SOME THEOREMS IN BIPOLAR VALUED MULTI FUZZY GRAPH

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ABSTRACT. In this paper, few basic definitions, results and theorems of bipolar valued multi fuzzy graph are studied and proved. Fuzzy graph is the simplification of the crisp graph, bipolar valued fuzzy graph is the simplification of fuzzy graph and bipolar valued multi fuzzy graph is the generalization of bipolar valued fuzzy graph. A new structure of bipolar valued multi fuzzy graph is introduced.

2010Mathematics subject classification. 03E72, 03F55, 05C72.

KEY WORDS. Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, strong bipolar valued multi fuzzy relation, bipolar valued multi fuzzy graph, bipolar valued multi fuzzy loop, bipolar valued multi fuzzy pseudo graph, bipolar valued multi fuzzy spanning subgraph, bipolar valued multi fuzzy induced subgraph, bipolar valued multi fuzzy underling graph, level set, degree of bipolar valued multi fuzzy vertex, order of the bipolar valued multi fuzzy graph, size of the bipolar valued multi fuzzy graph.

INTRODUCTION. In 1965, fuzzy set was introduced independently by Zadeh [14]. Lee [4] presented the view of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is distended from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 earnings that elements are inappropriate to the corresponding property, the membership degree (0, 1] indicates that elements fairly satisfy the property and the membership degree [−1, 0) shows that elements slightly satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets appearance comparable each other. Still, they are dissimilar each other [4, 5]. Bipolar valued multi fuzzy subset is defined in [13]. After that the generalized of crisp graph is fuzzy graphs which was introduced by Rosenfeld [11]. Fuzzy graphs are suitable to signify relationships which agreement with doubt. The fuzzy graph has numerous applications in the field of computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, etc. NagoorGani.A [9, 10] introduced a fuzzy graph and regular fuzzy graph. Multi fuzzy set was introduced by Sabu Sebastian, T.V.Ramakrishnan[12]. After that the fuzzy graph has been generalized with fuzzy loop and fuzzy multiple edges, this type of concepts was introduced by K.Arjunan and C.Subramani[1, 2]. The intuitionistic fuzzy graph with multiple edges and selfloops has been introduced by K.Arjunan and C.Subramani[3]. Multi fuzzy graph and intuitionistic multi fuzzy graph with loops and multi edges have been introduced by Marichamy.A et.al [7, 8]. In this paper a new structure is introduced that is bipolar valued multi fuzzy graph with loops and multiple boundaries and some results of bipolar valued multi fuzzy graph are stated and proved.

1.PRELIMINARIES.

Definition 1.1. ([14]) Let X be a non-empty set. A **fuzzy subset A** of X is a function A: X→ [0, 1].

Definition 1.2. ([4]) A bipolar valued fuzzy set (BVFS) A in X is defined as an objective of the form A = { < x, $A^+(x)$, $A^-(x)$ >/ x∈X}, where $A^+ : X \to [0, 1]$ and $A^- : X \to [-1, 0]$. The positive membership degree $A^+(x)$ signifies the agreement degree of an element x to the property matching to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the fulfilment degree of an element x to some implied counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.3. A = { < a, 0.75, –0.3 >, < b, 0.41, –0.7 >, < c, 0.5, –0.84 >} is a bipolar valued fuzzy subset of X = {a, b, c }.

Definition 1.4. ([13]) A bipolar valued multi fuzzy set (BVMFS) A in X is defined as an object of the form A = { < x, $A_1^+(x)$, $A_2^+(x)$, ..., $A_n^+(x)$, $A_1^-(x)$, $A_2^-(x)$, ..., $A_n^-(x)$ > / x∈X }, where for each i, $A_i^+ : X \to [0, 1]$ and $A_i^- : X \to [-1, 0]$. The positive membership degrees $A_i^+(x)$ denote the satisfaction degree of an element x to the property analogous to a bipolar valued multi fuzzy set A and the negative membership degrees $A_i^-(x)$ mean the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued multi fuzzy set A.

Example 1.5. A = { < a, 0.6, 0,6, 0.7, –0.3, –0.9, –0.5 >, < b, 0.2, 0.4, 0.7, –0.71, –0.3, –0.6 >, < c, 0.15, 0.3, 0.8, –0.4, –0.35, –0.3 >} is a bipolar valued multi fuzzy subset of X = { a, b, c }.

Definition 1.6. ([13]) Let A = $\langle A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^- \rangle$ and B = $\langle B_1^+, B_2^+, \ldots, B_n^+, B_1^-, B_2^-, \ldots, B_n^- \rangle$ be two bipolar valued multi fuzzy subsets of a set X. We define the following relations and operations:

(i) A ⊂ B if and only if for all i, $A_i^+(x) \le B_i^+(x)$ and $A_i^-(x) \ge B_i^-(x)$ for all x∈X.

(ii) A = B if and only if for all i, $A_i^+(x) = B_i^+(x)$ and $A_i^-(x) = B_i^-(x)$ for all x∈X.

(iii) A∩B = { ⟨ x, min ($A_1^+(x)$, $B_1^+(x)$), min ($A_2^+(x)$, $B_2^+(x)$),…,min ($A_n^+(x)$, $B_n^+(x)$), max ($A_1^-(x)$, $B_1^-(x)$), max ($A_2^-(x)$, $B_2^-(x)$),…, max ($A_n^-(x)$, $B_n^-(x)$) ⟩ / x∈X }.

(iv) A∪B = { ⟨ x, max ($A_1^+(x)$, $B_1^+(x)$), max($A_2^+(x)$, $B_2^+(x)$),…,max($A_n^+(x)$, $B_n^+(x)$), min ($A_1^-(x)$, $B_1^-(x)$), min ($A_2^-(x)$, $B_2^-(x)$), …, min ($A_n^-(x)$, $B_n^-(x)$) ⟩ / x∈X }.

2.BIPOLAR VALUED MULTI FUZZY GRAPH.

Definition 2.1. Let A = ⟨ A^+, A^- ⟩ = ⟨ $A_1^+, A_2^+, \ldots, A_n^+, A_1^-, A_2^-, \ldots, A_n^-$ ⟩ be a bipolar valued multi fuzzy subset in a set V, the strongest bipolar valued multi fuzzy relation on V, that is a bipolar valued multi fuzzy relation on A is S = { ⟨ (x, y), $S_1^+(x, y)$, $S_2^+(x, y)$, ..., $S_n^+(x, y)$, $S_1^-(x, y)$, $S_2^-(x, y)$, ..., $S_n^-(x, y)$ ⟩ / x and y in V } given by for each i, $S_i^+(x, y) = \min \{ A_i^+(x), A_i^+(y) \}$ and $S_i^-(x, y) = \max \{ A_i^-(x), A_i^-(y) \}$ for all x and y in V.

