FOURIER SERIES INVOLVING G-FUNCTION OF TWO VARIABLES

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ABSTRACT

In the literature of special functions and boundary value problems, Fourier series for generalized hypergeometric functions occupies a prominent place. In the area of two-dimensional boundary value problems and theories of special functions, certain double Fourier series of generalized hypergeometric functions play an important role. The aim of this paper is to establish a Fourier series expansion involving G-Function of two variables.

Key Words: G-Function of two variables, G-function of one variable, Double Fourier Series, Fourier series expansion.

1. INTRODUCTION:

Certain number of Fourier series expansion involving generalized hypergeometric functions, recently Beg [3], Ayant Frédéric [1], Dubey [4] and others have evaluated. We shall try to obtain some new Fourier Series expansion involving G-function of two variables on the lines of above researchers.

Srivastava and Joshi [6, p. 471] in terms of Mellin-Barnes type integrals define G-function of two variables as follows:

$$G^{0,n_1:m_2,n_2;m_3,n_3}_{p_1,q_1:p_2,q_2;p_3,q_3}\left[\begin{matrix}x\\y\end{matrix}\middle|\begin{matrix}(a_j;1,1)_{1,p_1}:(c_j,1)_{1,p_2}:(e_j,1)_{1,p_3}\\(b_j;1,1)_{1,q_1}:(d_j,1)_{1,q_2}:(f_j,1)_{1,q_3}\end{matrix}\right]$$

$$= \frac{-1}{4\pi^2}\int_{L_1}\int_{L_2}\phi_1(\xi,\eta)\theta_2(\xi)\theta_3(\eta)x^{\xi}y^{\eta}\,d\xi\, d\eta \qquad (1)$$

where

$$\phi_1(\xi,\eta) = \frac{\prod_{j=1}^{n_1}\Gamma(1-a_j+\xi+\eta)}{\prod_{j=n_1+1}^{p_1}\Gamma(a_j-\xi-\eta)\prod_{j=1}^{q_1}\Gamma(1-b_j+\xi+\eta)},$$

$$\theta_2(\xi) = \frac{\prod_{j=1}^{m_2}\Gamma(d_j-\xi)\prod_{j=1}^{n_2}\Gamma(1-c_j+\xi)}{\prod_{j=m_2+1}^{q_2}\Gamma(1-d_j+\xi)\prod_{j=n_2+1}^{p_2}\Gamma(c_j-\xi)}$$

$$\theta_3(\eta) = \frac{\prod_{j=1}^{m_3}\Gamma(f_j-\eta)\prod_{j=1}^{n_3}\Gamma(1-e_j+\eta)}{\prod_{j=m_3+1}^{q_3}\Gamma(1-f_j+\eta)\prod_{j=n_3+1}^{p_3}\Gamma(e_j-\eta)}$$

x and y are not equal to zero, and an empty product is interpreted as unity p_i, q_i, n_i and m_j are non negative integers such that $p_i \geq n_i \geq 0$, $q_i \geq 0$, $q_j \geq m_j \geq 0$, $(i = 1, 2, 3; j = 2, 3)$.

The contour L_1 is in the ξ-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(d_j - \xi)$ (j = 1, ..., m_2) lie to the right, and the poles of $\Gamma(1 - c_j + \xi)$ (j = 1, ..., n_2), $\Gamma(1 - a_j + \xi + \eta)$ (j = 1, ..., n_1) to the left of the contour.

The countor L_2 is in the η-plane and runs from – i∞ to + i∞, with loops, if necessary, to ensure that the poles of $\Gamma(f_j - \eta)$ (j = 1, ..., m_3) lie to the right, and the poles of $\Gamma(1 - e_j + \eta)$ (j = 1, ..., n_3), $\Gamma(1 - a_j + \xi + \eta)$ (j = 1, ..., n_1) to the left of the contour.

and the double integral converges if

$$2(n_1 + m_2 + n_2) > (p_1 + q_1 + p_2 + q_2),\ 2(n_1 + m_3 + n_3) > (p_1 + q_1 + p_3 + q_3)$$

and $|\arg x| < \frac{1}{2} U\pi,\ |\arg y| < \frac{1}{2} V\pi.$

where $U = [n_1 + m_2 + n_2 - \frac{1}{2}(p_1 + q_1 + p_2 + q_2)],$

$V = [n_1 + m_3 + n_3 - \frac{1}{2}(p_1 + q_1 + p_3 + q_3)]$

These assumptions for the G-function of two variables will be adhered to throughout this research work.

The following formulae are required in the proof:

From Shukla [5]:

$$\int_0^\pi \int_0^\pi (\sin x)^{\lambda-1} (\sin y)^{\mu-1} \sin rx \sin ty\, g(x,y) dxdy$$

$$= \frac{\pi \sin\frac{r\pi}{2} \sin\frac{t\pi}{2}}{\sqrt{dh}} \psi(r,t); \qquad (2)$$

where $2(m_3 + n_3) > p_3 + q_3, |\arg z_2| < \left(m_3 + n_3 - \frac{1}{2}p_3 - \frac{1}{2}q_3\right)\pi,$

