Linear Incidence Edge Prime Labeling of Some Direct Triple Triangular and Double Quadrilateral Snake Graphs

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ABSTRACT

A graph G = (V,E) with p vertices and q edges is said to admit linear labeling, if there exist a bijection f : V → {0,1,2…,p-1}such that the induced function f_{lpl}^{*} : E → N defined by $f_{lpl}^{*}(uv)$ = 2f(v)+f(u) for every direct edge from u to v is one to one. A graph which admits linear labeling is called linear labeled graph. The greatest common incidence number ($gcin$) of a vertex of in degree greater than one is the greatest common divisor of the edges incident on that vertex. A linear labeled graph is called a linear incidence edge prime graph, if the gcin of each vertex of in degree greater than one is one. Major findings of this paper are direct triple triangular snake graph, direct alternate triple triangular snake graph and direct alternate double quadrilateral snake graph are linear incidence edge prime graphs.

Keywords: Graph labeling, linear, incidence edge prime labeling, prime graphs, di graphs, snake graphs.

1.INTRODUCTION

The concept of incidence edge prime labeling of graphs was introduced by Sunoj B S and Mathew Varkey T K in [5]. In this paper we prove that some direct triple triangular snake graph, direct alternate triple triangular snake graph, double quadrilateral snake graph and direct alternate double quadrilateral snake graph admit linear incidence edge prime labeling.

This paper contains three sections in which section I contains introduction to linear prime labeling, section II contains preliminaries and notations and section III contains main results and illustrations.

II. PRELIMINARIES AND NOTATIONS

In this paper we consider only those graphs which are simple, finite and direct. Graph is denoted by G, vertex set is denoted by V, edge set is denoted by E , p is the number of vertices of the graph G and q is the number of edges of the graph G. | V(G) | = number of vertices of graph G, | E(G) | = number of edges of graph G. Here the direction of the edge is from u to v if $f(v_i) < f(v_j)$.

Definition 2.1 If the vertices and edges of the graph are assigned values subject to certain conditions is known as graph labeling.

Definition 2.2 The greatest common incidence number of a vertex is defined as the greatest common divisor [1] of the labels of the edges incident on that vertex.

Definition 2.3 Let G = (V(G),E(G)) be a graph with p vertices and q edges . Define a bijection f : V(G) → {0,1,2,…, p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{lpl}^{*} : E(G) → set of natural numbers N by $f_{lpl}^{*}(uv) = 2f(v) + f(u)$ .The induced function f_{lpl}^{*} is said to be linear prime labeling, if for each vertex of in degree at least 2, the ***gcin*** of the labels of the incident edges is 1.

Definition 2.4 A Graph is said to be a di graph if each edge of G has a direction**.**

Definition 2.5 In-degree of a vertex in a digraph is the number of edges incident at that vertex.

Definition 2.6 If the gcin of each vertex of degree greater than one of a labeled graph is one, then the labeling is called incidence edge prime labeling.

Definition 2.7 A di graph which admits linear prime labeling is called linear prime di graph.

Definition 2.8 A triple triangular snake $T(T_n)$ is obtained from a path u_1u_2---u_n by joining u_i and $u_{i+1}$to three new vertices v_i, w_i and t_i for 1 ≤ i ≤ n-1.That is a $T(T_n)$ consists of three triangular snakes that have a common path. An alternate triple triangular snake $A[T(T_n)]$ consists of three alternate triangular snakes that have a common path.

Definition 2.9 A double quadrilateral snake D(Q_n) is obtained from a path u_1u_2---u_n by joining u_i to a_i, b_i and $u_{i+1}$to c_i, d_i then join a_i to c_i and b_i to d_i for $1 \le i \le n-1$.

III. MAIN RESULTS

Theorem 3.1 Direct triple triangular snake graph $\{T(T_n)\}^{\rightarrow}$ (n >2) admits linear incidence edge prime labeling.

Proof: Let G = $\{T(T_n)\}^{\rightarrow}$ and let $u_1,u_2,\ldots,u_{2n-1}$ are the vertices of G.

Here |V(G)| = 4n-3 and |E(G)| = 7n-7.

Define a function f : V → {0,1,2,…,4n-4} by

$f(v_j) = j-1$, j = 1,2,…,4n-3.