Note. $A^+(x) = (A_1^+(x), A_2^+(x), \ldots, A_n^+(x))$ and $A^-(x) = (A_1^-(x), A_2^-(x), \ldots, A_n^-(x))$.

Definition 2.2. Let V be any nonempty set, E be any set and f: E→V×V be any function. Then A is a bipolar valued multi fuzzy subset of V, S stands a bipolar valued multi fuzzy relation on V with respect to A and B is a bipolar valued multi fuzzy subset of E such that $\underset{e\in f^{-1}(x,y)}{B_i^+(e) \le S_i^+(x,y)}$ and $\underset{e\in f^{-1}(x,y)}{B_i^-(e) \ge S_i^-(x,y)}$ for all i. Formerly the ordered triple F = (A, B, f) is called a **bipolar valued multi fuzzy graph**, where the elements of A are called **bipolar valued multi fuzzy points** or **bipolar valued multi fuzzy vertices** and the elements of B are termed **bipolar valued multi fuzzy lines** or **bipolar valued multi fuzzy edges** of the bipolar valued multi fuzzy graph F. If f(e) = (x, y), then the bipolar valued multi fuzzy points (x, $A^+(x)$, $A^-(x)$), (y, $A^+(y)$, $A^-(y)$) are called **bipolar valued multi fuzzy adjacent points** and bipolar valued multi fuzzy point (x, $A^+(x)$, $A^-(x)$), bipolar valued multi fuzzy line (e, $B^+(e)$, $B^-(e)$) are called **incident** with each other. If two distinct bipolar valued multi fuzzy lines (e_1, $B^+(e_1)$, $B^-(e_1)$) and (e_2, $B^+(e_2)$, $B^-(e_2)$) are occurrence with a common bipolar valued multi fuzzy point, then they are so-called **bipolar valued multi fuzzy adjacent lines.**

Definition 2.3. A bipolar valued multi fuzzy line connection a bipolar valued multi fuzzy point to itself is called a **bipolar valued multi fuzzy loop**.

Definition 2.4. Let F = (A, B, f) be a bipolar valued multi fuzzy graph. If in excess of one bipolar valued multi fuzzy line joining two bipolar valued multi fuzzy vertices is allowed, then the bipolar valued multi fuzzy graph F is called a **bipolar valued multi fuzzy pseudo graph**.

Definition 2.5. F = (A, B, f) is called a **bipolar valued multi fuzzy simple graph** if it requires neither bipolar valued multi fuzzy multiple lines nor bipolar valued multi fuzzy loops.

Example 2.6. F = (A, B, f), where V = { v_1, v_2, v_3, v_4, v_5 }, E = {a, b, c, d, e, h, g } and f: $E \rightarrow V \times V$ is defined by f(a) = (v_1, v_2) , f(b) = (v_2, v_2), f(c) = (v_2, v_3), f(d) = (v_3, v_4), f(e) = (v_3, v_4), f(h) = (v_4, v_5), f(g) = (v_1, v_5). A bipolar valued multi fuzzy subset A = {(v_1, (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)), (v_2, (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), (v_3, (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), (v_4, (0.3, 0.3, 0.5), (–0.4, –0.2, –0.3)), (v_5, (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)) } of V. A bipolar valued multi fuzzy relation S = { ((v_1, v_1), (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)), ((v_1, v_2), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_1, v_3), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_1, v_4), (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)), ((v_1, v_5), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_2, v_1), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_2, v_2), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_2, v_3), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_2, v_4), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_2, v_5), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_3, v_1), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_3, v_2), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_3, v_3), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_3, v_4), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_3, v_5), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_4, v_1), (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)), ((v_4, v_2), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_4, v_3), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_4, v_4), (0.3, 0.3, 0.5), (–0.4, –0.2, –0.3)), ((v_4, v_5), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_5, v_1), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_5, v_2), (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)), ((v_5, v_3), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_5, v_4), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)), ((v_5, v_5), (0.3, 0.2, 0.5), (–0.3, –0.2, –0.2)) } on V relating to A and a bipolar valued multi fuzzy subset B = { (a, (0.1, 0.2, 0.3), (–0.2, –0.05, –0.1)), (b, (0.1, 0.2, 0.3), (–0.2, –0.05, –0.1)), (c, (0.1, 0.2, 0.3), (–0.2, –0.05, –0.1)), (d, (0.1, 0.2, 0.3), (–0.2, –0.1, –0.1)), (e, (0.2, 0.1, 0.2), (–0.2, –0.1, –0.1)), (h, (0.2, 0.2, 0.3), (–0.2, –0.1, –0.1)), (g, (0.2, 0.2, 0.3), (–0.2, –0.1, –0.1)) } of E.

