

Intuitionistic Fuzzy Almost β^{**} G Closed Mappings

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Abstract: The aim of this paper is to introduce and study the concepts of intuitionistic fuzzy almost β^{**} generalized closed mappings and intuitionistic fuzzy almost β^{**} generalized open mappings. We investigate some of their properties. Also we provide the relations between intuitionistic fuzzy almost β^{**} generalized closed mapping with other intuitionistic fuzzy closed mappings.

Keywords: Intuitionistic fuzzy topology, intuitionistic fuzzy point, intuitionistic fuzzy β^{**} generalized closed mappings, intuitionistic fuzzy β^{**} generalized open mappings, intuitionistic fuzzy almost β^{**} generalized closed mappings, intuitionistic fuzzy almost β^{**} generalized open mappings.

Subject Classification Code: 03F55, 54A40.

1. Introduction

Open maps and closed maps are very interesting concepts in topology. Noiri [8] has introduced semi-closed mappings which contain the class of closed mappings. Malghan [7] has introduced generalized closed maps in topology. Seok Jong Lee and EunPyo Lee [9] have introduced intuitionistic fuzzy open mapping and intuitionistic fuzzy closed mapping in intuitionistic fuzzy topological spaces. Sudha and Jayanthi [10] introduced intuitionistic fuzzy β^{**} generalized closed mappings in 2020 and now we have extend our idea towards intuitionistic fuzzy almost β^{**} generalized closed mappings, intuitionistic fuzzy almost β^{**} generalized open mappings. The interrelations with other already existing closed mappings with our newly defined closed mapping have been established.

2. Preliminaries

Definition 2.1: [1] An intuitionistic fuzzy set (IFS) A is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$$

where the functions $\mu_A : X \rightarrow [0, 1]$ and $\nu_A : X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $\text{IFS}(X)$, the set of all intuitionistic fuzzy sets in X . An intuitionistic fuzzy set A in X is simply denoted by $A = \langle x, \mu_A, \nu_A \rangle$ instead of denoting $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$.

Definition 2.2: [1] Let A and B be two IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle : x \in X \}$. Then,

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,
- (b) $A = B$ if and only if $A \subseteq B$ and $A \supseteq B$,
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle : x \in X \}$,
- (d) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle : x \in X \}$,
- (e) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle : x \in X \}$.

The intuitionistic fuzzy sets $0_- = \langle x, 0, 1 \rangle$ and $1_- = \langle x, 1, 0 \rangle$ are respectively the empty set and the whole set of X .

Definition 2.3: [2] An intuitionistic fuzzy topology (IFT) on X is a family τ of IFSs in X satisfying the following axioms :

- (i) $0_-, 1_- \in \tau$,
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- (iii) $\cup G_i \in \tau$ for any family $\{ G_i : i \in J \} \subseteq \tau$

In this case the pair (X, τ) is called the **intuitionistic fuzzy topological space** (IFTS) and any IFS in τ is known as an **intuitionistic fuzzy open set** (IFOS) in X . The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS) in X .

Definition 2.4: [6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

- (i) *intuitionistic fuzzy semi closed set* (IFSCS) if $\text{int}(\text{cl}(A)) \subseteq A$,
- (ii) *intuitionistic fuzzy pre closed set* (IFPCS) if $\text{cl}(\text{int}(A)) \subseteq A$,
- (iii) *intuitionistic fuzzy regular closed set* (IFRCS) if $\text{cl}(\text{int}(A)) = A$,
- (iv) *intuitionistic fuzzy α closed set* (IF α CS) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

The respective complements of the above IFCSs are called their respective IFOSs.

Definition 2.5: [11] An IFS A of an IFTS (X, τ) is said to be an **intuitionistic fuzzy β^{**} generalized closed set** (IF β^{**} GCS) if $\text{cl}(\text{int}(\text{cl}(A))) \cap \text{int}(\text{cl}(\text{int}(A))) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ) .

The complement A^c of an IF β^{**} GCS A in an IFTS (X, τ) is called an **intuitionistic fuzzy β^{**} generalized open set** (IF β^{**} GOS) in X .

