DETERMINATION OF THE LAMBERT'S LAW INVOLVING I-FUNCTION

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ABSTRACT

The aim of this paper is to represent the equation of Lambert's law by I–Function of one variable.

Keywords: I-Function, Lambert's law.

1. INTRODUCTION:

The I-function of one variable is defined by Saxena [3, p.366-375] and we will represent here in the following manner:

$$I^{m,\,n}_{p_i,\,q_i:\,r}\left[x\left|\begin{matrix}[(a_j,\alpha_j)_{1,\,n}],\,[(a_{ji},\alpha_{ji})_{n+1,\,p_i}]\\ [(b_j,\beta_j)_{1,\,m}],\,[(b_{ji},\beta_{ji})_{m+1,\,q_i}]\end{matrix}\right.\right] = (1/2\pi i)\int_L \theta(s)\,x^s\,ds \qquad (1)$$

where $i = \sqrt{(-1)}$,

$$\theta(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j s)\ \prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j s)}{\sum_{i=1}^{r}\left\{\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}+\beta_{ji}s)\ \prod_{j=n+1}^{p_i}\Gamma(a_{ji}-\alpha_{ji}s)\right\}},$$

integral is convergent, when $(B > 0,\ A \le 0)$, where

$$B = \sum_{j=1}^{n}\alpha_j - \sum_{j=n+1}^{p_i}\alpha_{ji} + \sum_{j=1}^{m}\beta_j - \sum_{j=m+1}^{q_i}\beta_{ji}, \qquad (2)$$

$$A = \sum_{j=1}^{p_i}\alpha_{ji} - \sum_{j=1}^{q_i}\beta_{ji}, \qquad (3)$$

$$|\arg x| < \tfrac{1}{2}B\pi,\ \forall\ i \in (1, 2, \ldots, r). \qquad (4)$$

2. MAIN RESULT:

The equation of Lambert's law in terms of I-function of one variable to be represented is:

$$\int I^{m+1,n}_{p_i+1,q_i+1:r}\,[x|^{\ldots\ldots\ldots,(1+I,I_1)}_{(I,I_1),\ldots\ldots\ldots}]dI$$

$$= -k\, I^{m,n+1}_{p_i+1,q_i+1:r}[x|^{(-t,t_1),\ldots\ldots}_{\ldots\ldots,(1-t,t_1)}] + c\, I^{m,n}_{p_i,q_i:r}[x], \quad (5)$$

valid for $I > I_1$, $t > t_1$ and $|\arg x| < ½\,\pi B$, where B is given in (2).

Proof of the Formula:

Let I be the intensity of incident light of wave length *l*, t be the thickness of medium, then Lambert's law mathematically [1] denoted as

$$dI/dt = -kI$$

or $$dI/I = -k\,dt \quad (6)$$

On integrating, (6) provides

$$\int dI/I = -k\int dt + c$$

or $$\int [\Gamma(I)/\,\Gamma(I+1)] = -kt + c$$

or $$\int [\Gamma(I)/\,\Gamma(I+1)] = -k\,[\Gamma(t+1)/\,\Gamma(t)] + c \quad (7)$$

where c is integral constant.

Again put $t = t + t_1 s$, $I = I - I_1 s$ (since as thickness of medium increases, intensity of light decrease) in (7) and multiply both side by $(1/2\pi i)\theta(s)x^s$, further integrate with respect to s in the direction of contour L and use (1), we get (5).

3. SPECIAL CASE:

On choosing r = 1 in (5), we get following formula for Lambert's law in terms of H-function [2] of one variable as follows:

$$\int H^{m+1,n}_{p+1,q+1}\,[x|^{(a_j,\alpha_j)_{1,p},(1+I,I_1)}_{(I,I_1),(b_j,\beta_j)_{1,q}}]dI$$

$$= -k\, H^{m,n+1}_{p+1,q+1}[x|^{(-t,t_1),(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q},(1-t,t_1)}] + c\, H^{m,n}_{p,q}\,[x|^{(a_j,\alpha_j)_{1,p}}_{(b_j,\beta_j)_{1,q}}], \quad (8)$$

valid for $I > I_1$, $t > t_1$ and $|\arg x| < \frac{1}{2}\pi A$, where A is given by

$$\sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j \equiv A > 0,$$

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