BIPOLAR VALUED MULTI I-FUZZY SUBSEMIRINGS OF A SEMIRING BY HOMOMORPHISM AND ANTI HOMOMORPHISM

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ABSTRACT: In this paper, bipolar valued multi I-fuzzy subsemiring of a semiring is studied by homomorphism and anti homomorphism and some properties are discussed. These properties are useful to further research.

KEY WORDS: Interval valued fuzzy subset, bipolar valued fuzzy subset, bipolar valued I-fuzzy subset, bipolar valued multi I-fuzzy subset, bipolar valued multi I-fuzzy subsemiring, bipolar valued multi I-fuzzy normal subsemiring, image and pre image.

INTRODUCTION: In 1965, Zadeh [16] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc. Lee [10] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [10, 11]. Fuzzy subgroup was introduced by Azriel Rosenfeld [5]. Anitha.M.S., et.al.[1, 2] defined as bipolar valued fuzzy subgroups of a group and homomorphism and anti homomorphism. After that K.Murugalingam and K.Arjunan[12] have discussed about interval valued fuzzy subsemiring of a semiring and then bipolar valued multi fuzzy subsemirings of a semiring have been introduced by Yasodara.B and KE.Sathappan[15]. Kodimalar.K, et.al.[8] have defined the bipolar valued multi I-fuzzy subsemirings of a semiring. Here, the concept of bipolar valued multi I-fuzzy subsemiring of a semiring is studied by homomorphism and anti homomorphism and estaiblished some results.

1.PRELIMINARIES:

Definition 1.1. [16] Let X be any nonempty set. A mapping $[M] : X \to D[0, 1]$ is called an interval valued fuzzy subset (I-fuzzy subset) of X, where D[0, 1] denotes the family of all closed subintervals of [0, 1] and $[M](x) = [M^-(x), M^+(x)]$, for all x in X, where M^- and M^+ are fuzzy subsets of X such that $M^-(x) \leq M^+(x)$, for all x in X. Thus $M^-(x)$ is an interval (a closed subset of [0, 1]) and not a number from the interval [0, 1] as in the case of fuzzy subset. Note that [0] = [0, 0] and [1] = [1, 1].

Definition 1.2. [10] A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) >/ x\in X\}$, where $A^+ : X\to [0, 1]$ and $A^- : X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.3. A = { < a, 0.5, −0.2 >, < b, 0.7, −0.8 >, < c, 0.4, −0.9 >} is a bipolar valued fuzzy subset of X = {a, b, c }

Definition 1.4. [15] A bipolar valued multi fuzzy set (BVMFS) A in X is defined as an object of the form $A = \{ < x, A_1^+(x), A_2^+(x), \ldots, A_n^+(x), A_1^-(x), A_2^-(x), \ldots, A_n^-(x) > / x\in X \}$, where $A_i^+ : X\to[0, 1]$ and $A_i^- : X\to[-1, 0]$ for all i = 1, 2, …, n. The positive membership degrees $A_i^+(x)$ denote the satisfaction degrees of an element x to the property corresponding to a bipolar valued multi fuzzy set A and the negative membership degrees $A_i^-(x)$ denote the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A.

Example 1.5. A = { < a, 0.2, 0,4, 0.7, −0.4, −0.5, −0.9 >, < b, 0.2, 0.6, 0.2, −0.9, −0.5, −0.1 >, < c, 0.2, 0.3, 0.4, −0.2, −0.9, −0.3 > } is a bipolar valued multi fuzzy subset of X = { a, b, c }

Definition 1.6. [8] A bipolar interval valued fuzzy subset (bipolar valued I-fuzzy subset) [A] in X is defined as an object of the form $[A] = \{ < x, [A]^+(x), [A]^-(x) > / x\in X \}$, where $[A]^+ : X\to D[0, 1]$ and $[A]^- : X\to D[-1, 0]$, where D[0, 1] denotes the family of all closed subintervals of [0, 1] and D[−1, 0] denotes the family of all closed subintervals of [−1, 0]. The positive interval membership degree $[A]^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued I-fuzzy subset [A] and the negative interval membership degree $[A]^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued I-fuzzy subset [A]

Example 1.7. [A] = { < a, [0.6, 0,7], [−0.3, −0.2] >, < b, [0.2, 0.4], [−0.6, −0.3] >, < c, [0.3, 0.6], [−0.5, −0.2] > } is a bipolar valued I-fuzzy subset of X = {a, b, c }