Result 2.6: [11] Every IFCS, IFRCS, IFSCS, IFPCS, IF β CS, IF α CS, IFGCS is an IF β^{**} GCS but the converses may not true in general.

Definition 2.7: [12] A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an **intuitionistic fuzzy β^{**} generalized continuous** (IF β^{**} G continuous) **mapping** if $f^{-1}(V)$ is an IF β^{**} GCS in (X, τ) for every IFCS V of (Y, σ) .

Definition 2.8: [14] An IFTS (X, τ) is an **intuitionistic fuzzy $\beta^{**}pT_{1/2}$** (IF $\beta^{**}pT_{1/2}$) **space** if every IF β^{**} GCS is an IFPCS in X .

Definition 2.9: [14] An IFTS (X, τ) is an **intuitionistic fuzzy $\beta^{**}gT_{1/2}$** (IF $\beta^{**}gT_{1/2}$) **space** if every IF β^{**} GCS is an IFGCS in X .

Definition 2.10: [9] Let $p_{(\alpha, \beta)}$ be an IFP of an IFTS (X, τ) . An IFS A of X is called an **intuitionistic fuzzy neighbourhood** (IFN) of $p_{(\alpha, \beta)}$ if there exists an IFOS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Definition 2.11: [5] Let $p_{(\alpha, \beta)}$ be an IFP in (X, τ) . An IFS A of X is called an **intuitionistic fuzzy β neighbourhood** (IF β N) of $p_{(\alpha, \beta)}$ if there exists an IF β OS B in X such that $p_{(\alpha, \beta)} \in B \subseteq A$.

Definition 2.12: [3] An IFS A is said to be an **intuitionistic fuzzy dense** (IFD) in another IFS B in an IFTS (X, τ) , if $\text{cl}(A) = B$.

Definition 2.13: [13] Let A be an IFS in an IFTS (X, τ) . Then the β^{**} generalized interior and β^{**} generalized closure of A are defined as

$$\beta^{**} \text{gint}(A) = \cup \{ G / G \text{ is an IF}\beta^{**}\text{GOS in } X \text{ and } G \subseteq A \} \text{ and}$$

$$\beta^{**} \text{gcl}(A) = \cap \{ K / K \text{ is an IF}\beta^{**}\text{GCS in } X \text{ and } A \subseteq K \}$$

It is to be noted that for any IFS A in (X, τ) , we have $\beta^{**} \text{gcl}(A^c) = (\beta^{**} \text{gint}(A))^c$ and $\beta^{**} \text{gint}(A^c) = (\beta^{**} \text{gcl}(A))^c$.

Definition 2.14: [4] A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an **intuitionistic fuzzy closed mapping** (IFCM) if $f(V)$ is an IFCS in Y for every IFCS V in X .

Definition 2.15: [6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an

- (i) **intuitionistic fuzzy pre closed mapping** (IFPCM) if $f(A)$ is an IFPCS in Y for each IFCS A in X ,
- (ii) **intuitionistic fuzzy α closed mapping** (IF α CM) if $f(A)$ is an IF α CS in Y for each IFCS A in X ,
- (iii) **intuitionistic fuzzy semi closed mapping** (IFSCM) if $f(A)$ is an IFSCS in Y for each IFCS A in X .

3. Intuitionistic Fuzzy Almost β^{**} Generalized Closed Mappings

In this section we have introduced intuitionistic fuzzy almost β^{**} generalized closed mappings, intuitionistic fuzzy almost β^{**} generalized open mappings and investigated some of their properties.

Definition 3.1 : A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is called an **intuitionistic fuzzy almost β^{**} generalized** (IF almost β^{**} G) **closed mapping** if $f(V)$ is an IF β^{**} GCS in Y for every IFRCS V of X .

Example 3.2 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b) \rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v) \rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, G_3, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost β^{**} G closed mapping.

