Q-SEMIRING HOMOMORPHISM USING FUZZY SOFT SET

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Abstract: In this paper, we lead the idea of fuzzy soft Q −S-ring homomorphism and take a look at a few families of homomorphism image of fuzzy soft Q− S-ring.

Keywords: fuzzy soft Q−semiring (Q-s-ring), fuzzy soft ideal, fuzzy soft k-ideal over Q−semi ring, fuzzy soft Q−semi ring homomorphism.

1. Introduction

Semiring is the great algebraic form which is a commonplace generalization of rings and distributive lattices, become first delivered through Vandiver [25] in 1934 but non trivial examples of s-rings had appeared within the research on the idea of commutative ideals of ring via Dedekind in nineteenth century. s-ring is an accepted algebra with binary operations called addition and multiplication in which one in every of them distributive over the alternative. Bounded distributive lattices are commutative s-rings which might be each additively idempotent and multiplicatively idempotent. If in a hoop, we do away with the requirement of getting additive inverse of every element then the resulting algebraic structure will become s-ring. Most of the s-rings have an order structure in addition to their algebraic shape. An instance of s-ring is the set of all numbers below common $''+''$ and $''.''$ of operations . In unique, if [0,1] is the unit interval of IIVF period at the actual mark then $(\vee, \wedge)$ wherein $''zero''$ is the $''+''$ identity and $''one''$ is the $''.''$ identity. The idea of ring after the concept of semi-groups have substantial effect at the growth of the idea of s-rings. Now form s-rings invention among semi-groups and rings. The have a look at of ring shows that $''.''$ form of a hoop is unbiased of $''+''$ form while in s-ring $''.''$ form of an s-ring isn't free of $''+''$ limit of an s-ring. $''+''$ and $''.''$ systems of an s-ring show a crucial function in figuring out the shape of an s-ring.

s-ring is very beneficial for solving issues in carried out arithmetic and facts sciences due to the fact s-ring gives an algebraic frame paintings for modeling. s-ring because the fundamental algebraic construction, become used inside the areas of hypothetical discipline in addition to inside the results of graph idea, optimization concept and in specific for analyzing mechanisms, coding concept and formal languages. S-ring concept has many packages in other branches. It is widely recognized that ideals show a vital position inside the study of any algebraic structures, particularly s-rings. Lajos, Iseki characterized the beliefs of semi-groups and ideals of s-rings respectively. Though s-ring is a generalization of a ring, ideals of s-ring do now not coincide with ring beliefs. For illustration a countless of an s-ring desires now not be the kernel of some s-ring homomorphism. To solve this hassle, Henriksen [8] described k−ideals in s-rings to obtain analogues of ring effects for s-ring.

The knowledge of fuzzy set is the most suitable principle for production with uncertainty, become head delivered with the aid of Zadeh [27]. The idea of fuzzy subgroup became delivered by way of Rosenfeld [22]. Several papers on FSs looked displaying the significance of the idea and its programs to reason, set concept, group idea, ring theory, actual study, topology, grade idea etc. Uncertain records in lots of crucial applications within the regions together with economics, engineering, surroundings, clinical sciences and enterprise control can be due to records randomness, records incompleteness, limitations of measuring tool, not on time information updates and so on. Molodtsov [16] delivered the idea of soft set concept as a new accurate device for coping with uncertainties, best partially resolves the trouble is that objects in regularly occurring set frequently does now not exactly fulfil the factors related to each of the factors inside the set. Then Maji et al. [13] prolonged soft set principle to fuzzy soft set theory. Akta¸s and C¸ a˘gman [2] defined the SSs and soft businesses. Majumdar [15] lengthy SSs to FSSs. Acar et al. [1], gave the simple concept of soft ring. Further greater, Shah and Medhit [24] gave the concept of primary decomposition in a soft ring and a soft module. Ghosh et al. [7] initiated the examine of FS ring and FS ideals. O zturk et al. [20], [21] studied soft Q –s-rings and fuzzy sub near-rings. Jun and Lee [9] studied fuzzy Q -rings. Feng et al. [6] studied soft s-rings via the usage of the soft set principle. ¨O. Bekta¸s et al. [4] studied soft Q−s-rings. Attanassov [3] deliberate intuitionistic FSs. Zhou et Al. [28] persistent the concept of intuitionistic FS set to s-group theory. Maji et al. [14] presented the idea of IFS set which is an extension to soft set and IFS. Murali Krishna [18] brought and studied fuzzy soft beliefs and fuzzy tender ok−ideals over a Q −S-ring. Murali Krishna and Venkateswarlu [19] delivered intuitionistic ordinary fuzzy soft ok−ideals over a Q −s-ring and studied their residences. In continuation of paper [18], we present the notion of fuzzy soft Q- s-ring homomorphism and study a few families of homomorphism image of FS Q − s-ring in this paper.