From the definition itself it is clear that f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{lpl} is defined as follows

$f^*_{lpl}(v_{4j-3}\ v_{4j-2})$	= 12j-10,	j = 1,2,…,n-1.
$f^*_{lpl}(v_{4j-3}\ v_{4j-1})$	= 12j-8,	j = 1,2,…,n-1.
$f^*_{lpl}(v_{4j-3}\ v_{4j})$	= 12j-6,	j = 1,2,…,n-1.
$f^*_{lpl}(v_{4j-3}\ v_{4j+1})$	= 12j-4,	j = 1,2,…,n-1.
$f^*_{lpl}(v_{4j-2}\ v_{4j+1})$	= 12j-3,	j = 1,2,…,n-1.
$f^*_{lpl}(v_{4j-1}\ v_{4j+1})$	= 12j-2,	j = 1,2,…,n-1.
$f^*_{lpl}(v_{4j}\ v_{4j+1})$	= 12j-1,	j = 1,2,…,n-1.

Here the edge labels are arranged in strictly increasing order. Hence f^*_{lpl} is clearly an injection

gcin of (v_{4j+1}) = gcd of $\{f^*_{lpl}(v_{4j-2}\ v_{4j+1}), f^*_{lpl}(v_{4j-1}\ v_{4j+1})\}$
= gcd of {12j-3, 12j-2}
= 1, j = 1,2,…,n-1.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence $\{T(T_n)\}^{\rightarrow}$,admits linear incidence edge prime labeling.

Example 3.1 G = $\{T(T_4)\}^{\rightarrow}$

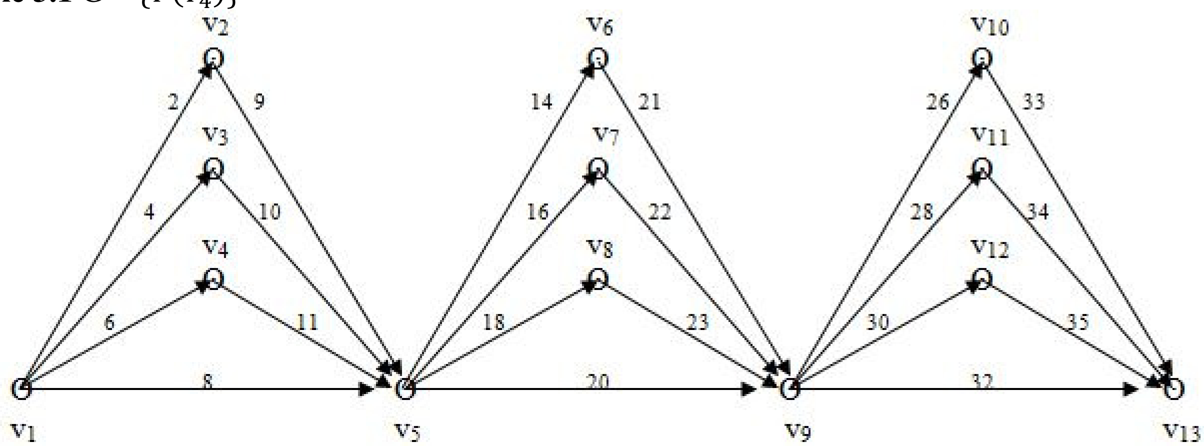

fig -3.1

Theorem 3.2 Direct alternate triple triangular snake graph $A\{T(T_n)\}^{\rightarrow}$ (n >3) admits linear incidence edge prime labeling, if n is odd and triple triangles start from the first vertex.

Proof: Let G = $A\{T(T_n)\}^{\rightarrow}$ and let $u_1,u_2,\ldots,u_{\frac{5n-3}{2}}$ are the vertices of G.

Here |V(G)| = $\frac{5n-3}{2}$ and |E(G)| = 4n-4.

Define a function f : V → $\{0,1,2,\ldots,\frac{5n-5}{2}\}$ by

$f(v_j) = j-1$, $j = 1,2,\ldots,\frac{5n-3}{2}$.

From the definition itself it is clear that f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{5j-4}\, v_{5j-3}) = 15j-13,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

$f_{lpl}^*(v_{5j-4}\, v_{5j-2}) = 15j-11,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

$f_{lpl}^*(v_{5j-4}\, v_{5j-1}) = 15j-9,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

$f_{lpl}^*(v_{5j-4}\, v_{5j}) = 15j-7,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

$f_{lpl}^*(v_{5j-3}\, v_{5j}) = 15j-6,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

$f_{lpl}^*(v_{5j-2}\, v_{5j}) = 15j-5,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

$f_{lpl}^*(v_{5j-1}\, v_{5j}) = 15j-4,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

$f_{lpl}^*(v_{5j}\, v_{5j+1}) = 15j-1,\quad j = 1,2,\ldots,\frac{n-1}{2}$.