Proposition 3.3 : Every IF closed mapping is an IF almost $\beta^{**}G$ closed mapping but not conversely in general.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF closed mapping. Let V be an IF RCS in X . Since every IF RCS is an IFCS in X , V is an IFCS in X . Then $f(V)$ is an IFCS in Y , by hypothesis. Since every IFCS is an IF $\beta^{**}GCS$, $f(V)$ is an IF $\beta^{**}GCS$ in Y . Hence f is an IF almost $\beta^{**}G$ closed mapping.

Example 3.4 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b) \rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v) \rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, G_3, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost $\beta^{**}G$ closed mapping but not an IF closed mapping, since $G_1^c = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b) \rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFCS in Y , as $\text{cl}(f(G_1^c)) = 1_- \neq f(G_1^c)$.

Proposition 3.5 : Every IF semi closed mapping is an IF almost $\beta^{**}G$ closed mapping in (X, τ) but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF semi closed mapping. Let V be an IF RCS in X . Since every IF RCS is an IFCS in X , V is an IFCS in X . Then $f(V)$ is an IFSCS in Y , by hypothesis. Since every IFSCS is an IF $\beta^{**}GCS$, $f(V)$ is an IF $\beta^{**}GCS$ in Y . Hence f is an IF almost $\beta^{**}G$ closed mapping.

Example 3.6 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b) \rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v) \rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, G_3, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost $\beta^{**}G$ closed mapping but not an IF semi closed mapping, since $G_1^c = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b) \rangle$ is an IFCS in X but $f(G_1^c)$ is not an IFSCS in Y , as $\text{int}(\text{cl}(f(G_1^c))) = 1_- \not\subseteq f(G_1^c)$.

Proposition 3.7 : Every IF pre closed mapping is an IF almost $\beta^{**}G$ closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF pre closed mapping. Let V be an IFRCS in X . Since every IFRCS is an IFCS in X , V is an IFCS in X . Then $f(V)$ is an IFPCS in Y , by hypothesis. Since every IFPCS is an $IF\beta^{**}GCS$, $f(V)$ is an $IF\beta^{**}GCS$ in Y . Hence f is an IF almost $\beta^{**}G$ closed mapping.

Example 3.8 : Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5_a, 0.6_b), (0.5_a, 0.4_b) \rangle$, $G_2 = \langle x, (0.4_a, 0.3_b), (0.6_a, 0.7_b) \rangle$, and $G_3 = \langle y, (0.6_u, 0.6_v), (0.4_u, 0.4_v) \rangle$. Then $\tau = \{0_-, G_1, G_2, 1_-\}$ and $\sigma = \{0_-, G_3, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost $\beta^{**}G$ closed mapping, but not an IF pre closed since G_2^c is an IFCS in X but $f(G_2^c)$ is not an IFPCS in Y , as $cl(int(f(G_2^c))) = cl(G_3) = 1_-\underline{\not\subseteq} f(G_2^c)$.

Proposition 3.9: Every $IF\alpha$ closed mapping is an IF almost $\beta^{**}G$ closed mapping but not conversely in general.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an $IF\alpha$ closed mapping. Let V be an IFRCS in X . Since every IFRCS is an IFCS in X , V is an IFCS in X . Then $f(V)$ is an $IF\alpha CS$ in Y , by hypothesis. Since every $IF\alpha CS$ is an $IF\beta^{**}GCS$, $f(V)$ is an $IF\beta^{**}GCS$ in Y . Hence f is an IF almost $\beta^{**}G$ closed mapping.

Example 3.10: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.3_a, 0.2_b), (0.7_a, 0.8_b) \rangle$, $G_2 = \langle y, (0.8_u, 0.6_v), (0.2_u, 0.4_v) \rangle$ and $G_3 = \langle y, (0.3_u, 0.4_v), (0.5_u, 0.6_v) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, G_3, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost $\beta^{**}G$ closed mapping but not an $IF\alpha$ closed mapping, since $G_1^c = \langle x, (0.7_a, 0.8_b), (0.3_a, 0.2_b) \rangle$ is an IFCS in X but $f(G_1^c)$ is not an $IF\alpha CS$ in Y , as $cl(int(cl(f(G_1^c)))) = 1_-\underline{\not\subseteq} f(G_1^c)$.

