on the observation that the excitation originates at the cathode, N. Rashevsky, developed a theory which postulates that two different kinds of cations are responsible for the process. One is exciting and the other kind is inhibiting. These two kinds are said to be antagonistic factors.

2. MATHEMATICAL MODEL:

Let E = E(t) be the concentration of the exciting cations and F = F(t) be the concentration of the inhibiting cations near the cathode at any time t. The theory then states that excitation occurs whenever the ratio E/F exceeds a certain value. Denoting this value by c, we have excitation when $E/F \geq C$ and there will be no excitation if $E/F < C$. Let E_0 and F_0 be the concentrations at rest of exciting and inhibiting cations, respectively. When E increases and F remains limited, there is excitation. When E does not increase as fast as F, then there is no excitation.

Let I be the intensity of the stimulant current. For convenience sake, assume that I is constant during a certain time interval. Rashevsky showed that the Excitation of nerves can be described by the differential equations

$$dE/dt = JI - K(E - E_0) \quad (5)$$

and

$$dF/dt = LI - M(F - F_0) \quad (6)$$

where J, K, L, M are positive constants.

The above equations can be easily solved for E and F, and finally the ratio E/F determines whether excitation occurs and when.

3. SOLUTION IN TERMS OF I-FUNCTION:

Choose E(t) and F(t) concentration of the exciting cations and concentration of the inhibiting cations respectively in terms of I-function as

$$E(t) = I^{m+1,n}_{p_i+1,q_i+1:r}\left[xt^{\lambda}\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]\end{matrix}\right.\right] \quad (7)$$

$$F(t) = I^{m+1,n}_{p_i+1,q_i+1:r}\left[xt^{\mu}\left|\begin{matrix}[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]\\ [(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]\end{matrix}\right.\right] \quad (8)$$

where $\lambda > 0$, $\mu > 0$, $|\arg z| < \frac{1}{2}\pi B$, where B is given in (2).

Now differentiate (7) and (8) with respect to t, we get

$$dE/dt = (1/t)\ I^{m,n+1}_{p_i+1,q_i+1:r}\left[xt^{\lambda}\Big|^{(0,\lambda),[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(1,\lambda)}\right] \quad (9)$$

and

$$dF/dt = (1/t)\ I^{m,n+1}_{p_i+1,q_i+1:r}\left[xt^{\mu}\Big|^{(0,\mu),[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(1,\mu)}\right] \quad (10)$$

Now after using (7), (8), (9) and (10) in (5) and (6), we get following results:

$$(1/t)\ I^{m,n+1}_{p_i+1,q_i+1:r}\left[xt^{\lambda}\Big|^{(0,\lambda),[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(1,\lambda)}\right]$$

$$= JI - K\left(I^{m,n}_{p_i,q_i:r}\left[xt^{\lambda}\Big|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]}\right] - E_0\right) \quad (11)$$

$$(1/t)\ I^{m,n+1}_{p_i+1,q_i+1:r}\left[xt^{\mu}\Big|^{(0,\mu),[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}],(1,\mu)}\right]$$

$$= LI - M\left(I^{m,n}_{p_i,q_i:r}\left[xt^{\mu}\Big|^{[(a_j,\alpha_j)_{1,n}],[(a_{ji},\alpha_{ji})_{n+1,p_i}]}_{[(b_j,\beta_j)_{1,m}],[(b_{ji},\beta_{ji})_{m+1,q_i}]}\right] - F_0\right) \quad (12)$$

where $\lambda > 0$, $\mu > 0$, $|\arg x| < \frac{1}{2}\pi B$, where B is given in (2).

3. SPECIAL CASE:

On choosing r = 1 in (11) and (12), we get following equation of Nerve Excitation in terms of H-function [1] of one variable as follows:

$$(1/t)\ H^{m,n+1}_{p+1,q+1}\left[xt^{\lambda}\Big|^{(0,\lambda),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q},(1,\lambda)}\right]$$

$$= JI - K\left(H^{m,n}_{p,q}\left[xt^{\lambda}\Big|^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right] - E_0\right) \quad (13)$$

$$(1/t)\ H^{m,n+1}_{p+1,q+1}\left[xt^{\mu}\Big|^{(0,\mu),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q},(1,\mu)}\right]$$

$$= LI - M\left(H^{m,n}_{p,q}\left[xt^{\mu}\Big|^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}\right] - F_0\right) \quad (14)$$

where $\lambda > 0$, $\mu > 0$, $|\arg x| < \frac{1}{2} \pi A$, where A is given by

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv A > 0,$$

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