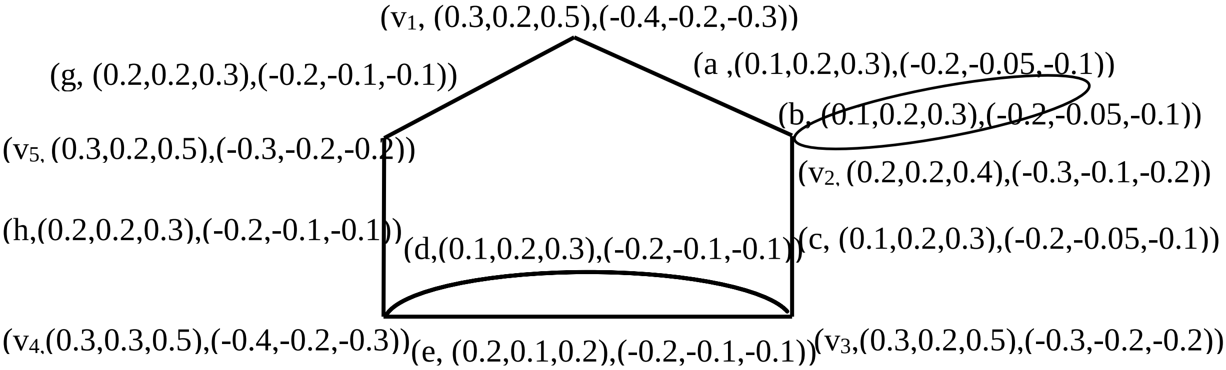

Fig. 1.1

In figure 1.1, (i) (v_1, (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)) is a bipolar valued multi fuzzy point. (ii) (a ,(0.1, 0.2, 0.3), (–0.2, –0.05, –0.1)) is a bipolar valued multi fuzzy edge. (iii) (v_1, (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)) and (v_2, (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)) are bipolar valued multi fuzzy adjacent points. (iv) (a ,(0.1, 0.2, 0.3), (–0.2, –0.05, –0.1)) join with (v_1, (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)) and (v_2, (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)) and therefore it is incident with (v_1, (0.3, 0.2, 0.5), (–0.4, –0.2, –0.3)) and (v_2, (0.2, 0.2, 0.4), (–0.3, –0.1, –0.2)) .(v) (a ,(0.1, 0.2, 0.3), (–0.2, –0.05, –0.1)) and (g, (0.2, 0.2, 0.3), (–0.2, –0.1, –0.1)) are bipolar valued multi fuzzy adjacent lines. (vi) (b, (0.1, 0.2, 0.3), (–0.2, –0.05, –0.1)) is a bipolar valued multi fuzzy loop. (vii) (d, (0.1, 0.2, 0.3), (–0.2, –0.1, –0.1)) and (e, (0.2, 0.1, 0.2), (–0.2, –0.1, –0.1)) are bipolar valued multi fuzzy multiple edges. (viii) The given graph stays not a bipolar valued multi fuzzy simple graph. (ix) The given graph is a bipolar valued multi fuzzy pseudo graph.

Definition 2.7. The bipolar valued multi fuzzy graph H = (C, D, f), where C = $\langle C_1^+, C_2^+, \ldots, C_n^+, C_1^-, C_2^-, \ldots, C_n^- \rangle$ and D = $\langle D_1^+, D_2^+, \ldots, D_n^+, D_1^-, D_2^-, \ldots, D_n^- \rangle$ is called a **bipolar valued multi fuzzy subgraph** of F = (A, B, f) if $C \subseteq A$ and $D \subseteq B$.

Definition 2.8. The bipolar valued multi fuzzy subgraph H = (C, D, f) is said to be a **bipolar valued multi fuzzy spanning subgraph** of F = (A, B, f) if C = A.

Definition 2.9. The bipolar valued multi fuzzy subgraph H = (C, D, f) is said to be a **bipolar valued multi fuzzy induced sub graph** of F = (A, B, f) if H is the maximal bipolar valued multi fuzzy subgraph of F with bipolar valued multi fuzzy point set C.

Definition 2.10. Let F = (A, B, f) be a bipolar valued multi fuzzy graph with respect to the sets V and E. Let C = $\langle C_1^+, C_2^+, \ldots, C_n^+, C_1^-, C_2^-, \ldots, C_n^- \rangle$ be a bipolar valued multi fuzzy subset of V, the bipolar valued multi fuzzy subset D = $\langle D_1^+, D_2^+, \ldots, D_n^+, D_1^-, D_2^-, \ldots, D_n^- \rangle$ of E is defined as for each i, $D_i^+(e) = \min\{ C_i^+(u), C_i^+(v), B_i^+(e) \}$, $D_i^-(e) = \max\{ C_i^-(u), C_i^-(v), B_i^-(e) \}$, where f(e) = (u, v) for all e in E. Then H = (C, D, f) is called **bipolar valued multi fuzzy partial subgraph** of F.

Definition 2.11. Let F = (A, B, f) be a bipolar valued multi fuzzy graph. Let H is a bipolar valued multi fuzzy sub graph of F obtained by removing the bipolar valued multi fuzzy point (x, $A^+(x)$, $A^-(x)$) and all the bipolar valued multi fuzzy lines incident with (x, $A^+(x)$, $A^-(x)$). It is denoted H = F–(x, $A^+(x)$, $A^-(x)$). Thus F–(x, $A^+(x)$, $A^-(x)$) = (C, D, f), where C = A–{(x, $A^+(x)$, $A^-(x)$) } and D = {(e, $B^+(e)$, $B^-(e)$) / (e, $B^+(e)$, $B^-(e)$)$\in$B and (x, $A^+(x)$, $A^-(x)$) is not incident with (e, $B^+(e)$, $B^-(e)$) }. Then H = F–(x, $A^+(x)$, $A^-(x)$) is called a **bipolar valued multi fuzzy induced subgraph** of F. Here $A^+(x) = (A_1^+(x), A_2^+(x), \ldots, A_n^+(x))$ and $A^-(x)$

= ($A_1^-(x)$, $A_2^-(x)$, …, $A_n^-(x)$). Let (e, $B^+(e)$, $B^-(e)$)∈B. Then F–(e, $B^+(e)$, $B^-(e)$) = (A, D, f) = H is called bipolar valued multi fuzzy sub graph of F obtained by the removal of the bipolar valued multi fuzzy line (e, $B^+(e)$, $B^-(e)$), where D = B–{ (e, $B^+(e)$, $B^-(e)$) }. Then H = F –(e, $B^+(e)$, $B^-(e)$) is called a **bipolar valued multi fuzzy spanning sub graph** of F which contains all the lines of F except (e, $B^+(e)$, $B^-(e)$). Here $B^+(e)$ = ($B_1^+(e)$, $B_2^+(e)$, …, $B_n^+(e)$) and $B^-(e)$ = ($B_1^-(e)$, $B_2^-(e)$, …, $B_n^-(e)$).

Definition 2.12. By deleting from a bipolar valued multi fuzzy graph F all bipolar valued multi fuzzy loops and in each collection of bipolar valued multi fuzzy multiple edges all bipolar valued multi fuzzy edge but one bipolar valued multi fuzzy edge in the collection we obtain a bipolar valued multi fuzzy simple spanning subgraph F, is called **bipolar valued multi fuzzy underling simple graph of F.**

Example 2.13.