Proposition 3.11 : Every $IF\beta$ closed mapping is an IF almost $\beta^{**}G$ closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an $IF\beta$ closed mapping. Let V be an IFRCS is an IFCS in X , V is an IFCS in X . Then $f(V)$ is an $IF\beta CS$ in Y by hypothesis. Since every $IF\beta CS$ is an $IF\beta^{**}GCS$, $f(V)$ is an $IF\beta^{**}GCS$ in Y . Hence f is an IF almost $\beta^{**}G$ closed mapping.

Example 3.12: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.4_a, 0.4_b), (0.6_a, 0.6_b) \rangle$, $G_2 = \langle x, (0.6_u, 0.6_v), (0.4_u, 0.4_v) \rangle$, $G_3 = \langle y, (0.5_u, 0.7_v), (0.5_u, 0.3_v) \rangle$ and $G_4 = \langle y, (0.5_u, 0.6_v), (0.5_u,$

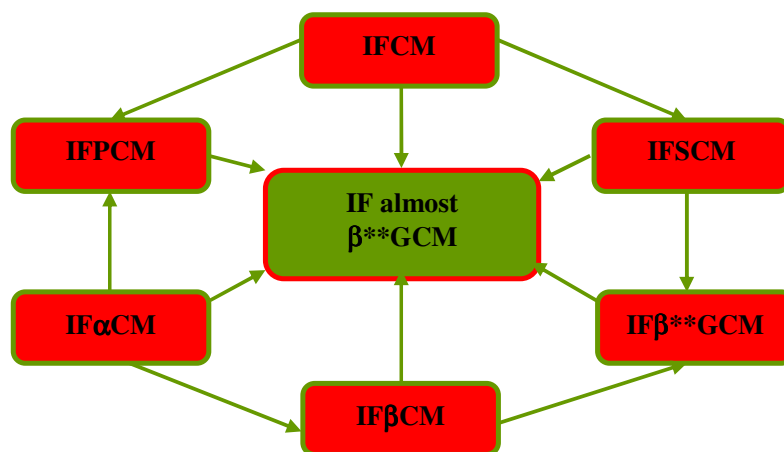
0.4_v). Then $\tau = \{0_-, G_1, G_2, 1_-\}$ and $\sigma = \{0_-, G_3, G_4, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost $\beta^{**}G$ closed mapping but not an IF β closed mapping, since $G_1^c = \langle x, (0.6_a, 0.6_b), (0.4_a, 0.4_b) \rangle$ is an IFCS in X but $f(G_1^c)$ is not an IF β CS in Y , as $\text{int}(\text{cl}(\text{int}(f(G_1^c)))) = 1_-\not\subseteq f(G_1^c)$.

Proposition 3.13 : Every IF $\beta^{**}G$ closed mapping is an IF almost $\beta^{**}G$ closed mapping but not conversely in general.

Proof : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF $\beta^{**}G$ closed mapping. Let V be an IFRCS in X . Since every IFRCS is an IFCS in X , V is an IFCS in X . Then $f(V)$ is an IF $\beta^{**}G$ CS in Y by hypothesis. Hence f is an IF almost $\beta^{**}G$ closed mapping.

Example 3.14 : Let $X = \{a, b\}$, $Y = \{u, v\}$, $G_1 = \langle x, (0.2_a, 0.2_b), (0.5_a, 0.8_b) \rangle$, $G_2 = \langle x, (0.2_u, 0.2_v), (0.5_u, 0.6_v) \rangle$ and $G_3 = \langle y, (0.5_u, 0.8_v), (0.2_u, 0.2_v) \rangle$. Then $\tau = \{0_-, G_1, G_2, 1_-\}$ and $\sigma = \{0_-, G_3, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IF almost $\beta^{**}G$ closed mapping but not an IF $\beta^{**}G$ closed mapping, since $G_1^c = \langle x, (0.5_a, 0.8_b), (0.2_a, 0.2_b) \rangle$ is an IFCS in X but $f(G_1^c)$ is not an IF $\beta^{**}G$ CS in Y , as $f(G_1^c) \subseteq G_3$ whereas $\text{int}(\text{cl}(\text{int}(f(G_1^c)))) \cap \text{cl}(\text{int}(\text{cl}(f(G_1^c)))) = 1_-\not\subseteq G_3$.