Definition 1.8. [8] A bipolar interval valued multi fuzzy subset (bipolar valued multi I-fuzzy subset) [A] in X is defined as an object of the form $[A] = \{ < x, [A]_1^+(x), [A]_2^+(x), \ldots, [A]_n^+(x), [A]_1^-(x), [A]_2^-(x), \ldots, [A]_n^-(x) > / x\in X \}$, where for each i, $[A]_i^+ : X\to D[0, 1]$ and $[A]_i^- : X\to D[-1, 0]$, where D[0, 1] denotes the family of all closed subintervals of [0, 1] and D[−1, 0] denotes the family of all closed subintervals of [−1, 0]. The positive interval membership degrees $[A]_i^+(x)$ denotes the satisfaction degrees of an element x to the property corresponding to a bipolar valued multi I-fuzzy subset [A] and the negative interval membership degrees $[A]_i^-(x)$ denotes the satisfaction degrees of an element x to some implicit counter-property corresponding to a bipolar valued multi I-fuzzy subset [A]. Note that [0] = ([0, 0], [0, 0], …, [0, 0]), [1] = ([1, 1], [1, 1] , …, [1, 1]) and [−1] = ([−1, −1], [−1, −1], …, [−1, −1]). It is denoted as $[A] = \langle [A]_1^+, [A]_2^+, \ldots, [A]_n^+, [A]_1^-, [A]_2^-, \ldots, [A]_n^- \rangle$.

Example 1.9. [A] = { < a, [0.4, 0,6], [0.4, 0,5], [0.4, 0,8], [−0.3, −0.2], [−0.3, −0.1], [−0.5, −0.2] >, < b, [0.2, 0.4], [0.2, 0.5], [0.2, 0.6], [−0.7, −0.2], [−0.6, −0.2], [−0.5, −0.2] >, < c, [0.3, 0.6], [0.3, 0.7], [0.3, 0.8], [−0.4, −0.2], [−0.7, −0.3], [−0.9, −0.3] > } is a bipolar valued multi I-fuzzy subset of X = {a, b, c }

Definition 1.10. [8] Let R be a semiring. A bipolar valued multi I-fuzzy subset $[A] = \langle [A]_1^+, [A]_2^+, \ldots, [A]_n^+, [A]_1^-, [A]_2^-, \ldots, [A]_n^- \rangle$ of R is said to be a bipolar valued multi I-fuzzy subsemiring of R (BVMIFSSR) if the following conditions are satisfied for each i,

(i) $[A]_i^+(x+y) \geq \text{rmin}\{ [A]_i^+(x), [A]_i^+(y) \}$

(ii) $[A]_i^+(xy) \geq \text{rmin}\{ [A]_i^+(x), [A]_i^+(y) \}$

(iii) $[A]_i^-(x+y) \leq \text{rmax}\{ [A]_i^-(x), [A]_i^-(y) \}$

(iv) $[A]_i^-(xy) \leq \text{rmax}\{ [A]_i^-(x), [A]_i^-(y) \}$ for all x and y in R.

Example 1.11. Let $R = Z_3$ = { 0, 1, 2 } be a semiring with respect to the ordinary addition and multiplication. Then [A] = { < 0, [0.5, 0.6], [0.6, 0.7], [0.7, 0.8], [− 0.6, − 0.5], [− 0.7, − 0.6], [− 0.8, − 0.7] >, < 1, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7], [− 0.5, − 0.4], [− 0.6, − 0.5] , [− 0.7, −0.6] >, < 2, [0.4, 0.5], [0.5, 0.6], [0.6, 0.7], [− 0.5, − 0.4], [− 0.6, − 0.5] , [− 0.7, − 0.6] > } is a bipolar valued multi I-fuzzy subsemiring of R.

Definition 1.12. Let R be a semiring. A bipolar valued multi I-fuzzy subsemiring [A] of R is said to be a bipolar valued multi I-fuzzy normal subsemiring of R if for each i, $[A]_i^+(x+y) = [A]_i^+(y+x)$, $[A]_i^+(xy) = [A]_i^+(yx)$, $[A]_i^-(x+y) = [A]_i^-(y+x)$ and $[A]_i^-(xy) = [A]_i^-(yx)$ for all x and y in R.

