On αws-connected Space in Topological Spaces

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Abstract – *Aim of this paper is to introduce a new type of connected space namely αws-connected space. Also we obtain some properties of αws-connected space.*

AMS Classification 2010: 54D05
Key words: αws-connected space, $T_{\alpha ws}$–space.

1. INTRODUCTION

In topology and related branches of mathematics a connected space is a topological space that cannot be represented as the union of two disjoint non empty open subsets. Connectedness is one of the principal topological properties that are used to distinguish topological spaces. In this paper we introduce αws-connected space.

2. PRELIMINARIES

Throughout this paper (X,τ), (Y,σ) and (Z, μ) represents the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space (X,τ), *cl*A and *int*A denote the closure of A and the interior of A respectively. X – A denotes the complement of A in X. We recall the following definitions

Definition 2.1: A subset A of a space X is called

(i) α-open [9] if A ⊆ *int cl int*A and α-closed if *cl intcl*A⊆ A.
(ii) regular open [10] if A = *intcl*A and regular closed if *cl int*A = A.

(iii) π-open [16] if A is the union of regular open sets and π-closed if A is the intersection of regular closed sets.

The alpha-closure of a subset A of X is the intersection of all alpha-closed sets containing A and is denoted by αclA.

Definition 2.2: A subset A of a space X is called

(i) semi generalized-closed (briefly sg-closed) [3] if sclA ⊆ U whenever A ⊆ U and U is semi-open.

(ii) genralized pre regular-closed (briefly gpr-closed) [4] if pclA⊆ U whenever A ⊆ U and U is regular-open.

(iii) generalized b-closed (briefly gb-closed) [1] if bclA ⊆ U whenever A ⊆ U and U is open.

(iv) semi generalized b-closed (briefly sgb-closed) [6] if bclA ⊆ U whenever A ⊆ and U is semi-open.

(v) alpha Weakly Semi closed [11] (briefly αws-closed) if αclA ⊆ U whenever A⊆ U and U is ws-open.

The complements of the above mentioned closed sets are their respective open sets. For example a subset B of a space X is semi generalized open (briefly sg-open) if X – B is sg-closed.

Definition 2.3: [13] A function f: (X,τ) → (Y,σ) is called

i) αws-continuous function if f^{-1}(V) is αws-closed in (X,τ) for every closed subset V of (Y,σ).

ii) αws-irresolute if f^{-1}(V) is αws-closed in (X,τ) for every αws-closed subset V of (Y,σ).

Definition 2.4: A topological space (X, τ) is said to be

i) connected if X cannot be expressed as the union of two disjoint nonempty open sets in X [15]

ii) gpr-connected if X cannot be expressed as the union of two disjoint nonempty gpr-open sets in X [5]

iii) gb- connected if X cannot be expressed as the union of two disjoint nonempty gb-open sets in X [2]

iv) sgb-connected if X cannot be expressed as the union of two disjoint nonempty sgb-open sets in X [7]

v) sg- connected if X cannot be expressed as the union of two disjoint nonempty sg-open sets in X [8]

Definition 2.5: [14] A function f: (X, τ) → (Y, σ) is called contra αws-continuous function if f^{-1}(V) is αws-closed in (X, τ) for every open subset V of (Y, σ).

Lemma 2.6:[13]

Let us assume f: $(X, \tau) \rightarrow (Y,\sigma)$ be a function. Then the following three statements are equivalent.

i) f is an αws-continuous function.

ii) The inverse image of each open sets in (Y, σ) is an αws-open set in (X, τ).

iii) The inverse image of each closed sets in (Y, σ) is an αws-closed set in (X, τ).

Remark 2.7:[13]

A function f: $(X, \tau) \rightarrow (Y, \sigma)$ is called αws-irresolute function iff the inverse image of every αws-open set in (Y, σ) is αws-open in (X, τ).

3. αws-connected Space

A topological space X is said to be an αws-connected space if X cannot be expressed as the union of two disjoint nonempty αws-open sets in X.

Otherwise, it's αws-disconnected space.

Theorem 3.1:

For a topological space X, the following are equivalent

i) X is αws-connected.

ii) X and ϕ are only subsets of X which are both αws-open and αws-closed.

iii) Each αws-continuous map of X into a discrete space Y with at least two points is constant map.