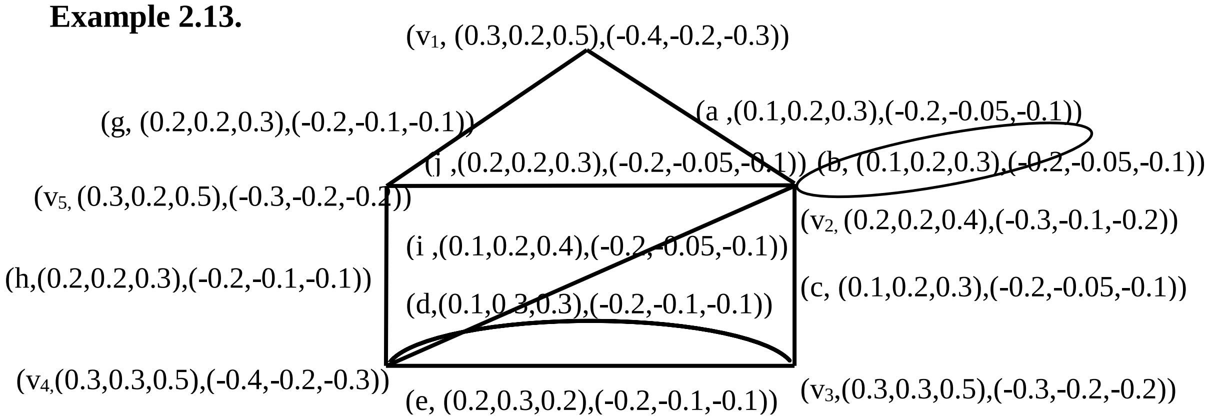

Fig. 1.2 A bipolar valued multi fuzzy pseudo graph F = (A, B, f)

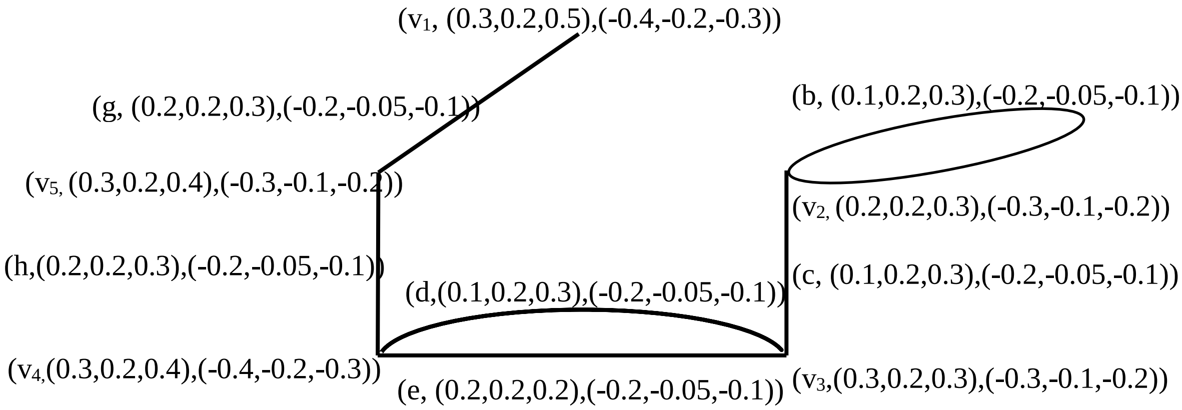

Fig. 1.3 A bipolar valued multi fuzzy subgraph of F

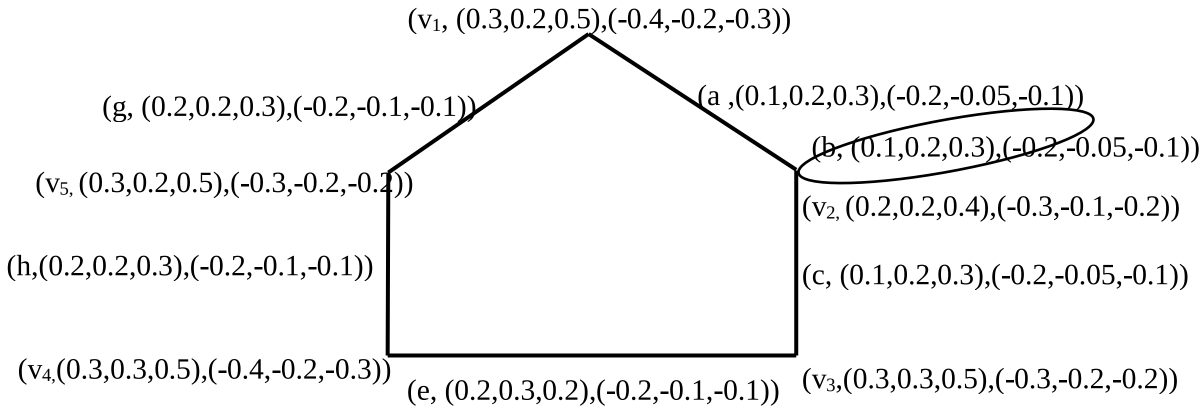

Fig. 1.4 A bipolar valued multi fuzzy spanning subgraph of F

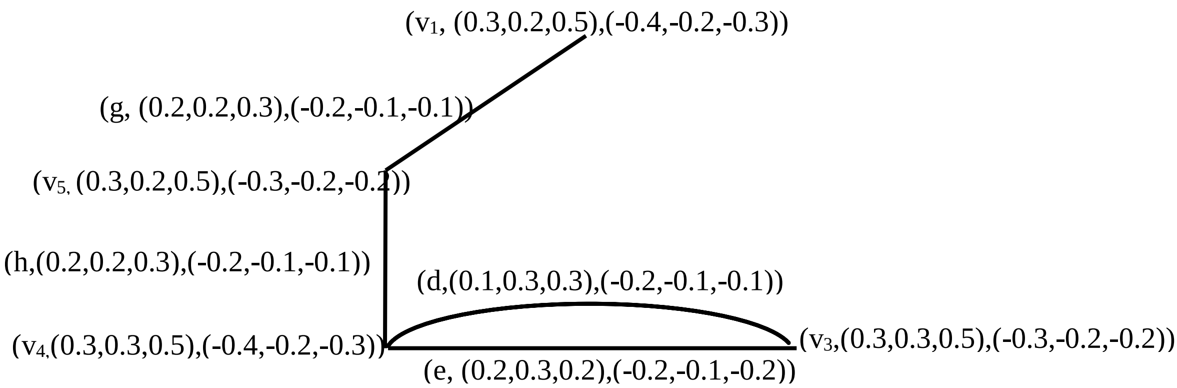

Fig.1.5 A bipolar valued multi fuzzy subgraph induced by P = { v_1, v_3, v_4,v_5 }