Definition 1.13. [2] Let R and R^l be any two semirings. Then the function $f: R \to R^l$ is said to be an antihomomorphism if f(x+y) = f(y)+f(x) and f(xy) = f(y)f(x) for all x and y in R.

Definition 1.14. Let X and X^l be any two sets. Let $f : X \to X^l$ be any function and let [A] be a bipolar valued multi I-fuzzy subset in X, [V] be a bipolar valued multi I-fuzzy subset in $f(X) = X^l$, defined for each i, by $[V]_i^+(y) = r\sup_{x\in f^{-1}(y)} [A]_i^+(x)$ and $[V]_i^-(y) = r\inf_{x\in f^{-1}(y)} [A]_i^-(x)$, for all x in X and y in X^l. [A] is called a preimage of [V] under f and is defined as $[A]_i^+(x) = [V]_i^+(f(x))$, $[A]_i^-(x) = [V]_i^-(f(x))$ for all x in X and is denoted by $f^{-1}([V])$.

2. SOME THEOREMS:

Theorem 2.1. Let R and R^{l} be any two semirings. The homomorphic image of a bipolar valued multi I-fuzzy subsemiring of R is a bipolar valued multi I-fuzzy subsemiring of R^{l}.

Proof: Let f : $R\rightarrow R^{l}$ be a homomorphism. Let [V] = f([A]) where [A] is a bipolar valued multi I-fuzzy subsemiring of R. We have to prove that [V] is a bipolar valued multi I-fuzzy subsemiring of R^{l}. Now for f(x), f(y) in R^{l}, for each i, $[V]_i^+(f(x)+f(y)) = [V]_i^+(f(x+y)) \geq [A]_i^+(x+y) \geq rmin\{[A]_i^+(x), [A]_i^+(y)\} = rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$ which implies that $[V]_i^+(f(x)+f(y)) \geq rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$. And $[V]_i^+(f(x)f(y)) = [V]_i^+(f(xy)) \geq [A]_i^+(xy) \geq rmin\{[A]_i^+(x), [A]_i^+(y)\} = rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$ which implies that $[V]_i^+(f(x)f(y)) \geq rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$. Also $[V]_i^-(f(x)+f(y)) = [V]_i^-(f(x+y)) \leq [A]_i^-(x+y) \leq rmax\{[A]_i^-(x), [A]_i^-(y)\} = rmax\{[V]_i^-(f(x)), [V]_i^-(f(y))\}$ which implies that $[V]_i^-(f(x)+f(y)) \leq rmax\{[V]_i^-(f(x)), [V]_i^-(f(y))\}$. And $[V]_i^-(f(x)f(y)) = [V]_i^-(f(xy)) \leq [A]_i^-(xy) \leq rmax\{[A]_i^-(x), [A]_i^-(y)\} = rmax\{[V]_i^-(f(x)), [V]_i^-(f(y))\}$ which implies that $[V]_i^-(f(x)f(y)) \leq rmax\{[V]_i^-(f(x)), [V]_i^-(f(y))\}$. Hence [V] is a bipolar valued multi I-fuzzy subsemiring of R^{l}.

2.2 Theorem: Let R and R^{l} be any two semirings. The homomorphic preimage of a bipolar valued multi I-fuzzy subsemiring of R^{l} is a bipolar valued multi I-fuzzy subsemiring of R.

Proof: Let f : $R \rightarrow R^{l}$ be a homomorphism. Let [V] = f([A]) where [V] is a bipolar valued multi I-fuzzy subsemiring of R^{l}. We have to prove that [A] is a bipolar valued multi I-fuzzy subsemiring of R. Let x and y in R. For each i, $[A]_i^+(x+y) = [V]_i^+(f(x+y)) = [V]_i^+(f(x)+f(y)) \geq rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\} = rmin\{[A]_i^+(x), [A]_i^+(y)\}$ which implies that $[A]_i^+(x+y) \geq rmin\{[A]_i^+(x), [A]_i^+(y)\}$. And $[A]_i^+(xy) = [V]_i^+(f(xy)) = [V]_i^+(f(x)f(y)) \geq rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\} = rmin\{[A]_i^+(x), [A]_i^+(y)\}$ which implies that $[A]_i^+(xy) \geq rmin\{[A]_i^+(x), [A]_i^+(y)\}$. Also $[A]_i^-(x+y) = [V]_i^-(f(x+y)) = [V]_i^-(f(x)+f(y)) \leq rmax\{[V]_i^-(f(x)), [V]_i^-(f(y))\} = rmax\{[A]_i^-(x), [A]_i^-(y)\}$ which implies that $[A]_i^-(x+y) \leq rmax\{[A]_i^-(x), [A]_i^-(y)\}$. And $[A]_i^-(xy) = [V]_i^-(f(xy)) = [V]_i^-(f(x)f(y)) \leq rmax\{[V]_i^-(f(x)), [V]_i^-(f(y))\} = rmax\{[A]_i^-(x), [A]_i^-(y)\}$ which implies that $[A]_i^-(xy) \leq rmax\{[A]_i^-(x), [A]_i^-(y)\}$. Hence [A] is a bipolar valued multi I-fuzzy subsemiring of R.