2. **Preliminaries**

Definition 2.1. A set M collectively with associative binary operations called $+$ and $.$ could be known as a s-ring furnished

(i) $+$ Is a commutative operation.

(ii) $\exists$ 0 ∈ S then $x + 0 = x$ and $x \cdot 0 = 0 \cdot x = 0$, $x \in M$.

(iii) . Distributes over $+$ each from the left.

Definition 2.2. Let $(L, +)$ and $(Q, +)$ be commutative s-groups. Then we namely L as a Q-s-ring, if ∃ a function $L \times Q \times L \rightarrow L$ written (a, δ, b)as ab such that it satisfies the subsequent conditions $\forall\ a, b, c \in L$ and $\nu, \delta \in Q$

(i) $a\delta(b + c) = a\delta b + a\delta c$

(ii) $(a + b)\delta c = a\delta c + b\delta c$

(iii) $a(\delta + \nu)b = a\delta b + a\nu b$

(iv) $a\delta(b\nu c) = (a\delta b)\nu c.$

Definition 2.3. Let Mbe a Q−s-ring and A be a non-empty subset of M. A is named a Q−sub s-ring of M if A is a sub-semigroup of $(M, +)$ and $AQA \subseteq A$.

Definition 2.4. Let M be a Q−s-ring. A Subset A of M is referred to as a left (right) ideals of M if A is closed below addition and $MQA \subseteq A(AQM \subseteq A)$. A is named an ideal of M if it's far each a left ideal and a right ideal.

Definition 2.5. Let $M \neq \varphi$ set.A function $f : M \rightarrow [0, 1]$ is called a FSS of$M$.

Definition 2.6. Let $g$be a FSS of M. For $t \in [0, 1]$ the set g_t= $\{a \in M \mathbin{/} f(a) \geq t\}$ is stated as a level subset of M under g.

Definition 2.7. An ideal I of a Q−s-ring M is stated as an k−ideal if for $, b \in M$, $a + b \in I, b \in I \Rightarrow a \in I$.

Definition 2.8. Let the two FSS f and g of M . Then $f \cup g, f \cap g$ are FSSs of M stated as

$$(f \sqcup g)a = \max\{f(a), g(a)\}, (f \sqcap g)(a) = \min\{f(a), g(a)\}, \forall a \in M.$$

Definition 2.9. A fuzzy sub set μ of $M \neq \varphi$ is stated as FSS if μ isn't the steady function.

Definition 2.10. Let γ and $\mu \in M$ be two FSS the $, \gamma \subseteq \mu \Rightarrow \gamma(a) \leq \mu(a)$ $,\forall a \in M$.

Definition 2.11. Let two FSS f and g of $Q-$s-ring$S$. Then $f \circ g$is stated as

$(f \circ g)c = \{ \sup_{c=a\delta b} \{\min\{f(a), g(b)\}\}$, and zero otherwise for all $a, b, c \in S$ and$\delta \in Q$.

Definition 2.12. Let the two FS L and M and define the function$\tau: L \to M$. A FSS μ of M is said to be an τ invertible if $(a) = \tau(b) \Rightarrow \mu(a) = \mu(b)$.

Definition 2.13. $f : L \to M$ Where L and M are $Q-$s-rings is said to be $Q-$s-ring homomorphism if $f(x + y) = f(x) + f(y)$ and $f(x\delta y) = f(x)\delta f(y)$ for all$x, y \in L, M \in Q$.

Definition 2.14. Let two FS L and M and $Q-$s-rings and f be a function from L into$M$. If μ is a FSS of M then the inverse of μ under f is the FSS of$L$, defined by$f^{-1}(\mu)(a) = \mu(f(a)$,$\forall a \in L$.

Definition 2.15. Letτ: L →M be a homomorphism of s-rings and μ be a FSS of L. We define a FSS τ(μ) of M by $\tau(\mu)(a) = \{ \sup_{b\in\tau^{-1}(a)} \mu(b), \tau^{-1}(a) \neq \emptyset$, and zero otherwise for all $a \in M$.

Definition 2.16. Let M be a $Q-$s-ring. A FSS μ of M is called to be fuzzy $Q-$sub s-ring of M if it fulfils the following axioms

(i) $\mu(a + b) \geq min\{\mu(a), \mu(b)\}$

(ii) $\mu(a\delta b) \geq min\{\mu(a), \mu(b)\}\ for\ all\ a, b \in M, \delta \in Q$.