Here the edge labels are arranged in strictly increasing order. Hence f_{lpl}^* is clearly an injection

gcin of (v_{5j}) $= \text{gcd of } \{f_{lpl}^*(v_{5j-2}\, v_{5j}), f_{lpl}^*(v_{5j-1}\, v_{5j})\}$

$= \text{gcd of } \{15j-5, 15j-4\}$

$= 1,\quad j = 1,2,\ldots,\frac{n-1}{2}$..

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence $A\{T(T_n)\}^{\rightarrow}$ admits linear incidence edge prime labeling.

Example 3.2 G = $A\{T(T_5)\}^{\rightarrow}$

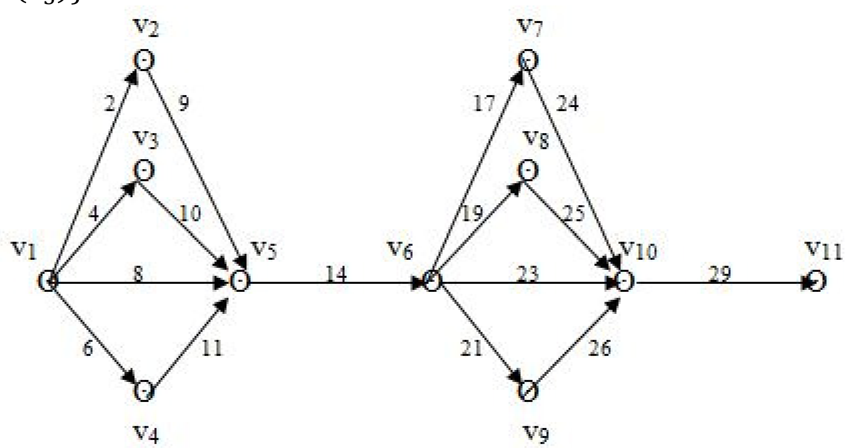

fig – 3.2

Theorem 3.3 Direct alternate triple triangular snake graph $A\{T(T_n)\}^{\rightarrow}$ (n >3) admits linear incidence edge prime labeling, if n is even and triple triangles start from the first vertex.

Proof: Let G = $A\{T(T_n)\}^{\rightarrow}$ and let $u_1,u_2,\ldots,u_{\frac{5n}{2}}$ are the vertices of G.

Here $|V(G)| = \frac{5n}{2}$ and $|E(G)| = 4n-1$.

Define a function $f : V \rightarrow \{0,1,2,\ldots,\frac{5n-2}{2}\}$ by

$f(v_j) = j-1$, $j = 1,2,\ldots,\frac{5n}{2}$.

From the definition itself it is clear that f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^* is defined as follows

$f_{lpl}^*(v_{5j-4}\, v_{5j-3}) = 15j-13,\quad j = 1,2,\ldots,\frac{n}{2}$.

$f_{lpl}^{*}(v_{5j-4}\ v_{5j-2})$ = 15j-11, j = 1,2,…,$\frac{n}{2}$.
$f_{lpl}^{*}(v_{5j-4}\ v_{5j-1})$ = 15j-9, j = 1,2,…,$\frac{n}{2}$.
$f_{lpl}^{*}(v_{5j-4}\ v_{5j})$ = 15j-7, j = 1,2,…,$\frac{n}{2}$.
$f_{lpl}^{*}(v_{5j-3}\ v_{5j})$ = 15j-6, j = 1,2,…,$\frac{n}{2}$.
$f_{lpl}^{*}(v_{5j-2}\ v_{5j})$ = 15j-5, j = 1,2,…,$\frac{n}{2}$.
$f_{lpl}^{*}(v_{5j-1}\ v_{5j})$ = 15j-4, j = 1,2,…,$\frac{n}{2}$.
$f_{lpl}^{*}(v_{5j}\ v_{5j+1})$ = 15j-1, j = 1,2,…,$\frac{n-2}{2}$.

