Study of New Concept of an Excess of a Player by Extending the Concept of an Excess of a Coalition in a Non Fuzzy Games

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Abstract: *In this paper we present by considering a payoff vector which minimizes the excess of a player in the lexicographical order, we propose a new solution concept in a fuzzy game.*

Keywords:Fuzzy set theory, Topological space, Game theory, Core and stable sets.

1. Introduction

The concept of excess in concerned with a coalition. We here introduce a new concept of an excess of a player by extending the concept of an excess of a coalition in a non fuzzy games as well as a fuzzy game. By considering a payoff vector which minimizes the excess of a player in the lexicographical order, we propose a new solution concept in a fuzzy game.

2. Excess in Fuzzy Games

The concept of excess was used to study the dynamics games by M.Davis and M. Maschter. Here we define the excess of a coalition and the excess of a player in a non fuzzy game as well as fuzzy games.

Let A = {S/S CN} be a family of coalitions

and $X = (X_1,X_2,X_3, \ldots,X_n)$ be a payoff vectors.

The coalition S can be defined by $\tau^S_{(i)} = \begin{cases} 1, & if\ i \in S \\ 0 \ldots, & if\ i \in S \end{cases}$

and represented by $\tau^S = \left(\tau^S_{(1)}, \tau^S_{(2)}, \tau^S_{(3)}, \ldots . \tau^S_{(n)}\right)$

Definition 1.*Excess of a Coalition in a Non-Fuzzy Game*

The excess of a coalition S W.R. to a payoff vector X is defined by

where $\tau^S_i = \begin{cases} 1, & if\ i \in S \\ 0 \ldots, & if\ i \in S \end{cases}$ for any player $i \in N$

Definition 2.*Excess of a Fuzzy Coalition*

For a fuzzy game FG = (N,f)

Let X be a payoff vector and G be a fuzzy game.

Then the excess of the fuzzy coalition $\tau \in [0,1]n$ w.r. to a payoff vector X is defined by e $(\tau,X)= f(\tau) - X\tau$ where $f(\tau)$ is a value of the characteristic function representing the gain that the fuzzy coalition τ can obtain from a game alone and X. τ is the amount of payoff of the fuzzy coalition τ for the payoff vector X.

3. Excess of a Player in a Non-Fuzzy Game

For a game g = (N,U)

Let e(S,X) be a excess of a coalition w.r. to a payoff vector X. Then, an excess of a player w.r. to X is defined by

$$e(i, X) = \sum_{i \in S \in A} e(S, X) = \sum_{i \in S \in A} (V(S) - X.\tau^S)$$

i.e. $e(i, X) = \sum_{S \in A} \tau_i^S e(S, X) = \sum_{S \in A} \tau_i^S (V(S) - X.\tau^S)$

where $\tau^S_{(i)}$ is written as $\tau^S_{(i)}$

and hence $\tau^S_{(i)} = \begin{cases} 1, & if\ i \in S \\ 0 \ldots, & if\ i \in S \end{cases}$

First of all, we consider a fuzzy game with a finite number of fuzzy coalitions. A Fuzzy coalition is represented by an n - dimensional vector,

Let a set of finite number of coalitions τ be T.

$$\tau \in [0,1]^n$$

Then an excess e* (i,X) of player I w.r. to a payoff vector X is

$$e * (i, e) = \sum_{\tau \in T} \tau_i\, e(\tau, X)$$

Consequently, an excess e (i,X) of player i in a non fuzzy game can be viewed as a special case of an excess e*((i,x) of player i in a fuzzy game.Secondly, we consider a fuzzy game with an infinite number of fuzzy coalitions. The excess e* (i,x) in a fuzzy game, which permits all of the fuzzy coalitions $\tau \in [0,1]^n$, is defined by multiplying an excess e (τ,x) by a rate of participation τ of player i and integrating it from 0 to 1.

i.e.$e * (i,x) = \int_0^1 \tau_i e(\tau,x) d\tau$

where$\int_0^1 d\tau = \int_0^1 \dots \dots \int_0^1 d\tau_1 . d\tau_2 . d\tau_3 \dots . . d\tau_n$

The domain of τ is limited to

$D = \{\tau/\alpha_i \leq \tau_i \leq \beta_i , 0 \leq \alpha_i \leq \beta_i \leq 1, \forall i \in N\}$ instead of $[0,1]^n$.