Definition 2.17. A FSS μ of $Q-$s-ring M is referred to as a fuzzy left (proper) ideal of M if it satisfies the subsequent situations

$(i)\ \mu(a + b) \geq min\{\mu(a), \mu(b)\}$

$(ii)\mu(a\delta b) \geq \mu(b)\big(\mu(a)\big), \forall a, b \in M, \delta \in Q$.

Definition 2.18. A FSS μ of $Q-$S-ring M is said to be a fuzzy ideal of M if it fulfils the following axioms

$(i)\ \mu(a + b) \geq min\{\mu(a), \mu(b)\}$

$(ii)\ \mu(x\delta y) \geq max\{\mu(a), \mu(b)\} \forall a, b \in M, \delta \in Q$.

Definition 2.19. Let U be an initial set and E be the set of parameters. Let $P(U)$ denotes the power set of$U$. A pair (f, E) is referred to as a soft set over U in which f is a mapping given with the aid of $f : E \to P(U)$.

Definition 2.20. Let (f, A) be soft set, the set $\{a \in A \mid f(a) \neq \emptyset\}$ is said be a sustenance of (f, A), denoted by$Sus(f, A)$. If $Sus(f, A) \neq \emptyset$ then (f, A) is called a not a null soft set.

Definition 2.21. Let (f, S), (g, T) be FSS over$U$. Then (f, S) is stated to be FSS of (g, T), meant by $(f, S) \subseteq (g, T)$ if $S \subseteq T$ and $f(x) \subseteq g(x)$ for all$x \in A$.

Definition 2.22. Let U be an initial big set,E is parameters set and$A \subseteq E$. A pair (f, A) is identified as a FS set over U in which f is a function given by way of $f : A \rightarrow [0,1]_u$ where $[0,1]_u$ denotes the gathering of all fuzzy subsets of$U$.

Definition 2.23. Let X be a group and (f, A) be a SS over$X$. Then (f, A) is stated to be soft institution over X if and simplest if $f(a)$ is a subgroup of X for every$a \in A$.

Definition 2.24. Let X be a set and (f, A) be FSS over$X$. Then (f, A) is called FS organization over X iff for every $x \in A, a, b \in X$ (i) $f_x(a * b) \geq f_x(a) * f_x(b)$

(ii) $f_x(a^{-1}) \geq f_x(a)$ Where f_x is the FSS of X equivalent to the parameter$x \in A$.

Definition 2.25. Let Mbe a Q-S-ring,E be a parameter set and A ⊆ E. Let f be a function given by using $f : A \rightarrow P(M)$ in which $P(M)$the power is set of$M$. Then (f, A) is called a soft $Q-$S-ring over S if and handiest if for every $x \in A$, $f(x)$ is $Q-$sub S-ring of$M$. That is

(i) $a, b \in M \Rightarrow a + b \in f(x)$

(ii) (ii) $a. b \in M, \delta \in Q \Rightarrow a\delta b \in f(a)$.

3. Fuzzy Soft $Q-$ S-ring homomorphism

In this segment, the concept of fuzzy tender Q−S-ring homomorphism is brought
And studied their residences.

Definition 3.1.Let (f, A) and (g, B) be FSs over $Q-$s-rings L and M respectively. Let$\tau : L \rightarrow M$ and $\rho: A \rightarrow B$ be functions wherein A and B are parameter sets for the crisp units L and S respectively. Then the pair (τ, ρ) is said to be a FS distinctive from L to M.

Definition.3.2. Let (f, A) and(g, B) be fuzzy SS over $Q-$ s-rings L and M respectively and (τ, ρ) be FS feature from L to$M$. Then (τ, ρ) is stated to be FS $Q-$ s-ring homomorphism if the subsequent situations preserve.

(i) τ is a $Q-$ s-ring homomorphism from L onto S.

(ii) ρ is a mapping from A onto B.

(iii) $\tau(f_x) = g\ (x)$ for all $x \in A$.

Definition 3.3. If ∃ a FS $Q-$ s-ring homomorphism among (f, A) and (g, B) FS s-rings, we say that (f, A) is soft homomorphic to (g, B).

Theorem 3.1. Let (f, A) be a FS $Q-$ s-ring over a $Q-$s-ring S. If $\vartheta : L \rightarrow M$ be an onto homomorphism and for each $x \in A$, define $(\vartheta f)_x(a) = f_x(\vartheta(a))$, $\forall\, a \in L$ then $(\vartheta f, A)$ is a FS $Q-$s-ring over M.