The relation between various types of intuitionistic fuzzy closed mappings is given in the following diagram. In this diagram ‘CM’ means closed mappings. The reverse implications are not true in general in the below diagram.



Definition 3.15: A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be an **intuitionistic fuzzy almost β^{**} generalized (IF almost β^{**} G) open mapping** if $f(A)$ is an IF β^{**} GOS in Y for each IFROS A in X .

Proposition 3.16 : A bijective mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF almost β^{**} G closed mapping if and only if the inverse image of each IFROS in X is an IF β^{**} GOS in Y .

Proof : Necessity : Let A be an IFROS in X . This implies A^c is an IFRCS in X . Then $f(A^c)$ is an IF β^{**} GCS in Y , by hypothesis. Since $f(A^c) = (f(A))^c$, for bijective mapping $f(A)$ is an IF β^{**} GOS in Y .

Sufficiency : Let A be an IFRCS in X . Then A^c is an IFROS in X . By hypothesis $f(A^c)$ is IF β^{**} GOS in Y . Therefore $f(A)$ is an IF β^{**} GCS in Y . Hence f is an IF almost β^{**} G closed mapping.

Proposition 3.17 : Let $p_{(\alpha, \beta)}$ be an IFP in X . A mapping $f : X \rightarrow Y$ is an IF almost β^{**} G open mapping if for every IFOS A in X with $f^{-1}(p_{(\alpha, \beta)}) \in A$, there exists an IFOS B in Y with $p_{(\alpha, \beta)} \in B$ such that $f(A)$ is IFD in B .

Proof : Let A be an IFROS in X . Then A is an IFOS in X . Let $f^{-1}(p_{(\alpha, \beta)}) \in A$, then there exists an IFOS B in Y such that $p_{(\alpha, \beta)} \in B$ and $\text{cl}(f(A)) = B$. Since B is an IFOS, $\text{cl}(f(A)) = B$ is also an IFOS in Y . Therefore $\text{int}(\text{cl}(f(A))) = \text{cl}(f(A))$. Now $f(A) \subseteq \text{cl}(f(A)) = \text{int}(\text{cl}(f(A))) \subseteq \text{cl}(\text{int}(\text{cl}(f(A))))$. This implies $f(A)$ is an IF β OS in Y and hence an IF β^{**} GOS in Y . Thus f is an IF almost β^{**} G open mapping.

Proposition 3.18: Let $f : X \rightarrow Y$ be a mapping. Then f is an IF almost β^{**} G open mapping if for each IFP $p_{(\alpha, \beta)} \in Y$ and for each IF β OS B in X such that $f^{-1}(p_{(\alpha, \beta)}) \in B$, $\beta \text{cl}(f(B))$ is an IF β N of $p_{(\alpha, \beta)} \in Y$.

Proof : Let $p_{(\alpha, \beta)} \in Y$ and let A be an IFROS in X . Then A is an IF β OS in X . By hypothesis $f^{-1}(p_{(\alpha, \beta)}) \in A$, that is $p_{(\alpha, \beta)} \in f(A)$ in Y and $\beta \text{cl}(f(A))$ is an IF β N of $p_{(\alpha, \beta)}$ in Y . Therefore there exists an IF β OS B in Y such that $p_{(\alpha, \beta)} \in B \subseteq \beta \text{cl}(f(A))$. We have $p_{(\alpha, \beta)} \in f(A) \subseteq \beta \text{cl}(f(A))$. Now $B = \bigcup \{p_{(\alpha, \beta)} / p_{(\alpha, \beta)} \in B\} = f(A)$. Therefore $f(A)$ is an IF β OS in Y and hence an IF β^{**} GOS in Y . Thus f is an IF almost β^{**} G open mapping.