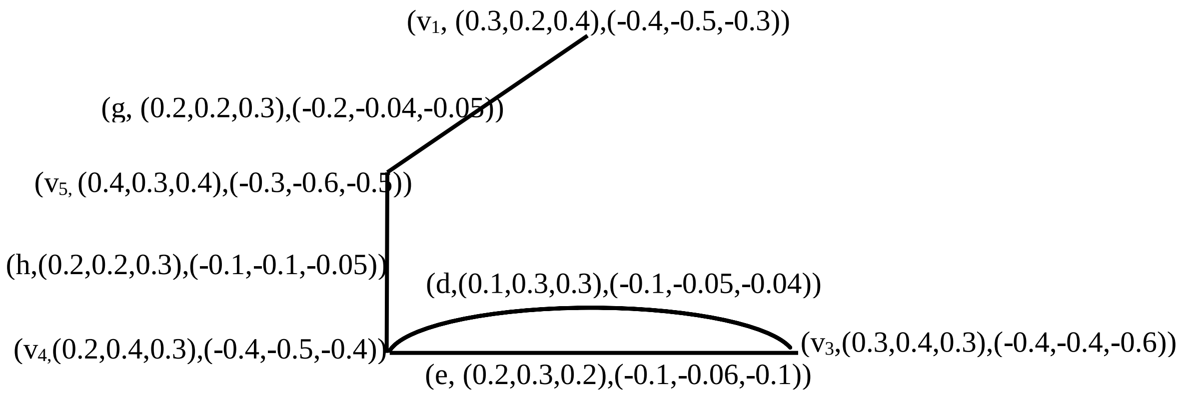

Fig.1.6 A partial bipolar valued multi fuzzy subgraph induced by C, where
$C(v_1)$ = ((0.3, 0.2, 0.4), (-0.4, -0.5, -0.3)), $C(v_3)$ = ((0.3, 0.4, 0.3), (-0.4, -0.4,-0.6)),
$C(v_4)$ = ((0.2, 0.4, 0.3), (-0.4, -0.5, -0.4)), $C(v_5)$ = ((0.4, 0.3, 0.4), (-0.3, -0.6, -0.5)).

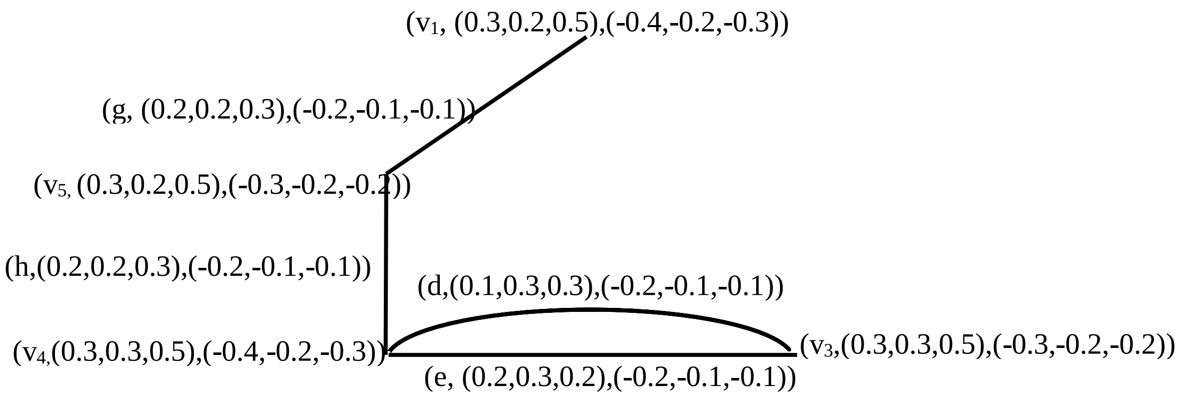

Fig. 1.7 F – ((v_2, (0.2,0.2, 0.4), (-0.3, -0.1, -0.2)))

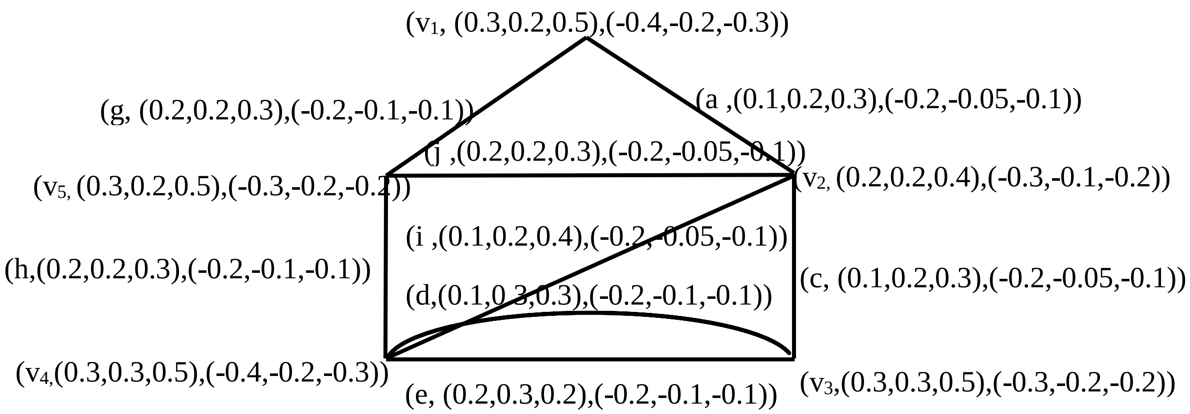

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Fig. 1.8 F– ((b, (0.1, 0.2, 0.3), (-0.2, -0.05, -0.1)))

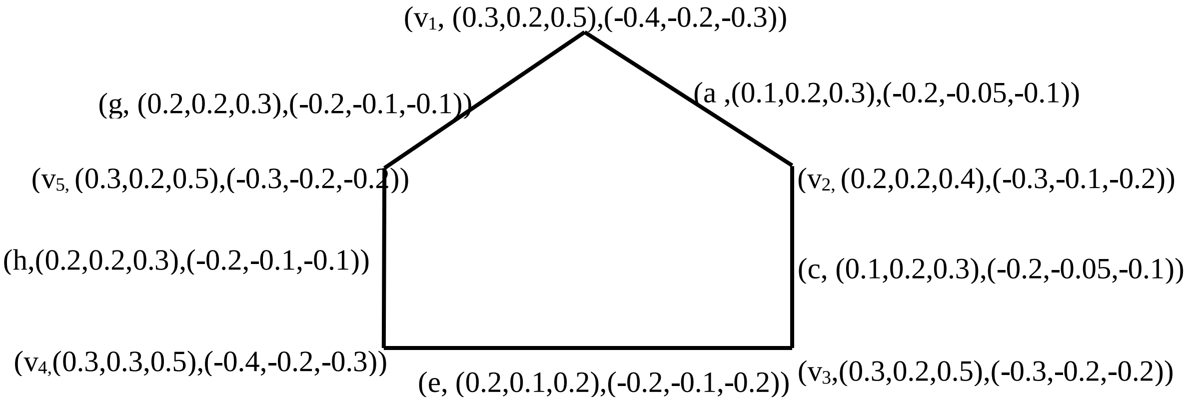

Fig .1.9 Underling bipolar valued multi fuzzy simple graph of F.