2.3 Theorem: Let R and R^{l} be any two semirings. The antihomomorphic image of a bipolar valued multi I-fuzzy subsemiring of R is a bipolar valued multi I-fuzzy subsemiring of R^{l}.

Proof: Let f : $R \rightarrow R^{l}$ be an antihomomorphism. Let [V] = f([A]) where [A] is a bipolar valued multi I-fuzzy subsemiring of R. We have to prove that [V] is a bipolar valued multi I-fuzzy subsemiring of R^{l}. Now for f(x), f(y) in R^{l}, for each i, $[V]_i^+(f(x)+f(y)) = [V]_i^+(f(y+x)) \geq [A]_i^+(y+x) \geq rmin\{[A]_i^+(x), [A]_i^+(y)\} = rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$ which implies that $[V]_i^+(f(x)+f(y)) \geq rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$. And $[V]_i^+(f(x)f(y)) = [V]_i^+(f(yx)) \geq [A]_i^+(yx) \geq rmin\{[A]_i^+(x), [A]_i^+(y)\} = rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$ which implies that $[V]_i^+(f(x)f(y)) \geq rmin\{[V]_i^+(f(x)), [V]_i^+(f(y))\}$. Also $[V]_i^-(f(x)+f(y)) = [V]_i^-(f(y+x)) \leq [A]_i^-(y+x) \leq rmax\{[A]_i^-(x), [A]_i^-(y)\} = rmax\{[V]_i^-(f(x)), [V]_i^-(f(y))\}$ which implies that $[V]_i^-(f(x)+f(y)) \leq$

$\text{rmax}\{ [V]_i^-(f(x)), [V]_i^-(f(y)) \}$. And $[V]_i^-(f(x)f(y)) = [V]_i^-(f(yx)) \leq [A]_i^-(yx) \leq \text{rmax}\{ [A]_i^-(x), [A]_i^-(y) \} = \text{rmax}\{ [V]_i^-(f(x)), [V]_i^-(f(y)) \}$ which implies that $[V]_i^-(f(x)f(y)) \leq \text{rmax}\{ [V]_i^-(f(x)), [V]_i^-(f(y)) \}$. Hence [V] is a bipolar valued multi I-fuzzy subsemiring of R^I.

2.4 Theorem: Let R and R^I be any two semirings. The antihomomorphic preimage of a bipolar valued multi I-fuzzy subsemiring of R^I is a bipolar valued multi I-fuzzy subsemiring of R.

Proof: Let $f : R \to R^I$ be an antihomomorphism. Let [V] = f([A]) where [V] is a bipolar valued multi I-fuzzy subsemiring of R^I. We have to prove that [A] is a bipolar valued multi I-fuzzy subsemiring of R. Let x and y in R. For each i, $[A]_i^+(x+y) = [V]_i^+(f(x+y)) = [V]_i^+(f(y)+f(x)) \geq \text{rmin}\{ [V]_i^+(f(x)), [V]_i^+(f(y)) \} = \text{rmin}\{ [A]_i^+(x), [A]_i^+(y) \}$ which implies that $[A]_i^+(x+y) \geq \text{rmin}\{ [A]_i^+(x), [A]_i^+(y) \}$. And $[A]_i^+(xy) = [V]_i^+(f(xy)) = [V]_i^+(f(y)f(x)) \geq \text{rmin}\{ [V]_i^+(f(x)), [V]_i^+(f(y)) \} = \text{rmin}\{ [A]_i^+(x), [A]_i^+(y) \}$ which implies that $[A]_i^+(xy) \geq \text{rmin}\{ [A]_i^+(x), [A]_i^+(y) \}$. Also $[A]_i^-(x+y) = [V]_i^-(f(x+y)) = [V]_i^-(f(y)+f(x)) \leq \text{rmax}\{ [V]_i^-(f(x)), [V]_i^-(f(y)) \} = \text{rmax}\{ [A]_i^-(x), [A]_i^-(y) \}$ which implies that $[A]_i^-(x+y) \leq \text{rmax}\{ [A]_i^-(x), [A]_i^-(y) \}$. And $[A]_i^-(xy) = [V]_i^-(f(xy)) = [V]_i^-(f(y)f(x)) \leq \text{rmax}\{ [V]_i^-(f(x)), [V]_i^-(f(y)) \} = \text{rmax}\{ [A]_i^-(x), [A]_i^-(y) \}$ which implies that $[A]_i^-(xy) \leq \text{rmax}\{ [A]_i^-(x), [A]_i^-(y) \}$. Hence [A] is a bipolar valued multi I-fuzzy subsemiring of R.