Proposition 3.19 : Let $f : X \rightarrow Y$ be a mapping. If f is an IF almost $\beta^{**}G$ closed mapping, then $\beta^{**}gcl(f(A)) \subseteq f(cl(A))$ for every IF β OS A in X .

Proof : Let A be an IF β OS in X . Then $cl(A)$ is an IFRC in X . By hypothesis $f(cl(A))$ is an IF $\beta^{**}GCS$ in Y . Then $\beta^{**}gcl(f(cl(A))) = f(cl(A))$. Now $\beta^{**}gcl(f(A)) \subseteq \beta^{**}gcl(f(cl(A))) = f(cl(A))$. That is $\beta^{**}gcl(f(A)) \subseteq f(cl(A))$.

Corollary 3.20 : Let $f : X \rightarrow Y$ be a mapping. If f is an IF almost $\beta^{**}G$ closed mapping, then $\beta^{**}gcl(f(A)) \subseteq f(cl(A))$ for every IFSOS A in X .

Proof : Since every IFSOS is an IF β OS, the proof directly follows from the Proposition 3.19.

Corollary 3.21 : Let $f : X \rightarrow Y$ be a mapping. If f is an IF almost $\beta^{**}G$ closed mapping, then $\beta^{**}gcl(f(A)) \subseteq f(cl(A))$ for every IFPOS A in X .

Proof : Since every IFPOS is an IF β OS, the proof directly follows from the Proposition 3.19.

Proposition 3.22 : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. If f is an IF almost $\beta^{**}G$ closed mapping, then $\beta^{**}gcl(f(A)) \subseteq f(cl(\beta_{int}(A)))$ for every IF β OS A in X .

Proof : Let A be an IF β OS in X . Therefore $\beta_{int}(A) = A$ and $cl(A)$ is an IFRC in X . By hypothesis, $f(cl(A))$ is an IF $\beta^{**}GCS$ in Y . Then $\beta^{**}gcl(f(A)) \subseteq \beta^{**}gcl(f(cl(A))) = f(cl(A)) = f(cl(\beta_{int}(A)))$.

Corollary 3.23 : Let $f : X \rightarrow Y$ be a mapping. If f is an IF almost $\beta^{**}G$ closed mapping, then $\beta^{**}gcl(f(A)) \subseteq f(cl(\beta_{int}(A)))$ for every IFSOS A in X .

Proof : Since every IFSOS is an IF β OS, the proof directly follows from the Proposition 3.22.

Corollary 3.24 : Let $f : X \rightarrow Y$ be a mapping, If f is an IF almost $\beta^{**}G$ closed mapping, then $\beta^{**}gcl(f(A)) \subseteq f(cl(\beta_{int}(A)))$ for every IFPOS A in X .

Proof : Since every IFPOS is an IF β OS, the proof directly follows from the Proposition 3.22.

Proposition 3.25 : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a bijective mapping. Then the following are equivalent.

- (i) f is an IF almost $\beta^{**}G$ open mapping.
- (ii) f is an IF almost $\beta^{**}G$ closed mapping.
- (iii) f^{-1} is an IF almost $\beta^{**}G$ continuous mapping.

Proof :(i) \Leftrightarrow (ii) is obvious as for a bijective mapping, $f(A^c) = f(A)^c$.

(ii) \Rightarrow (iii) Let $A \subseteq X$ be an IFRCS. Then by hypothesis, $f(A)$ is an IF $\beta^{**}GCS$ in Y . That is $(f^{-1})^{-1}(A)$ is an IF $\beta^{**}GCS$ in Y . This implies f^{-1} is an IF almost $\beta^{**}G$ continuous mapping.

(iii) \Rightarrow (ii) Let $A \subseteq X$ be an IFRCS. Then by hypothesis, $(f^{-1})^{-1}(A)$ is an IF $\beta^{**}GCS$ in Y . That is $f(A)$ is an IF $\beta^{**}GCS$ in Y . Hence f is an IF almost $\beta^{**}G$ closed mapping.

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