HORN'S FUNCTION OF TWO VARIABLES AND ITS APPLICATION IN A BOUNDARY VALUE PROBLEMS:

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Abstract - In this paper ,we evaluate an integral involving Horn's function of two variables and then we make its application to solve a boundary value problem on heat conduction.An expansion formula involving above function has also been obtained

Keywords: Horn's function of two variables ,Heat conduction, Boundary value problem.

M.S.C.: 33C45, 33C60, 26D20

1. Introduction

In this paper,we obtain a solution of a simple problem of Horn's function of two variables and then we make its application to solve a boundary value problem on heat conduction.

Here,we use the Horn's function of two variables.H_7 and H_9 are given in Eede'lyi [1952,p.244-226] and make application the following modified form of the integral Ronghe [1992,p.54 (2.4)].

$$\int_{-\infty}^{+\infty} x^{2\alpha}\, e^{-x^2}\, H_m(x)\, \mathrm{dx} = \frac{\sqrt{\pi}\ 2^{m-2}\ \Gamma(2\alpha+1)}{\Gamma(\alpha-\frac{m}{2}+1)}\quad, \tag{1.1}$$

Where $\alpha = 0,1,2,\ldots$

INTEGRAL:

The integral involving Horn's function of two variables to be evaluated is :

$$\int_{-\infty}^{+\infty} x^{2\alpha}\, e^{-x^2}\, H_m(x)\, H_9(\lambda, \alpha-\frac{m}{2}+1; \delta; z_1, z_2x^2)\mathrm{dx}$$

$$= \frac{\sqrt{\pi}\ 2^{m-2}\ \Gamma(2\alpha+1)}{\Gamma(\alpha-\frac{m}{2}+1)}\ H_7(\lambda, , \alpha+\frac{m}{2}, \alpha+1; \delta;\ z_1, z_2). \tag{1.2}$$

PROOF OF THE INTEGRAL:

Consider

$$\varphi = \int_{-\infty}^{+\infty} x^{2\alpha}\, e^{-x^2} H_m(x) H_9\,(\lambda, \alpha-\frac{m}{2}+1; \delta; z_1, z_2x^2)\mathrm{dx}\ .$$

Now,express H_9 in series form and change the order of integration and summation,

$$\varphi = \sqrt{\pi}\ 2^{m-2\alpha} \sum_{r,s=0}^{\infty} \frac{(\alpha)_{2r-s}\,(\alpha-\frac{m}{2}+1)_r\,\Gamma\{2(\alpha+k)+1\}}{(r)!(k)!(\delta)_r\,\Gamma(\alpha+k-\frac{m}{2}+1)}\ z_1^r\ (\frac{z_2}{4})^k.$$

Again using the results

$$(\lambda)_n = \frac{\Gamma(\lambda+m)}{\Gamma(\alpha)},$$

And $(\alpha)_{2m} = 2^{2m}(\frac{\lambda}{2})_m(\frac{\alpha+1}{2})_m$.

We get,

$$\varphi = \frac{\sqrt{\pi}\, 2^{m-2\alpha}\ \Gamma(2\alpha+1)}{\Gamma(\alpha-\frac{m}{2}+1)}\ \sum_{r,s=0}^{\infty} \frac{(\alpha)_{2r-k}(\alpha+\frac{1}{2})_k(\alpha+1)_k}{(r)!(k)!(\delta)_r}.$$

By using the definition of H_7

APPLICATION OF HEAT CONDUCTION:

In this ,section we consider the following partial differential equation,

$$\left(\frac{\partial C}{\partial t}\right) = k\left(\frac{\partial^2 c}{\partial m^2} + 2x\frac{\partial c}{\partial x} + 2c\right), \quad (-\infty < x < \infty) \qquad \textbf{(1.3)}$$

Where the boundary condition is

$\lim C_{(n)} = 0$, and $|x| \to \infty$.

Equation (1.3) is related to the following equation Carslaw

and Jaeger [1986,p.148(1)]:

$$\frac{\partial^2 s}{dx^2} - \frac{4}{k}\frac{\partial s}{dx} - \frac{s(s-s)}{k} - \frac{1}{k}\frac{\partial s}{\partial t} = 0, \qquad \textbf{(1.4)}$$

Where c $=2k_x$ $s_0= 0$,s = -2k. $(-\infty < x < \infty)$.

The solution of equation (1.3) is given by Bajpai [1993,p.44(2.10)] as follows:

$$C(x,t) = \sum_{m=0}^{\infty} u_m\, e^{-2kmt-x^2} H_m(x), \qquad \textbf{(1.5)}$$

Where H_m(x) is Hermite polynomial and

$$U_n = \frac{1}{2^n (n)! \sqrt{\pi}} \int_{-\infty}^{+\infty} c(x) H_m(x)\, dx. \qquad \textbf{(1.6)}$$

Now ,we shall consider to the problem of determining c(x,t), where

$$C(x) = x^{2\alpha} e^{-x^2} H_9\left(\lambda, \alpha - \tfrac{m}{2} + 1, \delta; z_1, z_2 x^2\right). \qquad \textbf{(1.7)}$$

Combining (1.6) and (1.7) and making the use of the integral (1.2),we derive

$$U_m = \frac{\Gamma(2\alpha+1)}{(m)!\,\Gamma(\alpha - \frac{m}{2}+1) 2^{2\alpha}}\, H_7\left(\lambda, , \alpha + \tfrac{m}{2}, \alpha+1; \delta; z_1, z_2\right). \qquad \textbf{(1.8)}$$

Putting the value of c_m from (1.8) in (1.5),we get the following required solution of the problem.

$$C(x,t) = \frac{\Gamma(2\alpha+1)}{2^{2\alpha}} \sum_{m=0}^{\infty} \frac{1}{(m)!\,\Gamma(\alpha - \frac{m}{2}+1)}\, H_m(x) e^{-2k_m t - x^2} \times H_7\left(\lambda, , \alpha + \tfrac{m}{2}, \alpha+1; \delta; z_1, z_2\right). \qquad \textbf{(1.9)}$$

2. EXPANSION FORMULA:

Similarly, making an use of (1.7) and (1.8) in (1.5),we derive the following expansion formula.

$$x^{2\alpha} H_9(\lambda, , \alpha - \tfrac{m}{2} + 1, \delta; z_1, z_2 x^2) = \frac{\Gamma(2\alpha+1)}{2^{2\alpha}} \sum_{m=0}^{\infty} \frac{1}{(m)!\,\Gamma(\alpha - \frac{m}{2} + 1)} H_m (x)\, H_7(\lambda, , \alpha + \tfrac{m}{2}, \alpha + 1; \delta; z_1, z_2) \,. \quad \textbf{(2.1)}$$

_2. CONCLUSION:

Specializing the parameters of the Horn's function, we can obtain a large number of results involving various special functions of one and two variables useful in Mathematics analysis, Applied Mathematics, Physics and Mechanics . The result derived in this paper is of general character and may prove to be useful in several interesting situations appearing in the literature of sciences.

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