Evolution of Numeral Concept & Theory of Numbers

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Abstract:*In this paper firstly we have tried our best to make attempts to evolution of numerical concept and theory of numbers.*

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1. Initial Turning Stages of Human Race

Our first conceptions of number and form data back to times as far removed as the Old Stone Age, the Paleolithicum. Throughout the hundreds or more milleania of this period men lived in caves, under conditions differing little from those of animals, and their main energies were directed towards the elementary process of collecting food wherever they could get it. The made weapons for hunting and fishing, developed a language to communicate with each other, and in the later Paleolithic ages enriched their lives with creative art forms, statuettes and paintings. The paintings in cases of France and Spain (perhaps 15000 years ago) may have had some ritual significance; certainly they reveal a remarkable understanding of the form. Little progress was made in understanding numerical values and space relations until the transition occurred from the mere gathering of food to its actual production, from hunting and fishing to agriculture. With this fundamental change, a revolution in which the passive attitude of man towards nature turned into an active one, we enter the New stone Age, the Neolithicum.

This great event in the history of mankind occurred perhaps ten thousand years ago, when the ice sheet which covered Europe and Asia began to melt and made for forests and deserts. Nomadic wandering in search of food came slowly to an end. Fisherman and hunters were in large part replaced by primitive farmers. Such farmers remaining in one place as long as the soil stayed fertile began to build more permanent dwelling; villages emerged as protection against climate and against predatory enemies. Many such Neolithic settlements have been excavated. The remains show how gradually elementary crafts such as pottery, carpentry, and weaving developed. There were granaries, so that the inhabitants were able to provide against hard times by establishing a surplus. Bread was baked beer was brewed and in late Neolithic times copper and bronze were smelted and prepared. Inventions were made, notably of the potters' wheel and the wagon wheel; boats and shelters were improved. All these remarkable innovations occurred only within local areas and did not always spread to other localities. The American Indian, for example, did not learn of the existence of the wagon wheel until coming in touch of the white man. Nevertheless, as compared with the Paleolithic times, the tempo of technical improvement was enormously accelerated. Between the villages a considerable trade existed, which so expanded that connections can be traced between places hundreds of miles apart. The discovery of the arts of smelting and manufacturing, first copper, the bronze tools and weapons, strongly stimulated this commercial activity. This again promoted the further formation of languages. The words of these languages expressed very concrete things and very few abstractions, but there was already some room for simple numerical terms and some form relations.

2. Number-sense in Mankind

With the appearance of the human race there developed an opportunity for conscious efforts for the recognition of number-concept.

In the beginning of human civilization a herdsman was in his element to recognize that one of his sheep was missing without being able to count his flock. He had number-sense, but not the sense of counting. Thus the idea of number did not have to await the development of spoken language. In the infancy of its civilization human race possessed only the number-sense. The sense of counting developed latter on.

For a long time after the advent of man such simple numbers as two and three were sufficient for all purposes. The number-sense developed with the growing needs of mankind. Generally, the development of number senses keeps space with the growth of a number-language. This can be inferred from the speech of lower types of savages which have been studied during the last century. Chiquito is a tribal race of Bolivia, a South American Country. There is no numeral terminology in the language of this race. Whenever a person of this race wants to express his sense of unity (One), he uses a word 'Aitma' which resembles the Sanskrit work 'Atman' He cannot count even upto two.

Amsablada is another tribal race of America 'Te' and 'Kayapa' are only two numeral words the language of this race 'Te' means one and 'Kayapa' means two. In the mobokobi language of America also there are only two numeral terminology, 'Yantvaka' which means one and 'Yamka' which means two.

Botosudo is a tribal race of Guayak family of American tribes. 'Mokenam' and 'Uruh' are the only two numeral terminology in the language of this race. 'Mokenam' means one and 'Uruh' means many.

The YahganFuegian is a tribal race of Tierradel Fuego. The numeral names of this tribe are Kaueli (for one), Kombai (for two) and Maten (for three). Thus the numberal terminology of this tribal is limited to three only. In the language of Baroro tribe also there are only three number names Kaue (for one), Makaue (for two) and Uaue (for three).

These facts lead us to the conclusion that all living beings of the world have the concept of one. Is this because of every one being self-conscious? Whatever the reason may be, the notion of one is universal.

While scrutinizing his own body, man must have noticed that he has only one nose but a pair of ears. He has only one mouth but a pair of hands, a pair of feet and a pair of eyes. Thus the pair of ears, pair of hands, pair of feet and pair of eyes provided the notion of duality or the concept of two.

Thus, the number-sense developed further with growing needs of the primitive people. Many Australian, American, and African tribes were in this stage at the period of their first contact with the white men; some tribes are still living in these conditions so that it is possible to study their habits and forms of expression.