Definition 2.14. Let A be a bipolar valued multi fuzzy subset of X. Then the **level subset** or **(α,β)-cut** of A is $A_{(\alpha,\beta)} = \{ x \in X \,/\, A_i^+(x) \geq \alpha_i$ and $A_i^-(x) \leq \beta_i \}$, where $\alpha_i \in [0, 1]$, $\beta_i \in [-1, 0]$ for all i. Here $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ and $\beta = (\beta_1, \beta_2, ..., \beta_n)$.

Theorem 2.15. Let F = (A, B, f) be a bipolar valued multi fuzzy graph with regard to the set V and E. Let α_i, $\beta_i \in [0,1]$, λ_i, $\eta_i \in [-1, 0]$ and $\alpha_i \leq \beta_i$ and $\lambda_i \geq \eta_i$. Then $(A_{(\beta,\eta)}, B_{(\beta,\eta)}, f)$ is a subgraph of $(A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$. Here $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$, $\beta = (\beta_1, \beta_2, ..., \beta_n)$, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ and $\eta = (\eta_1, \eta_2, ..., \eta_n)$.

Proof. The proof follows from the definition 2.14.

Theorem 2.16. Let F = (A, B, f) be a bipolar valued multi fuzzy graph with respect to the set V and E, the level subsets $A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}$ of A and B subset of V and E respectively. Then $F_{(\alpha,\lambda)} = (A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$ is a subgraph of G = (V, E, f). Here $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ and $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$.

Proof. The proof follows from the definition 2.14 and the theorem 2.15.

Theorem 2.17. Let H = (C, D, f) be a bipolar valued multi fuzzy subgraph of F = (A, B, f), $\alpha_i \in [0,1]$ and $\lambda_i \in [-1, 0]$. Then $H_{(\alpha,\lambda)} = (C_{(\alpha,\lambda)}, D_{(\alpha,\lambda)}, f)$ is a subgraph of $F_{(\alpha,\lambda)} = (A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$. Here $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ and $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$.

Proof. Let H = (C, D, f) be a bipolar valued multi fuzzy subgraph of F = (A, B, f). That is for each i, $C_i^+(u) \leq A_i^+(u)$ and $C_i^-(u) \geq A_i^-(u)$ for all u in V. Also $D_i^+(e) \leq B_i^+(e)$ and $D_i^-(e) \geq B_i^-(e)$ for all e in E. We have to prove that ($C_{(\alpha,\lambda)}$, $D_{(\alpha,\lambda)}$, f) is a subgraph of ($A_{(\alpha,\lambda)}$, $B_{(\alpha,\lambda)}$, f). It is enough to prove that $C_{(\alpha,\lambda)} \subseteq A_{(\alpha,\lambda)}$ and $D_{(\alpha,\lambda)} \subseteq B_{(\alpha,\lambda)}$. Let $u \in C_{(\alpha,\lambda)}$ which implies that for each i, $C_i^+(u) \geq \alpha_i$ and $C_i^-(u) \leq \lambda_i$ implies that $A_i^+(u) \geq C_i^+(u) \geq \alpha_i$ and $A_i^-(u) \leq C_i^-(u) \leq \lambda_i$ implies that $u \in A_{(\alpha,\lambda)}$. Therefore $C_{(\alpha,\lambda)} \subseteq A_{(\alpha,\lambda)}$. Let $e \in D_{(\alpha,\lambda)}$ implies that $D_i^+(e) \geq \alpha_i$ and $D_i^-(e) \leq \lambda_i$ implies that $B_i^+(e) \geq D_i^+(e) \geq \alpha_i$ and $B_i^-(e) \leq D_i^-(e) \leq \lambda_i$ implies that $e \in B_{(\alpha,\lambda)}$. Therefore $D_{(\alpha,\lambda)} \subseteq B_{(\alpha,\lambda)}$. Hence $H_{(\alpha,\lambda)}$ is a subgraph of $F_{(\alpha,\lambda)}$.

Definition 2.18. Let A be a bipolar valued multi fuzzy subset of X. Then the **strong level subset** or **strong(α,β)-cut** of A is $A_{(\alpha+,\beta+)} = \{x \in X \,/\, A_i^+(x) > \alpha_i$ and $A_i^-(x) < \beta_i\}$ for all i, where $\alpha_i \in [0,1]$, $\beta_i \in [-1, 0]$. Here $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$ and $\beta = (\beta_1, \beta_2, ..., \beta_n)$.

Theorem 2.19. Let F = (A, B, F) be a bipolar valued multi fuzzy graph with respect to the set V and E. Let α_i, $\beta_i \in [0,1]$, λ_i, $\eta_i \in [-1, 0]$ and $\alpha_i \le \beta_i$ and $\lambda_i \ge \eta_i$. Then $(A_{(\beta+,\eta+)}, B_{(\beta+,\eta+)}, f)$ is a subgraph of $(A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$. Here $\alpha = (\alpha_1, \alpha_2, ..., \alpha_n)$, $\beta = (\beta_1, \beta_2, ..., \beta_n)$, $\lambda = (\lambda_1, \lambda_2, ..., \lambda_n)$ and $\eta = (\eta_1, \eta_2, ..., \eta_n)$.

Proof. The proof follows from definition 2.18.

Theorem 2.20. Let F = (A, B, f) be a bipolar valued multi fuzzy subgraph with respect to the set V and E, the level subsets $A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}$ of A and B subset of V and E respectively. Then $F_{(\alpha+,\lambda+)} = (A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$ is a subgraph of G = (V, E, f).

Proof. The proof follows from the definition 2.18 and the theorem 2.19.