Proof: i) ⇒ ii) Suppose X is an αws-connected space. Let S be a proper subset which is both αws-open and αws-closed in X. Its complement X – S is also αws-open and αws-closed. Then $X = S \cup (X - S)$, the disjoint union of two non-empty αws-open sets, which is a contradiction to our assumption. Therefore $S = \phi$ or X.

ii) ⇒ i)

Suppose that $X = A \cup B$ where A and B are disjoint nonempty αws-open subsets of X. Then A and B are both αws-open and αws-closed, because $A = X - B$ and $B = X - A$. By our assumption $A = \phi$ or X which is a contradiction. Therefore X is αws- connected.

ii) ⇒ iii)

Let f: $X \rightarrow Y$ be an αws-continuous function where Y is a discrete space with at least two points. To prove f is a constant map. Since f is an αws-continuous function, $f^{-1}(\{y\})$ is αws-closed and αws-open for each $y \in Y$. By our assumption, $f^{-1}(\{y\}) = \phi$ or X. If $f^{-1}(\{y\}) = \phi$ for all $y \in Y$, then f fails to be a function $\Longrightarrow f^{-1}(\{y\}) = X$ for some $y \in Y$. Therefore $f(x) = y$ for all $x \in X$. This shows that f is a constant map.

iii) ⇒ ii)

Let S be both αws-open and αws-closed in X. Let S ≠ ϕ. To prove that S = X. Let f : X → Y be an αws-continuous function by f (S) = {y_1} and f(X – S) = {y_2} for some distinct points y_1 and y_2 ∈ Y. By (iii) f is a constant function. Therefore S = X.

Theorem 3.2:

Every αws-connected space is connected.

Proof:
Let X be an αws-connected space. Suppose X is not connected. Then there exists a proper nonempty subset B of X which is both open and closed in X. Since every closed set is αws-closed set, B is a proper nonempty subset of X which is both αws-open and αws-closed in X. Then by using Theorem 3.1, X is not αws-Connected.

The converse of the above theorem is not true as seen in the following Example.

Example 3.3:

Let X = {a, b, c} with τ = { ϕ, {a}, {b}, {a, b}, {a, c}, X}. Then (X, τ) is connected but not αws-connected because {a, c} and {b} are both αws-open and αws-closed in (X, τ).

Theorem 3.4:

If f: X →Y is an αws-continuous and X is an αws-connected, then Y is connected.

Proof:
Given f is an αws-continuous and X is αws-connected. To prove that Y is connected. Suppose Y is disconnected. Then Y = A ∪ B, where A and B are disjoint non-empty open subsets of Y. Since f is an αws-continuous, by using Lemma 2. 6, X = f^{-1} (A) ∪ f^{-1} (B) where f^{-1} (A), f^{-1} (B) are disjoint non-empty αws-open subsets of X. This is a contradiction to our assumption. Therefore Y is connected.

Theorem 3.5:

If f: X →Y is an αws-irresolute and X is an αws-Connected, then Y is an αws-connected.

Proof:
Given f is an αws-irresolute and X is an αws-connected. To prove that Y is αws-connected. Suppose Y is not αws-connected. Then Y = A ∪ B, where A and B are disjoint non-empty αws-open subsets of Y. since f is an αws-irresolute, by using Remark 2.7, X = f^{-1} (A) ∪ f^{-1} (B) where f^{-1} (A), f^{-1} (B) are disjoint non-empty αws-open subsets of X. This is a contradiction to our assumption. Therefore Y is αws-connected.

Theorem 3. 6:

The contra αws-continuous image of an αws-connected space is connected.

Proof:

Let f : X → Y be contra αws-continuous function from an αws-connected space X onto a topological space Y. Assume Y is disconnected. Then Y = A ∪ B, where A and B are non-empty clopen subsets of Y with A ∩ B = ϕ. since f is contra αws-continuous, we have f^{-1} (A), f^{-1} (B) are non-empty αws-open sets in X with f^{-1} (A) ∪ f^{-1} (B) = f^{-1} (A ∪ B) = f^{-1} (Y) = X and f^{-1} (A) ∩ f^{-1} (B) = f^{-1} (A ∩ B) = f^{-1} (ϕ) = ϕ. This means that X is not αws-connected. This is a contradiction to our assumption. Therefore Y is Connected.

Theorem 3.7:

i) Every gpr-connected space is αws-connected.

ii) Every gb-connected space is αws-connected.

iii) Every sgb-connected space is αws-connected.

iv) Every sg-connected space is αws-connected.