3. Growth of the Concept of Counting among Aborigins

Numerical terms- expressing some of "the most of abstract ideas which the human mind is capable of forming", as Adam Smith has said-come only slowly into use. Their first occurrence was qualitative rather than quantitative, making a distinction only between one (or better "a" - "a man" - rather than "one man") and two and many. The ancient qualitative origin of numerical conceptions can still be detected in the special dual terms existing in certain languages such as Greek or Celtic. When the number concept was extended higher numbers were first formed by addition: 3 by adding 2 and 1, 4 by adding 2 and 2,5 by adding 2 and 3.

Let us consider an example from some Australian tribes:

Muray River : enea means 1, petcheval means 2, petcheval - enea means 3, petchevalpetcheval means 4.

and Kamilaroi : mal means 1, bulan means 2, guliba means 3, bulan bulan means 4,bula-guliba, means 5 and guliba guliba means 6.

Of course, necessity is the mother of invention. The needs of primitive people stirred their innate number - sense and compelled them to move in the direction of developing some sort of counting. It is logical to assume that the need for counting large number of things was not felt until people began to keep herds and flocks. The shepherds and herdsmen must had developed some sense of counting the numbers of their sheep and cows to see that none in missing, and long before men began to dwell is cities they had

hit on the device of counting them in groups. When the primitive people started counting, he could count only by pointing to the objects counted, one-by-one.

Primarily, primitive people used fingers of hands and feets, notches, cuts in trees, lines drawn on the soil of the earth's surface, pebbles, fruits and grains etc. for the purpose of counting's. We can see that the primitive words used in counting were primarily tied to concrete groups of objects. In the Malay and Aztec tongues, the number names mean literally one stone, two stones, three stones and so on. The Niue's of Southern Pacific use "one fruit, two fruits, three fruits." The Javans use "one grain, two grains, three grains." These are the relics of concrete stage of counting.

4. Counting on Scales in Primitive Stages

In our present number system, we group objects for enumeration in tens, tens of tens (hundreds), tens of tens of tens (thousands) and so on. Since we have ten figures in our hands, so it has been convenient to count on a scale of ten or a decimal scale. But ten was not, however, the primitive scale. Probablytwo was first used as a radix.The radix of two is still used by the African pygmies. They count a (one), oa (two), oa - a (two-one), oa - oa (two-two) and so on. The early Syrian numeral forms shown below are examples of binary scale:

1	1		
1	2	3	4

The members of a tribal race 'Kampa' are found in Peru, in South America. They people count on the radix of three. They say Paltio (one), Piltaini (two), Mahuani (three), Paltio - Mahuani (one - three), Piltaini- Mahuani (two - three), Mahuani -Mahuani (three-three) and so on.

There are certain South American tribes who have been using four as the radix of counting. Their counting beings with one, two, three four and then proceeds to four and one, four and two and so on. Here, on the soil of India, 'ganda' is also the relic of our counting on the scale of four. 'Ganda'[4] finds its place in the Kirtilata of Vidyapati also.

^^lsfdUnzlkfHkfuladHk, nSo u iqjc, vkjA
vggfddjmx.M´sxf.kvmiklAA**

The quinary scale was the first to be extensively used. The use of two, three and four as radices of counting were feeble attempts of the primitive race in the field of counting. This is due to the fact that primitive man, like a child, uses his fingers as a tally to check off the things he counts. The five fingers of the hand were brought into use as soon as mankind needed a basis for counting numbers of considerable size. The left hand was generally used, the right index finger pointing to the first object to be counted and then to a finger, repeating the process until the five fingers had been used and then repeating the operation, sometimes marking the fives by pebbles or sticks.

Certain South American tribes have been observed to count by hand: one, two, three, four, hand and one, hand and two and so on.

Mengo- Park (1771-1806) found a similar system in one of the tribes of Africa and in one part of Paraguay. Here, five is called the "fingers of one hand", ten being the "fingers of both hands" and twenty the "fingers of both hands and feet".

Sometimes a more primitive scale becomes blended with the scale of five, as in the case of the Yukaghirs of Siberia. These people count, "one, two, three, three and one, five, two threes, one more two threes, two fours, ten (with) one missing and ten.[5]

Russian Scientist Mikaluko - Makalai (1846-88) travelled Newgini in 1871 A.D. He found there that Papua aborigins closes fingers of a hand one after another and whisper 'Be-be-be…. ' during the course of counting. While counting at five, they whisper 'Ebon-be' which means 'one hand'. Further, during the course of counting, they closes fingers of the other hand one-by-one till they reach at ten for which they whisper 'both the hands'. Furthermore, during the course of counting, they take toes and again whisper 'Be-be-be' till they reach at 'fifteen' and 'twenty' for which they whisper 'Samba-be' and 'Samba-ali' respectively and these two terms mean 'one foot' and 'both the feet' respectively .