The excess e* (i,x) can also be

Considered as e* (i,x) = $\int_D \tau_i e(\tau,x) d\tau$

4. Excess of a Player in Fuzzy Game

For a fuzzy game FG = (N,f).

Let e (τ,x) be an excess of a fuzzy coalition τw.r. to a payoff vector x.

Also let D $[0,1]^n$ and T $[0,1]^n$.

Then an excess of a player in any fuzzy game is defined by

e* (i,x) = $\int_D \tau_i e(\tau,x) d\tau + \sum_{\tau \in T} \tau_i e(\tau, X)$

5. Lexicographical Order: Definition

Let γ(x) be a vector arranged in order of decreasing magnitude i.e.,

i.e., $i < j = \gamma_i (x) > \gamma_j (x)$

Then for any two payoff vectors x and y if x = y or for the first component h in which they differ i.e. $\gamma_n(x) < \gamma_n (y)$, x is smaller than y in the lexicographical order and denoted by $\leq_L$.

Using the concept of lexicographical order, we consider a solution which minimizes an excess of a player e (i,x) or e* (i,x).

5. 1. Solution Which Minimizes an Excess of Player

Let H :RnRn be a mapping which arranges components of an n-dimensional vector in order of decreasing magnitude. Then for a non fuzzy game G = (N,V) and a fuzzy game FG = (N,f) the lexicographical solutions which minimize the excess e (i,x) or e* (i,x) of a player can be defined as follows:

$L_s(G) = \{x/H (e(1,x), e(2,x), \dots\dots\dots . e(n,x)$
$\leq_L H (e(1,y), e(2,y), \dots\dots\dots . e(n,y)$
$\forall y \in X(G)\}$

$L_s(FG) = \{x/H (e(1,x), e(2,x), \dots\dots\dots . e(n,x)$
$\leq_L H (e(1,y), e(2,y), \dots\dots\dots . e(n,y)$
$\forall y \in X(FG)\}$

where X (G) and X (FG) are the sets of imputations defined by

$X (G) = \{x/x_i > V (\{i\}), \forall i \in N_i, \sum_{i \in N} X_i = V (N)\}$

$X (FG) = \{x/x_i > f (\tau^i), \forall i \in N_i, \sum_{i \in N} X_i = F (\tau^N)\}$

The solution $E_s(G)$ exists and coincides with the solution $L_s(G)$, if a payoff vector X satisfying.

$\min_{x \in X(G)} \max_{i \in N} e(i,x)$ is unique.

Similarly E_s (FG) exists and coincides with the solutions $L_s(FG)$, if a payoff vector X satisfying

$\min_{x \in X(FG)} \max_{i \in N} e^*(i,x)$ is unique.

The above first conditions is valid if x satisfying the equation.

e (1,x) = 2 (2,x) = e (3,x) = ………= e(n,x)

Lies in the set of the imputations X (G) and the solution $L_s(G)$ can be computed by solving the linear programming problem. Thus, if the solution $E_s(G)$ exists, $L_s(G)$ and $E_s(G)$ coincide with each other. For a fuzzy game FG = (N,f), the existence of the solution Es(FG) can be proved similarly.

The second conditions of the theorem is

$$2^{n-2}V(N) + n \sum_{i \in S} V(S) - \sum_{\substack{j \in N \\ j \neq i}} \sum_{\substack{i \in S \\ i \neq j}} V(S) \geq 0$$

i = 1,2,3,……..,n

The only term

$$\sum_{\substack{j \in N \\ j \neq i}} \sum_{\substack{i \in S \\ i \neq j}} V(S)$$

6. Conclusions

In this condition, is negative and if the value of coalition to which the player j belongs is not so small compared with others. First conditions of the theorem satisfied and hence the solution Es (G) exists.

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