2.5 Theorem: Let R and R^I be any two semirings. The homomorphic image of a bipolar valued multi I-fuzzy normal subsemiring of R is a bipolar valued multi I-fuzzy normal subsemiring of R^I.

Proof: Let $f : R \to R^I$ be a homomorphism. Let [V] = f([A]) where [A] is a bipolar valued multi I-fuzzy normal subsemiring of R. We have to prove that [V] is a bipolar valued multi I-fuzzy normal subsemiring of R^I. Now for f(x), f(y) in R^I, for each i, $[V]_i^+(f(x)+f(y)) = [V]_i^+(f(x+y)) \geq [A]_i^+(x+y) = [A]_i^+(y+x) \leq [V]_i^+(f(y+x)) = [V]_i^+(f(y)+f(x))$ which implies that $[V]_i^+(f(x)+f(y)) = [V]_i^+(f(y)+f(x))$. And $[V]_i^+(f(x)f(y)) = [V]_i^+(f(xy)) \geq [A]_i^+(xy) = [A]_i^+(yx) \leq [V]_i^+(f(yx)) = [V]_i^+(f(y)f(x))$ which implies that $[V]_i^+(f(x)f(y)) = [V]_i^+(f(y)f(x))$. Also $[V]_i^-(f(x)+f(y)) = [V]_i^-(f(x+y)) \geq [A]_i^-(x+y) = [A]_i^-(y+x) \leq [V]_i^-(f(y+x)) = [V]_i^-(f(y)+f(x))$ which implies that $[V]_i^-(f(x)+f(y)) = [V]_i^-(f(y)+f(x))$. And $[V]_i^-(f(x)f(y)) = [V]_i^-(f(xy)) \geq [A]_i^-(xy) = [A]_i^-(yx) \leq [V]_i^-(f(yx)) = [V]_i^-(f(y)f(x))$ which implies that $[V]_i^-(f(x)f(y)) = [V]_i^-(f(y)f(x))$. Hence [V] is a bipolar valued multi I-fuzzy normal subsemiring of R^I.

2.6 Theorem: Let R and R^I be any two semirings. The homomorphic preimage of a bipolar valued multi I-fuzzy normal subsemiring of R^I is a bipolar valued multi I-fuzzy normal subsemiring of R.

Proof: Let $f : R \to R^I$ be a homomorphism. Let [V] = f([A]) where [V] is a bipolar valued multi I-fuzzy normal subsemiring of R^I. We have to prove that [A] is a bipolar valued multi I-fuzzy normal subsemiring of R. Let x and y in R. For each i, $[A]_i^+(x+y) = [V]_i^+(f(x+y)) = [V]_i^+(f(x)+f(y)) = [V]_i^+(f(y)+f(x)) = [V]_i^+(f(y+x)) = [A]_i^+(y+x)$ which implies that $[A]_i^+(x+y) = [A]_i^+(y+x)$. And $[A]_i^+(xy) = [V]_i^+(f(xy)) = [V]_i^+(f(x)f(y)) = [V]_i^+(f(y)f(x)) = [V]_i^+(f(yx)) = [A]_i^+(yx)$ which implies that $[A]_i^+(xy) = [A]_i^+(yx)$.