Proof:

i) Let X be gpr-connected space. To prove X is αws-connected. Suppose X is not αws-connected. Then there exist disjoint non-empty αws-open sets A and B such that X = A ∪ B. Since every αws-open set is gpr-open set [12], A and B are gpr-open sets. This is a contradiction to X is gpr-connected. Therefore X is αws-connected.

ii) Let X be gb-connected space. To prove X is αws-connected. Suppose X is not αws-connected. Then there exist disjoint non-empty αws-open sets A and B such that X = A ∪ B. Since every αws-open set is gb-open set [12], A and B are gb-open sets. This is a contradiction to X is gb-connected. Therefore X is αws-connected.

iii) Let X be sgb-connected space. To prove X is αws-connected. Suppose X is not αws-connected. Then there exist disjoint non-empty αws-open sets A and B such that X = A ∪ B. Since every αws-open set is sgb-open set [12], A and B are sgb-open sets. This is a contradiction to X is sgb-connected. Therefore X is αws-connected.

iv) Let X be sg-connected space. To prove X is αws-connected. Suppose X is not αws-connected. Then there exist disjoint non-empty αws-open sets A and B such that X = A ∪ B. Since every αws-open set is sg-open set [12], A and B are sg-open sets. This is a contradiction to X is sg-connected. Therefore X is αws-connected.

The converse of the above theorem is need not be true as in the following example

Example 3.8:

Let X = {a, b, c} with τ = { ϕ, {b}, X}. Then (X, τ) is αws-connected but not gpr-connected, not gb-connected because {a, b} is both gpr-open and gpr-closed also both gb-closed and gb-open in (X , τ).

Example 3.9:
Let X = {a, b, c} with τ = { ϕ, {a}, {b}, {a, b}, X}. Then (X, τ) is αws-connected but not sgb-connected, not sg-connected because {a, c} is both sgb-open and sgb-closed also both sg-closed and sg-open in (X , τ).

Definition 3.10:

A topological Space X is said to be $T_{\alpha ws}$-space if every αws-closed Subsets of X is closed subsets of X.

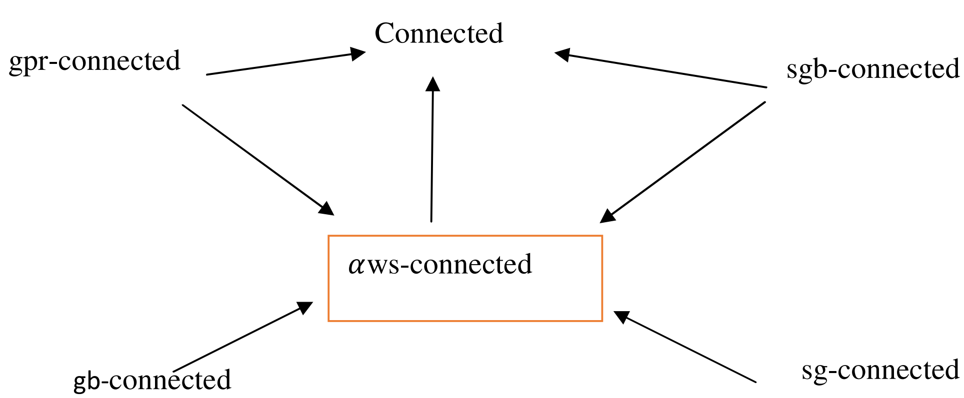
Theorem 3.11:

Suppose that X is $T_{\alpha ws}$-space then X is connected iff it is αws-connected.

Proof:
Suppose X is connected. Then X is cannot be expressed as disjoint union of two non-empty proper subsets of X. Suppose X is not αws-connected. Let A and B be any two αws-open subsets of X such that X = A∪ B, where A ∩ B = ϕ and A ⊂ X, B ⊂ X. Since X is $T_{\alpha ws}$-space and A, B are αws-open, A and B are open subsets of X. This is a contradiction to X is connected. Therefore X is αws-connected. Conversely, Assume X is αws-connected. To prove X is connected. Suppose X is not connected. Let A and B be any two open subsets of X such that X = A ∪ B, where A ∩ B = ϕ and A ⊂ X, B ⊂ X. since every open set is αws-open, A and B are αws-open subsets of X such that X = A ∪ B. This is a contradiction to X is αws-conneceted. Therefore X is connected.

Diagram 3.12:

Relation between αws-connected space and some generalized connected space.



Conclusion:

In this chapter we have introduced αws-connected space and studied some properties.

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