Here the edge labels are arranged in strictly increasing order. Hence f_{lpl}^{*} is clearly an injection

gcin of (v_{5j}) = gcd of $\{f_{lpl}^{*}(v_{5j-2}\ v_{5j}), f_{lpl}^{*}(v_{5j-1}\ v_{5j})\}$
= gcd of {15j-5, 15j-4}
= 1, j = 1,2,…, $\frac{n}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence $A\{T(T_n)\}^{\rightarrow}$ admits linear incidence edge prime labeling.

Example 3.3 G = $A\{T(T_4)\}^{\rightarrow}$

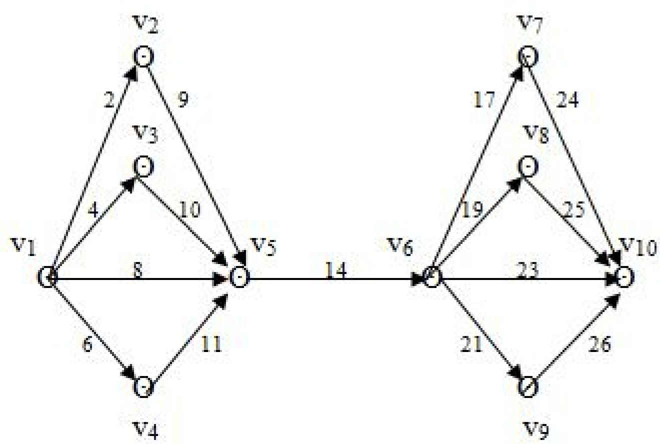

fig – 3.3

Theorem 3.4 Direct alternate triple triangular snake graph $A\{T(T_n)\}^{\rightarrow}$ (n >3) admits linear incidence edge prime labeling, if n is even and triple triangles start from the second vertex.

Proof: Let G = $A\{T(T_n)\}^{\rightarrow}$ and let $u_1, u_2, \ldots, u_{\frac{5n-6}{2}}$ are the vertices of G.

Here |V(G)| = $\frac{5n-6}{2}$ and |E(G)| = 4n-7.

Define a function f : V → {0,1,2,…, $\frac{5n-8}{2}$ } by

$f(v_j)$ = j-1 , j = 1,2,…, $\frac{5n-6}{2}$.

From the definition itself it is clear that f is a bijection.

For the vertex labeling f, the induced edge labeling f_{lpl}^{*} is defined as follows

$f_{lpl}^{*}(v_{5j-4}\ v_{5j-3})$ = 15j-13, j = 1,2,…,$\frac{n}{2}$.
$f_{lpl}^{*}(v_{5j-3}\ v_{5j-2})$ = 15j-10, j = 1,2,…,$\frac{n-2}{2}$.
$f_{lpl}^{*}(v_{5j-3}\ v_{5j-1})$ = 15j-8, j = 1,2,…,$\frac{n-2}{2}$.
$f_{lpl}^{*}(v_{5j-3}\ v_{5j})$ = 15j-6, j = 1,2,…,$\frac{n-2}{2}$.

$f_{lpl}^{*}(v_{5j-3}\ v_{5j+1})$ = 15j-4, j = 1,2,…,$\frac{n-2}{2}$.
$f_{lpl}^{*}(v_{5j-2}\ v_{5j+1})$ = 15j-3, j = 1,2,…,$\frac{n-2}{2}$.
$f_{lpl}^{*}(v_{5j-1}\ v_{5j+1})$ = 15j-2, j = 1,2,…,$\frac{n-2}{2}$.
$f_{lpl}^{*}(v_{5j}\ v_{5j+1})$ = 15j-1, j = 1,2,…,$\frac{n-2}{2}$.

Here the edge labels are arranged in strictly increasing order. Hence f_{lpl}^{*} is clearly an injection

gcin of (v_{5j+1}) = gcd of $\{f_{lpl}^{*}(v_{5j-2}\ v_{5j+1}), f_{lpl}^{*}(v_{5j-1}\ v_{5j+1})\}$
= gcd of {15j-3, 15j-2}
= 1, j = 1,2,…, $\frac{n-2}{2}$.

Each vertex of in degree greater than one have ***gcin*** one.
Hence $A\{T(T_n)\}^{\rightarrow}$ admits linear incidence edge prime labeling.

Example 3.4 G = $A\{T(T_6)\}^{\rightarrow}$.