Theorem 2.21. Let H = (C, D, f) be a bipolar valued multi fuzzy subgraph of F = (A, B, f), $\alpha_i \in [0,1]$ and $\lambda_i \in [-1, 0]$. Then $H_{\alpha+} = (C_{\alpha+}, D_{\alpha+}, f)$ is a subgraph of $F_{(\alpha+,\lambda+)} = (A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$.

Proof. The proof follows from the definition 2.18 and the theorem 2.20.

Theorem 2.22. Let F = (A, B, f) be a bipolar valued multi fuzzy subgraph with respect to the set V and E, let α_i, $\beta_i \in [0,1]$, λ_i, $\eta_i \in [-1, 0]$ and also $F_{(\alpha,\lambda)}$ and $F_{(\beta,\eta)}$ be two subgraphs of G. Then (i) $F_{(\alpha,\lambda)} \cap F_{(\beta,\eta)}$ is a subgraph of G. (ii) $F_{(\alpha,\lambda)} \cup F_{(\beta,\eta)}$ is a subgraph of G.

Proof. Since $A_{(\alpha,\lambda)}$ and $A_{(\beta,\eta)}$ are subset of V. Clearly $F_{(\alpha,\lambda)} \cap F_{(\beta,\eta)}$ is a subgraph of G. Also $F_{(\alpha,\lambda)} \cup F_{(\beta,\eta)}$ is a subgraph of G.

Definition 2.23. Let F = (A, B, f) be a bipolar valued multi fuzzy graph. Then the **degree of a bipolar valued multi fuzzy vertex** is defined by $d(v) = (d_1^+(v), d_2^+(v), ..., d_n^+(v), d_1^-(v), d_2^-(v), ..., d_n^-(v))$, for each i,

$$d_i^+(v) = \sum_{e \in f^{-1}(u,v)} B_i^+(e) + 2 \sum_{e \in f^{-1}(v,v)} B_i^+(e) \text{ and } d_i^-(v) = \sum_{e \in f^{-1}(u,v)} B_i^-(e) + 2 \sum_{e \in f^{-1}(v,v)} B_i^-(e).$$

Defination 2.24. The **minimum degree** of the bipolar valued multi fuzzy graph F = (A, B, f) is $\delta(F) = (\delta_1^+(F), \delta_2^+(F), ..., \delta_n^+(F), \delta_1^-(F), \delta_2^-(F), ..., \delta_n^-(F))$, for each i, $\delta_i^+(F) = \min\{ d_i^+(v) \,/\, v \in V \}$ and $\delta_i^-(F) = \min\{ d_i^-(v) \,/\, v \in V\}$ and the **maximum degree** of F is $\Delta(F) = (\Delta_1^+(F), \Delta_2^+(F), ..., \Delta_n^+(F), \Delta_1^-(F), \Delta_2^-(F), ..., \Delta_n^-(F))$, for each i, $\Delta_i^+(F) = \max\{ d_i^+(v) \,/\, v \in V \}$ and $\Delta_i^-(F) = \max\{ d_i^-(v) \,/\, v \in V \}$.

Definition 2.25. Let F = (A, B, f) be a bipolar valued multi fuzzy graph. Then the **order of bipolar valued multi fuzzy graph** F is defined to be $o(F) = (o_1^+(F), o_2^+(F), ..., o_n^+(F), o_1^-(F), o_2^-(F), ..., o_n^-(F))$, for each i, $o_i^+(F) = \sum_{v \in V} A_i^+(v)$ and $o_i^-(F) = \sum_{v \in V} A_i^-(v)$.

Definition 2.26. Let F = (A, B, f) be a bipolar valued multi fuzzy graph. Then the **size of the bipolar valued multi fuzzy graph F** is defined to be S(F) = (S_1^+(F), S_2^+(F), ..., S_n^+(F), S_1^-(F), S_2^-(F), ..., S_n^-(F)), for each i, $S_i^+(F) = \sum_{e \in f^{-1}(u,v)} B_i^+(e)$ and $S_i^-(F) = \sum_{e \in f^{-1}(u,v)} B_i^-(e)$

Example 2.27.

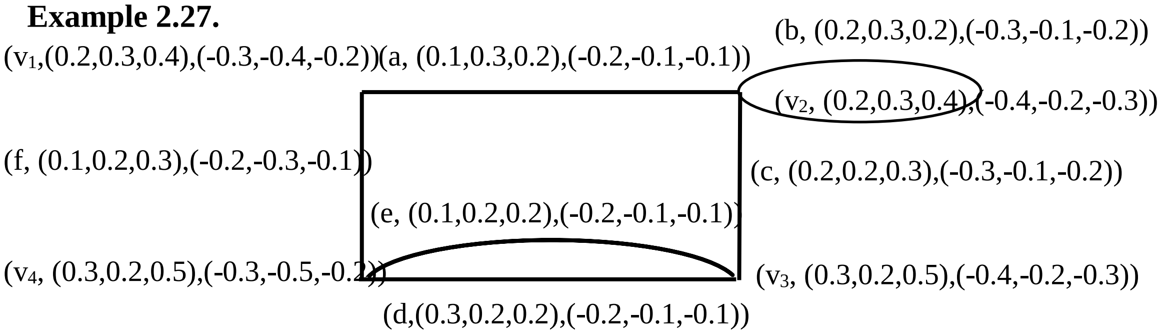

Fig.1.10 Bipolar valued multi fuzzy graph F

Here d(v_1) = ((0.2, 0.5, 0.5), (–0.4, –0.4, –0.2)), d(v_2) = ((0.7, 1.1, 0.9), (–1.1, –0.4, –0.7)), d(v_3) = ((0.6, 0.6, 0.7), (–0.7, –0.3, –0.4)), d(v_4) = ((0.5, 0.6, 0.7), (–0.6, –0.5, –0.3)), δ(F) = ((0.2, 0.5, 0.5), (–1.1, –0.5, –0.7)), Δ(F) = ((0.7, 1.1, 0.9), (–0.4, –0.3, –0.2)), o(F) = ((1.0, 1.0, 1.8), (–1.4, –1.3, –1.0)), S(F) = ((1.0, 1.4, 1.4), (–1.4, –0.8, –0.8)).