Also $[A]_i^-(x+y) = [V]_i^-(f(x+y)) = [V]_i^-(f(x)+f(y)) = [V]_i^-(f(y)+f(x)) = [V]_i^-(f(y+x)) = [A]_i^-(y+x)$ which implies that $[A]_i^-(x+y) = [A]_i^-(y+x)$. And $[A]_i^-(xy) = [V]_i^-(f(xy)) = [V]_i^-(f(x)f(y)) = [V]_i^-(f(y)f(x)) = [V]_i^-(f(yx)) = [A]_i^-(yx)$ which implies that $[A]_i^-(xy) = [A]_i^-(yx)$. Hence [A] is a bipolar valued multi I-fuzzy normal subsemiring of R.

2.7 Theorem: Let R and R^l be any two semirings. The antihomomorphic image of a bipolar valued multi I-fuzzy normal subsemiring of R is a bipolar valued multi I-fuzzy normal subsemiring of R^l.
Proof: Let $f : R \to R^l$ be an antihomomorphism. Let [V] = f([A]) where [A] is a bipolar valued multi I-fuzzy normal subsemiring of R. We have to prove that [V] is a bipolar valued multi I-fuzzy normal subsemiring of R^l. Now for f(x), f(y) in G^l, for each i, $[V]_i^+(f(x)+f(y)) = [V]_i^+(f(y+x)) \geq [A]_i^+(y+x) = [A]_i^+(x+y) \leq [V]_i^+(f(x+y)) = [V]_i^+(f(y)+f(x))$ which implies that $[V]_i^+(f(x)+f(y)) = [V]_i^+(f(y)+f(x))$. And $[V]_i^+(f(x)f(y)) = [V]_i^+(f(yx)) \geq [A]_i^+(yx) = [A]_i^+(xy) \leq [V]_i^+(f(xy)) = [V]_i^+(f(y)f(x))$ which implies that $[V]_i^+(f(x)f(y)) = [V]_i^+(f(y)f(x))$. Also $[V]_i^-(f(x)+f(y)) = [V]_i^-(f(y+x)) \leq [A]_i^-(y+x) = [A]_i^-(x+y) \geq [V]_i^-(f(x+y)) = [V]_i^-(f(y)+f(x))$ which implies that $[V]_i^-(f(x)+f(y)) = [V]_i^-(f(y)+f(x))$. And $[V]_i^-(f(x)f(y)) = [V]_i^-(f(yx)) \leq [A]_i^-(yx) = [A]_i^-(xy) \geq [V]_i^-(f(xy)) = [V]_i^-(f(y)f(x))$ which implies that $[V]_i^-(f(x)f(y)) = [V]_i^-(f(y)f(x))$. Hence [V] is a bipolar valued multi I-fuzzy normal subsemiring of R^l.

2.8 Theorem: Let R and R^l be any two semirings. The antihomomorphic preimage of a bipolar valued multi I-fuzzy normal subsemiring of R^l is a bipolar valued multi I-fuzzy normal subsemiring of R.
Proof: Let $f : R \to R^l$ be an antihomomorphism. Let [V] = f([A]) where [V] is a bipolar valued multi I-fuzzy normal subsemiring of R^l. We have to prove that [A] is a bipolar valued multi I-fuzzy normal subsemiring of R. Let x and y in R. For each i, $[A]_i^+(x+y) = [V]_i^+(f(x+y)) = [V]_i^+(f(y)+f(x)) = [V]_i^+(f(x)+f(y)) = [V]_i^+(f(y+x)) = [A]_i^+(y+x)$ which implies that $[A]_i^+(x+y) = [A]_i^+(y+x)$. And $[A]_i^+(xy) = [V]_i^+(f(xy)) = [V]_i^+(f(y)f(x)) = [V]_i^+(f(x)f(y)) = [V]_i^+(f(yx)) = [A]_i^+(yx)$ which implies that $[A]_i^+(xy) = [A]_i^+(yx)$. Also $[A]_i^-(x+y) = [V]_i^-(f(x+y)) = [V]_i^-(f(y)+f(x)) = [V]_i^-(f(x)+f(y)) = [V]_i^-(f(y+x)) = [A]_i^-(y+x)$ which implies that $[A]_i^-(x+y) = [A]_i^-(y+x)$. And $[A]_i^-(xy) = [V]_i^-(f(xy)) = [V]_i^-(f(y)f(x)) = [V]_i^-(f(x)f(y)) = [V]_i^-(f(yx)) = [A]_i^-(yx)$ which implies that $[A]_i^-(xy) = [A]_i^-(yx)$. Hence [A] is a bipolar valued multi I-fuzzy normal subsemiring of R.

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