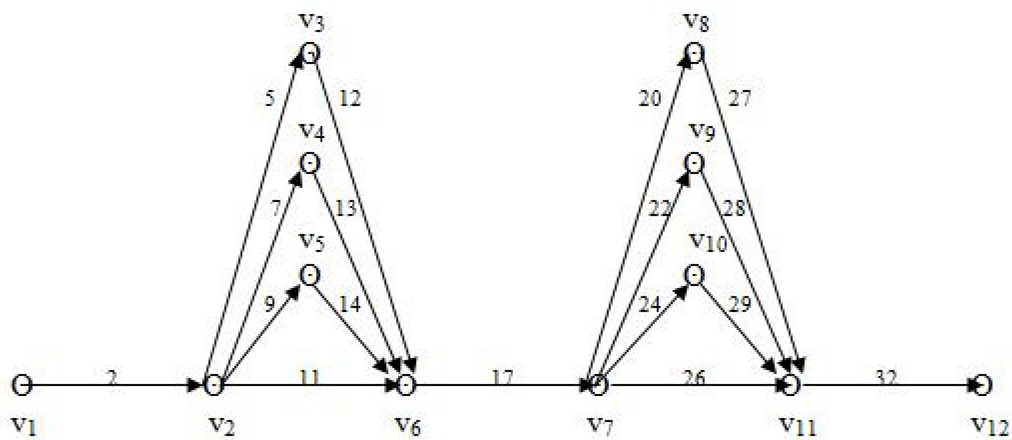

fig – 3.4

Theorem 3.5 Direct alternate double quadrilateral snake graph $A\{D(Q_n)\}^{\rightarrow}$ (n >2) admits linear incidence edge prime labeling, if n is odd and double quadrilateral start from the first vertex.

Proof: Let G =$A\{D(Q_n)\}^{\rightarrow}$ and let $u_1, u_2, \ldots, u_{3n-2}$ are the vertices of G.
Here |V(G)| = 3n-2 and |E(G)| = 4n-4.
Define a function f : V → {0,1,2,…,3n-3 } by

$f(v_j)$ = j-1 , j = 1,2,…,3n-2.

From the definition itself it is clear that f is a bijection.
Edge labels are defined as

$f_{lpl}^{*}(v_{6j-5}\ v_{6j-4})$ = 18j-16, j = 1,2,…,$\frac{n-1}{2}$.
$f_{lpl}^{*}(v_{6j-5}\ v_{6j-3})$ = 18j-14, j = 1,2,…,$\frac{n-1}{2}$.
$f_{lpl}^{*}(v_{6j-4}\ v_{6j-2})$ = 18j-11, j = 1,2,…,$\frac{n-1}{2}$.
$f_{lpl}^{*}(v_{6j-3}\ v_{6j-2})$ = 18j-10, j = 1,2,…,$\frac{n-1}{2}$.
$f_{lpl}^{*}(v_{6j-3}\ v_{6j-1})$ = 18j-8, j = 1,2,…,$\frac{n-1}{2}$.
$f_{lpl}^{*}(v_{6j-2}\ v_{6j})$ = 18j-5, j = 1,2,…,$\frac{n-1}{2}$.
$f_{lpl}^{*}(v_{6j-1}\ v_{6j})$ = 18j-4, j = 1,2,…,$\frac{n-1}{2}$.
$f_{lpl}^{*}(v_{6j-2}\ v_{6j+3})$ = 18j+1, j = 1,2,…,$\frac{n-3}{2}$.

$f_{lpl}^{*}(v_{3n-5}\, v_{3n-2})$ = 9n-12.

Here the edge labels are arranged in strictly increasing order. Hence f_{lpl}^{*} is clearly an injection

gcin of (v_{6j-2}) = gcd of $\{f_{lpl}^{*}(v_{6j-4}\, v_{6j-2}), f_{lpl}^{*}(v_{6j-3}\, v_{6j-2})\}$
= gcd of {18j-11, 18j-10}
= 1, j = 1,2,…, $\frac{n-1}{2}$.

gcin of (v_{6j}) = gcd of $\{f_{lpl}^{*}(v_{6j-2}\, v_{6j}), f_{lpl}^{*}(v_{6j-1}\, v_{6j})\}$
= gcd of {18j-5, 18j-4}
= 1, j = 1,2,…, $\frac{n-1}{2}$.

gcin of (v_{6j+3}) = gcd of $\{f_{lpl}^{*}(v_{6j-2}\, v_{6j+3}), f_{lpl}^{*}(v_{6j+1}\, v_{6j+3})\}$
= gcd of {18j+1, 18j+4}
= gcd of {3, 18j+1}
= 1, j = 1,2,…, $\frac{n-3}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence $A\{D(Q_n)\}^{\rightarrow}$ admits linear incidence edge prime labeling.