In many languages the same word is used for five as is used for hand.

In Persian five is called 'Panja and the open hand is also called 'Panja'.

Our 'gahi' is also an example of quinary scale. Gahi is, of course, a dialectical vicissitude of the word 'Grahi' which mean "something that clutches."

It may be taken to be the synonym of "Open hand". One more example warrants our attention. "Ainda" is a language of Florence. Some number- names of this language are as follows: Sa (one), Jwa (two), Lime (five) - (hand), Lima - sa (six), Lima - Jwa (Seven) and so on.[6]

Here five is term as 'Lima' which also means hand.

Thus, we see that the scale of five was very popular among the primitive people.

Of the scales from six to nine inclusive, few traces exist. There is, however, an occasional trace of the intermediate scales, as the Bolans or Buramans of the West Coast of African count by the radix of six. A relic of this radix also appears in South Bretagne, where the word "Triouech' (three sixes) is still used for eighteen.

As soon as the primitive people found that it could count by the aid of the fingers of one hand, it naturally switched on to the use of the fingers of both the hands, thus forming a scale of ten or the decimal scale. One reason for the wide adoption of the decimal scale is that few characters are needed in writing numbers with such a radix but writing came much later. The chief reason of its extensive use, however, is the fact that mam has ten fingers. How many of us realize that the vast edifice of modern mathematics is based on a number scale which arose from the fact that Nature decreed that man should have ten fingers?

When primitive men, by learning to count up to ten, proved that he was insome strange way different from all the rest of animal creation, he invented only ten number- sounds. The reason was that he counted in the way a small child counts to - day, one by one, making use of his fingers. Since those primeveal days, only five other basic number - sounds have been invented by the people. The needs and possessions of a primitive man were few. He required no large numbers. When he wished to express a number greater than ten, he simply combined certain of ten sound connected with his fingers. Thus if he wished to express "One more than ten, he said" one - ten" (the Latin undecim). Hence the word "eleven is simply a modern form of two - life, "two - over" - (the Romar due - decim, "two - ten). The word twenty is two - tig, "two - tens", "thirty" is thrie-tig," three tens" and so on. The only basic number - sounds in addition to the ten primary ones are "hundred", "thousand", "Million", "billion" and "trillion".[7]

Just because primitive people invented the same number of sounds as they were in possession of ten fingers, our number scale to - day is a decimal one, i.e., a scale based on ten and consisting of endless repetitions of the first ten basic number sounds. There is no way of telling when the world adopted this scale. But its wide geographical range leads to the belief that it must have been before the general migrations of races. An examination of more than twenty languages showed that ten was used as a radix in every case, and although this may be directly to the fingers, it is quite as likely to be due to a common linguistic origin.

There is reason to believe that the scale of tweleve, the due-decimal scale was favoured in pre-historic period in various parts of the world, but chiefly in relation to measurements. While it may have been suggested by the approximate number of lunations in years, it was undoubtedly its divisibility by two, three, four and six, thus allowing for simple fractional parts that made it attractive.

Its popularity is seen in the number of inches in a foot, the number of ounces in the ancient pound, the number of pence in a shilling and the number of units in a dozen. The scale of sixteen was also prevalent. The relic of this scale is still found in India in our 'Sorahi' which means sixteen. Even to -day we count own sheaves in 'Sorahi'. Sixteen sheaves of reaped corn - stalk are called a 'Sorahi'. Probably this had its origin in the sixteen phases of the moon.

The vigesimal scale i.e. the scale of twenty was not uncommon in pre-historic times. It is a reminder of the fare- foot days of the human race when men counted toes as well as fingers. Some trace of it is still found in the Malayan languages, but it was early carried to parts of the world far removed from the tropics. The Maya of Yucatan and the Aztecs of Mexico and elaborate system in which twenty was prominent.

But the Greenlanders who had' no such elaborate system, used the expression "one man" for twenty, "two men" for forty and so on.

In India, even to-day, old illiterate persons use the scale of twenty termed as 'Bisa () or Kori () for the sake of counting things.

With the help of fingers and toes man counted upto twenty only, but the need of mankind went on growing. He looked around and noticed pebbles scattered hither and thither. His inquistivie mind switched over to pebbles. He began to count with the help of pebbles also. This probably synchronized with the advent of the early stone age of human civilization.

In the primitive age some people counted dates with the help of pebbles. Every morning they put a pebble somewhere in a corner. If anyone asked the date, the pebbles in the corner were counted and the number of pebbles indicated the date. Some people counted dates by giving knots in a string or by drawing scratches on trees etc.

It is said that when Robinson Crusoe was left alone in an island, he made a scratch on a stick every day. Whenever he wanted to know the number of days, he had spent in that island; he used to count the number of scratches on the stick.