Theorem 2.28. (i)The amount of the degree of positive membership values of all bipolar valued multi fuzzy vertices in a bipolar valued multi fuzzy graph is equivalent to twice the sum of the positive membership values of all bipolar valued multi fuzzy edges. i.e., $\sum_{v \in V} d_i^+(v) = 2S_i^+(F)$. (ii)The quantity of the degree of negative membership values of all bipolar valued multi fuzzy vertices in a bipolar valued multi fuzzy graph is equal to twice the amount of the negative membership value of all bipolar valued multi fuzzy edges. i.e., $\sum_{v \in V} d_i^-(v) = 2S_i^-(F)$.

(iii) The amount of the degree of all bipolar valued multi fuzzy vertices in a bipolar valued multi fuzzy graph is equal to twice the quantity of the all bipolar valued multi fuzzy edges.i.e., $\sum_{v \in V} d(v) = 2S(F)$.

Proof. (i) Let F = (A, B, f) be a bipolar valued multi fuzzy graph with respect to the set V and E. Since degrees of a bipolar valued multi fuzzy vertex denote sum of the positive membership values of all bipolar valued multi fuzzy edges incident on it. Each bipolar valued multi fuzzy edge of F is incident with two bipolar valued multi fuzzy vertices. Hence positive membership value of each bipolar valued multi fuzzy edge contributes two to the sum of degrees of bipolar valued multi fuzzy vertices. Hence the amount of the degree of all bipolar valued multi fuzzy vertices in a bipolar valued multi fuzzy graph is equal to twice the amount of the positive membership value of all bipolar valued multi fuzzy edges. i.e., $\sum_{v \in V} d_i^+(v) = 2S_i^+(F)$.

(ii) Let F = (A, B, f) be a bipolar valued multi fuzzy graph with respect to the set V and E. Since degree of a bipolar valued multi fuzzy vertex denote sum of the negative membership values of all bipolar valued multi fuzzy edges incident on it. Each bipolar valued multi fuzzy edge of F is incident with two bipolar valued multi fuzzy vertices.

Hence negative membership value of each bipolar valued multi fuzzy edge contributes two to the amount of degrees of bipolar valued multi fuzzy vertices. Hence the quantity of the degree of all bipolar valued multi fuzzy vertices in a bipolar valued multi fuzzy graph is equal to twice the sum of the negative membership value of all bipolar valued multi fuzzy edges. i.e., $\sum_{v \in V} d_i^-(v) = 2S_i^-(F)$.

(iii) From (i) and (ii), the amount of the degree of all bipolar valued multi fuzzy vertices in a bipolar valued multi fuzzy graph is equal to twice the quantity of the all bipolar valued multi fuzzy edges. i.e., $\sum_{v \in V} d(v) = 2S(F)$.

Theorem 2.29. Let F = (A, B, f) be a bipolar valued multi fuzzy graph with number of bipolar valued multi fuzzy vertices n, all of whose bipolar valued multi fuzzy vertices have degree $s = (s_1^+, s_2^+, ..., s_n^+, s_1^-, s_2^-, ..., s_n^-)$ or $t = (t_1^+, t_2^+, ..., t_n^+, t_1^-, t_2^-, ..., t_n^-)$. If F has p bipolar valued multi fuzzy vertices of degree s and (n–p) bipolar valued multi fuzzy vertices of degree t, then 2S(F) = ps + (n–p)t.

Proof. Let V_1 be the set of all bipolar valued multi fuzzy vertices with degree s. Let V_2 be the set of all bipolar valued multi fuzzy vertices with degree t. Then $\sum_{v \in V} d(v)$ $= \sum_{v \in V_1} d(v) + \sum_{v \in V_2} d(v)$ which implies that 2S(F) $=(\sum_{v \in V_1} d^+ v), \sum_{v \in V_1} d^-(v)) +$ $(\sum_{v \in V_2} d^+(v), \sum_{v \in V_2} d^-(v))$ which implies that 2S(F) = p($s_1^+, s_2^+, ..., s_n^+, s_1^-, s_2^-, ..., s_n^-$) + (n–p) ($t_1^+, t_2^+, ..., t_n^+, t_1^-, t_2^-, ..., t_n^-$) which implies that 2S(F) = ps + (n–p)t.

REFERENCES:

1. Arjunan.K & Subramani.C, "Notes on fuzzy graph", *International Journal of Emerging Technology and Advanced Engineering*, Vol.5, Iss.3 (2015), 425–432.
2. Arjunan.K & Subramani.C, "A study on I-fuzzy graph", *International Journal of Mathematical Archive*, 6(4) (2015), 222–233.
3. Arjunan.K & Subramani.C,"Notes on intuitionistic fuzzy graphs", *Shanlax International Journal of Arts, Science and Humanities*, Vol. 5 (2017), 35–46.
4. K.M.Lee, Bipolar valued fuzzy sets and their operations. *Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand,* (2000), 307 - 312.
5. K.M.Lee, Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolarvalued fuzzy sets. *J. fuzzy Logic Intelligent Systems,* 14 (2) (2004), 125-129.
6. Mardeson.J.N and C.S.Peng, "Operation on Fuzzy graph", *Information science* 19 (1994), 159 – 170.
7. Marichamy.A, K.Arjunan and K.L.Muruganantha Prasad, "Multi fuzzy irregular graph", *Infokara,* Vol. 8, Iss. 8(2019), 636 – 649.
8. Marichamy.A, K.Arjunan and K.L.Muruganantha Prasad, "Notes on intuitionistic multi fuzzy graph", *International Journal of Mathematical Archive,* 10(12) (2019), 25 – 38.
9. NagoorGani, A. and BasheerAhamed.M.,"Order and size in fuzzy graphs", *Bulletin of Pure and Applied Sciences,* Vol 22E (No.1) (2003), 145–148.

10. NagoorGani.A and ShajithaBegum.S, "Degree, order and size in intuitionistic fuzzy graphs", *International Journal of Algorithms and mathematics*, Vol. 3 (2010), 11 – 16.
11. Rosenfeld,A., "Fuzzy Graphs, In:L.A.Zadeh,K.S.Fu, M.Shimura,Eds., Fuzzy Sets and their Applications", *Academic press*(1975), 77–95.
12. Sabu Sebastian, T.V.Ramakrishnan, "Multi fuzzy sets", *International Mathematical Forum*, 5, no.50 (2010), 2471–2476.
13. Santhi.V.K and K. Anbarasi, "Bipolar valued multi fuzzy subhemirings of a hemiring", *Advances in Fuzzy Mathematics*, Volume 10, Number 1 (2015), 55 –62.
14. L. A. Zadeh, "Fuzzy sets", *Information and Control*, 8 (1965) 338 –353.