Example 3.5 G = $A\{D(Q_5)\}^{\rightarrow}$

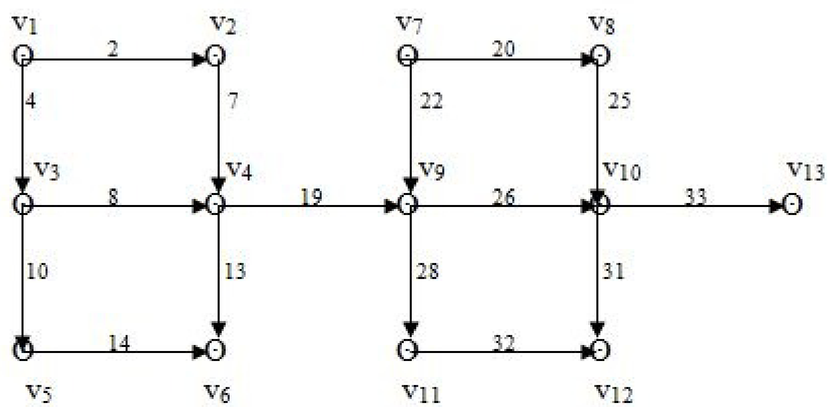

fig – 3.5

Theorem 3.6 Direct alternate double quadrilateral snake graph $A\{D(Q_n)\}^{\rightarrow}$ (n >3) admits linear incidence edge prime labeling, if n is even and double quadrilateral start from the first vertex.

Proof: Let G =$A\{D(Q_n)\}^{\rightarrow}$ and let $u_1, u_2, \ldots, u_{3n}$ are the vertices of G.

Here |V(G)| = 3n and |E(G)| = 4n-1.

Define a function f : V → {0,1,2,…,3n-1 } by

$f(v_j) = j-1$, j = 1,2,…,3n.

From the definition itself it is clear that f is a bijection.

Edge labels are defined as

$f_{lpl}^{*}(v_{6j-5}\, v_{6j-4})$ = 18j-16, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-5}\, v_{6j-3})$ = 18j-14, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-4}\, v_{6j-2})$ = 18j-11, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-3}\, v_{6j-2})$ = 18j-10, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-3}\, v_{6j-1})$ = 18j-8, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-2}\, v_{6j})$ = 18j-5, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-1}\, v_{6j})$ = 18j-4, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-2}\, v_{6j+3})$ = 18j+1, j = 1,2,…,$\frac{n-2}{2}$.

Here the edge labels are arranged in strictly increasing order. Hence f_{lpl}^{*} is clearly an injection

gcin of (v_{6j-2}) = gcd of $\{f_{lpl}^{*}(v_{6j-4}\, v_{6j-2}), f_{lpl}^{*}(v_{6j-3}\, v_{6j-2})\}$
= gcd of {18j-11, 18j-10}
= 1, j = 1,2,…, $\frac{n}{2}$.

gcin of (v_{6j}) = gcd of $\{f_{lpl}^{*}(v_{6j-2}\, v_{6j}), f_{lpl}^{*}(v_{6j-1}\, v_{6j})\}$
= gcd of {18j-5, 18j-4}
= 1, j = 1,2,…, $\frac{n}{2}$.

gcin of (v_{6j+3}) = gcd of $\{f_{lpl}^{*}(v_{6j-2}\, v_{6j+3}), f_{lpl}^{*}(v_{6j+1}\, v_{6j+3})\}$
= gcd of {18j+1, 18j+4}
= gcd of {3, 18j+1}
= 1, j = 1,2,…, $\frac{n-2}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.
Hence $A\{D(Q_n)\}^{\rightarrow}$ admits linear incidence edge prime labeling.