In Germany the primitive savage made signs with chalk for the sake of counting. Sometimes small pieces of straw were also used in counting.

5. Beginning Stage of Crystallization of Numeral Concept

The development of the crafts and of commerce stimulated this crystallization of the number concept. Numbers were arranged and bundled into larger units, usually by the use of the fingers of one hand or both hands, a natural procedure in trading. This led to numeration first with five, later with ten as a base, completed by addition and sometimes by subtraction, so that twelve was conceived as 10 + 2 or 9 as 10 - 1. Sometimes 20, the number of fingers and toges was selected as a base. Of 307 number system of primitive American people investigated by W.C.Eels, 146, were decimal, 106 quinary and quinary decimal, vigesimal ad quinary vigesimal.[8] The Vigesimal system in its most characteristics form occurred among the Mayas of Mexico and the Celts in Europe.

It is true that counting by fives and tens came only at a certain stage of social development. Once it was reached, numbers could be expressed with reference to a base, with the aid of which large numbers could be formed; thus originated a primitive type of arithmetic. Fourteen was expressed as 10+4, sometimes as 15 -1. Multiplication began where 20 was expressed as 10 + 10, but as 2 x 10. Such dyadic operations were used for millennia as a kind of middle path between addition and multiplication, notably in Egypt and in the pre-Aryan Civilization of Mohenjo-Daro on the Indus. Division began where 10 was expressed as "half of a body", though conscious formation of fractions remained extremely rare. Among North American tribes, for instance, only a few instances of such formations are known, and this is in almost all cases only of ½, although sometimes also of 1/3 or 1/4.[9] A curious phenomenon was the love of very large numbers, a love perhaps stimulated by the all - too- human desire to exaggerate the extent of herds or of enemies slain; remnants of this tendency appear in the Bible and in other sacred writings.

6. Need based Development of Numeral Concept

It also became necessary to measure the length and contents of objects. The standards were rough and often taken from parts of the human body, and in this way units originated like fingers, feet and hands. Names like ell, fathom, cubit also remind us of this custom. When houses were built, as among the agricultural Indians or the people house dewellers of Central Europe, rules were laid down for building along straight lines and at right angles. The word "straight" is related to "stretch", indicating operations with rope; the word "line" to "linen", showing the connection between the craft of weaving and the beginnings of geometry. This was one way in which interest in mensuration evolved. Neolithic people also developed a keen feeling for geometrical pattern. Numeral relationship might have entered into the geometrical figures as in certain prehistoric patterns which represent triangular numbers, sacred numbers.

The extent to which primitive people carried their number system was, of course, determined by their needs. The native Australians, for example, were little concerned to trade and so they felt the need of number names only to two, three or four, and for the Hottentot five was a sufficient limit. Among the Papuan people, those of Praido count to ten, while those of Pauwi, living in farther inland, find five sufficiently large for their simpler needs. The Kafirs, however, possessing herds of cattle, count to a

hundred or more, while the Nubians and Abyssinians, representing higher civilization, often use number to a thousand or even to a million, without apparent European influence. For a similar reason one of the Polynesian language has number names to thousands.

The ancient Indians, having need for large numbers in a semi -religious way, as appears form the Vedic writings, early developed a numerical system that is practically unlimited, the most extensive to be found among any ancient people.10

Unlike other ancient people, the Indians had vast conception of time and space. They had a long series of number names based on decimal system of numeration from the earliest known times.

The marvelous achievement of Indians in the field of numeral terminology at a very early period of civilization bear eloquent testimony to the antiquity of Indian civilization because the history of Mathematics in inter - related with the history of civilization.

We, to-day, inherit a culture based on mathematics revolutions. The very foundation of mathematics rests on the shoulder of integers including zero. The Indian invention in the hoary past laid the basis of zero and decimal system of numeration. It is the spiritual background of Ancient Hindus that mattes behind the discovery of zero. The invention of zero is an epoch - making event in the history of human civilization. It is rated as one of the greatest practical inventions of all the time.

7. Conclusions

It is a fact that the invention of zero with the conception of absence and fullness in guiding the Decimal place -value system of numerations must have occurred prior to 3000 B.C. But after the downfall of Vedic civilization, we find that a vast period of time in which the knowledge of zero and Decimal place - value system of numeration was pushed in darkness. This dark period is the period between the downfall of the Vedic Civilization and period to 500 B.C. We find here the re-discovery of zero and Decimal place - value system of numeration in the vicinity of 500 B.C., mainly in Buddha's and Jaina's works on the soil of India. After this rediscovery, we find the continuity in the flow of systematic recorded mathematical works which manifest the clear conception of zero and Decimal place - value system of numeration. We also see that India was alone in possession of the knowledge of zero and Decimal place -value system of numeration as 500 B.C.

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