$\mathrm{Re}(\lambda + 2df_j) > 0, Re(\mu + 2hf_j) > 0, j = 1, \dots, m_3;$

and
$$g(x,y) = G^{0,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[\begin{matrix} z_1 \\ z_2 (\sin x)^{2d} (\sin y)^{2h} \end{matrix}\middle| \begin{matrix} a_{p_1} : c_{p_2} : e_{p_3} \\ b_{q_1} : d_{q_2} : f_{q_3} \end{matrix}\right]$$

$$\psi(r,t) = G^{0,n_1;m_2,n_2;m_3,n_3+2d+2h}_{p_1,q_1;p_2,q_2;p_3+2d+2h,q_3+2d+2h}\left[\begin{matrix} z_1 \\ z_2 (\sin x)^{2d} (\sin y)^{2h} \end{matrix}\middle| \begin{matrix} a_{p_1} : c_{p_2} : \Delta(2d,1-\lambda), \Delta(2h,1-\mu), e_{p_3} \\ b_{q_1} : d_{q_2} : f_{q_3}, \Delta\left(d, \frac{1-\lambda\pm r}{2}\right), \Delta\left(h, \frac{1-\mu\pm t}{2}\right) \end{matrix}\right]$$

d and h are positive integers, the symbol $\Delta(d,\omega)$ represents the set of parameters $\frac{\omega}{d}, \frac{\omega+1}{d}, \dots, \frac{\omega+d-1}{d}$ and the expression $\Delta\left(d, \frac{1-\omega\pm m}{2}\right)$ stands for $\Delta\left(d, \frac{1-\omega+m}{2}\right), \Delta\left(d, \frac{1-\omega-m}{2}\right)$.

The following double orthogonality properties of sine and cosine function, which may be verified easily:

$$\int_0^\pi \int_0^\pi \sin mx \sin rx \sin ny \sin ty\, dxdy$$

$$= \begin{cases} \frac{\pi^2}{4}, m = r, n = t \\ 0, m \neq r \text{ or } n \neq t \end{cases} \qquad (3)$$

2. MAIN RESULT:

In this section, we shall establish following Fourier series:

$$f(x,y) = \frac{4}{\pi\sqrt{dh}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin\frac{m\pi}{2} \sin\frac{n\pi}{2} \psi(m,n) \sin mx \sin ny, \quad (4)$$

where $2(m_3+n_3) > p_3+q_3, |\arg z_2| < \left(m_3+n_3-\frac{1}{2}p_3-\frac{1}{2}q_3\right)\pi,$

$\mathrm{Re}(\lambda+2df_j) > 0, Re(\mu+2hf_j) > 0, j = 1, \ldots, m_3;$

and $f(x,y) = (\sin x)^{\lambda-1}(\sin y)^{\mu-1} g(x,y)$

where $g(x,y) = G^{0,n_1;m_2,n_2;m_3,n_3}_{p_1,q_1;p_2,q_2;p_3,q_3}\left[{}^{z_1}_{z_2(\sin x)^{2d}(\sin y)^{2h}}\Big|{}^{a_{p_1}:c_{p_2}:e_{p_3}}_{b_{q_1}:d_{q_2}:f_{q_3}}\right].$

Proof of (4):

To establish (4), let

$$f(x,y) = (\sin x)^{\lambda-1}(\sin y)^{\mu-1} g(x,y)$$
$$= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{m,n} \sin mx \sin ny \quad (5)$$

Equation (5) is valid since f(x,y) is continuous and of bounded variation in the open interval (0, π).

Multiplying both sides of (5) by sinrx sinty and integrating from 0 to π with respect to both x and y, we get

$$\int_0^{\pi}\int_0^{\pi} (\sin x)^{\lambda-1}(\sin y)^{\mu-1} \sin rx \sin ty\, g(x,y)\,dxdy$$
$$= \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{m,n} \int_0^{\pi}\int_0^{\pi} \sin mx \sin rx \sin ny \sin ty\, dxdy$$

Now using (2) and (3), we have

$$A_{r,t} = \frac{4}{\pi\sqrt{(dh)}}\left(\sin\frac{r\pi}{2}\sin\frac{t\pi}{2}\right)\psi(r,t) \quad (6)$$

where $\psi(r,t)$ is given in section 1.

Substituting the value of $A_{m,n}$ from (6) in (5), the double half-range Fourier series (4) is established.

3. SPECIAL CASES:

On specializing the parameters in (4), we get following Fourier series in terms of G-function of one variable, which is a result given by Bajpai [2, p. 32(3.1)]:

$$f(x,y) = \frac{4}{\pi\sqrt{dh}} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sin\frac{m\pi}{2} \sin\frac{n\pi}{2} \psi(m,n) \sin mx \sin ny \quad (7)$$

where $2(m+n) > p+q, |\arg z| < \left(m+n-\frac{1}{2}p-\frac{1}{2}q\right)\pi,$

$\mathrm{Re}(\lambda+2db_j) > 0, Re(\mu+2hb_j) > 0, j = 1, \ldots, m;$

$$\psi(r,t) = G^{m,n+2d+2h}_{p+2d+2h,q+2d+2h}\left[z(\sin x)^{2d}(\sin y)^{2h}\Big|{}^{\Delta(2d,1-\lambda),\Delta(2h,1-\mu),a_p}_{b_q,\Delta\left(d,\frac{1-\lambda\pm r}{2}\right),\Delta\left(h,\frac{1-\mu\pm t}{2}\right)}\right]$$

and $f(x,y) = (\sin x)^{\lambda-1}(\sin y)^{\mu-1} g(x,y)$

where $g(x,y) = G^{m,n}_{p,q}\left[z(\sin x)^{2d}(\sin y)^{2h}\Big|{}^{a_p}_{b_q}\right].$

4. CONCLUSION:

Since G-function of two variables may be reduced into several other higher transcendental functions by specializing the parameters. Therefore, the obtained result (4) is of general nature and may be reduced into different forms. So that in the literature on applied Mathematics and other branches, which will be useful for further study.

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