Example 3.6 G = $A\{D(Q_4)\}^{\rightarrow}$

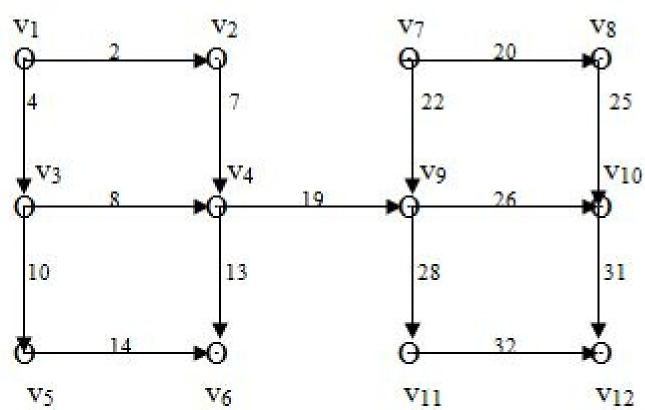

fig – 3.6

Theorem 3.7 Direct alternate double quadrilateral snake graph $A\{D(Q_n)\}^{\rightarrow}$ (n >3) admits linear incidence edge prime labeling, if n is even and double triangle start from the second vertex.
Proof: Let G =$A\{D(Q_n)\}^{\rightarrow}$ and let $u_1, u_2, \ldots, u_{3n-4}$ are the vertices of G.
Here |V(G)| = 3n-4 and |E(G)| = 4n-7.
Define a function f : V → {0,1,2,…,3n-5 } by

$f(v_j) = j-1$, j = 1,2,…,3n-4.

From the definition itself it is clear that f is a bijection.
Edge labels are defined as

$f_{lpl}^{*}(v_{6j-5}\, v_{6j-4})$ = 18j-16, j = 1,2,…,$\frac{n}{2}$.

$f_{lpl}^{*}(v_{6j-4}\, v_{6j-3})$ = 18j-13, j = 1,2,…,$\frac{n-2}{2}$.

$f_{lpl}^{*}(v_{6j-3}\, v_{6j-2})$ = 18j-10, j = 1,2,…,$\frac{n-2}{2}$.

$f_{lpl}^{*}(v_{6j-4}\, v_{6j-1})$ = 18j-9, j = 1,2,…,$\frac{n-2}{2}$.

$f_{lpl}^{*}(v_{6j-4}\, v_{6j+1})$ = 18j-5, j = 1,2,…,$\frac{n-2}{2}$.

$f^*_{lpl}(v_{6j-1}\, v_{6j})$ = 18j-4, j = 1,2,…,$\frac{n-2}{2}$.

$f^*_{lpl}(v_{6j-2}\, v_{6j+1})$ = 18j-3, j = 1,2,…,$\frac{n-2}{2}$.

$f^*_{lpl}(v_{6j}\, v_{6j+1})$ = 18j-1, j = 1,2,…,$\frac{n-2}{2}$.

Here the edge labels are arranged in strictly increasing order. Hence f^*_{lpl} is clearly an injection

gcin of (v_{6j+1}) = gcd of $\{f^*_{lpl}(v_{6j-4}\, v_{6j+1}), f^*_{lpl}(v_{6j-2}\, v_{6j+1})\}$

= gcd of {18j-5, 18j-3}

= gcd of {2, 18j-5}

= 1, j = 1,2,…, $\frac{n-2}{2}$.

So, ***gcin*** of each vertex of in degree greater than one is 1.

Hence $A\{D(Q_n)\}^{\rightarrow}$ admits linear incidence edge prime labeling.

Example 3.7 G = $A\{D(Q_4)\}^{\rightarrow}$

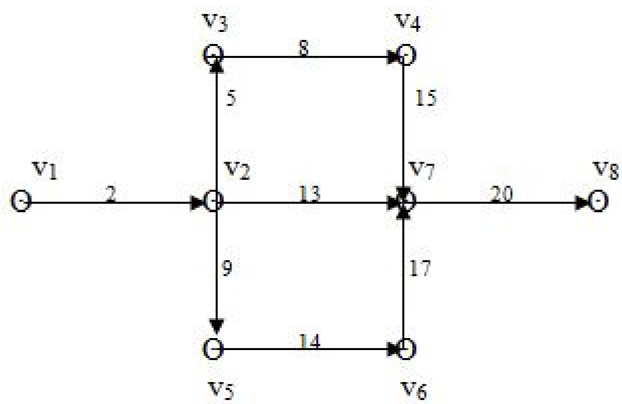

fig – 3.7

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