Proof: Let $a, b \in L, x \in A$ and $\lambda \in Q$.Then

$$(\vartheta f)_x(a+b) = f_x(\vartheta(a+b)$$
$$= f_x\big(\vartheta(a) + \vartheta(b)\big)$$
$$\geq \min\{f_x\big(\vartheta(a)\big), f_x\big(\vartheta(b)\big)\}$$
$$= \min\{(\vartheta f)_x(a), (\vartheta f)_x(b)\big)\}$$
$$\text{And } (\vartheta f)_x(a\lambda b) = f_x\big(\vartheta(a\lambda b)\big)$$
$$= f_x\big(\vartheta(a)\lambda\vartheta(b)\big)$$
$$\geq \min\{f_x\big(\vartheta(a)\big), f_x\big(\vartheta(b)\big)\}$$
$$= \min\{(\vartheta f)_x(a), (\vartheta f)_x(b)\}$$

Therefore $(\vartheta f)_x$ is a fuzzy Q-sub s-ring of M.Hence $(\vartheta f, A)$ is a FSQ-sub S-ring over M.

Theorem 3.2. Let (δ, A) be a FS s-ring over $Q-$ s-ring$L$. If ϑ is an endomorphism of L and define $(\delta\vartheta)_x = \delta_x\vartheta$ for each $x \in A$ then $(\delta\vartheta, A)$ is a FS Q- s-ring over Q- s-ring L.

Proof:

Let $a, b \in L, x \in A$ and $\lambda \in Q$.Then

$$(\vartheta\delta)_x(a+b) = \delta_x(\vartheta(a+b)$$
$$= \delta_x\big(\vartheta(a) + \vartheta(b)\big)$$
$$\geq \min\{\delta_x\big(\vartheta(a)\big), f_x\big(\vartheta(b)\big)\}$$
$$= \min\{(\vartheta\delta)_x(a), (\vartheta\delta)_x(b)\big)\}$$

And
$$(\vartheta\delta)_x(a\lambda b) = \delta_x\big(\vartheta(a\lambda b)\big)$$
$$= \delta_x\big(\vartheta(a)\lambda\vartheta(b)\big)$$
$$\geq \min\{\delta_x\big(\vartheta(a)\big), \delta_x\big(\vartheta(b)\big)\}$$
$$= \min\{(\vartheta\delta)_x(a), (\vartheta\delta)_x(b)\}$$

Therefore $(\vartheta\delta)_x$ is a fuzzy Q-sub s-ring of M.Hence $(\vartheta\delta, A)$ is a FSQ-sub s-ring over M.

Theorem 3.3. Let the onto homomorphism $\delta : L \rightarrow M$ be a function of $Q-$s-rings and (δ, A) be a FSo left ideal over Q -s-ring$M$. If for each$x \in A$, $\gamma_x = \delta^{-1}(\theta_x)$ then (δ, A) is an FSo left ideal over Q -s-ring L.

Proof: Let $x \in A$ and $\lambda \in Q$ then θ_x be the FSo left ideal of Q-s-ring M.

Let $a, b \in L$ and $\lambda \in Q$ then

$$\delta^{-1}(\theta_x)(a+b) = \theta_x\big(\delta(a+b)\big)$$
$$= \theta_x\{(\delta(a)+\delta(b)\}$$
$$\geq \min\{\theta_x\big(\delta(a)\big), \theta_x\big(\delta(b)\big)\}$$
$$= \min\{\delta^{-1}(\theta_x)(a), \delta^{-1}(\theta_x)(b)\}, \text{And}$$
$$\delta^{-1}(\theta_x)(a\lambda b) = \theta_x\big(\delta(a\lambda b)\big)$$
$$= \theta_x(\delta(a)\lambda\delta(b)\}$$
$$\geq \theta_x\big(\delta(b)\big)$$
$$= \delta^{-1}(\theta_x)(b)$$

Hence , $\gamma_x = \delta^{-1}(\theta_x)$ is a fuzzy ideal of Q-s-ring L.Hence (γ, A) is a FS ideal over Q-s-ring L.

Lemma 3.4. Let L and M be $Q-$ s-rings, $\delta : L \rightarrow M$ be a Q- s-ring homomorphism and f be a δ invertible FS of L. If $a = \delta(x)$ then$\delta f(a) = f(x), x \in L$.

Proof: Let $L$and M be a Q-s-rings,τ: $L \rightarrow M$ be a Q-s-ring momomorphism and f be δ invertible fuzzy ideal of L.Assume $x \in L$ and $\delta(a) = x$ then $\delta^{-1}(a) = x$.Let $t \in \delta^{-1}(a)$. Then $\delta(t) = a = \delta(x)$.Since f is δ invertible fuzzy subset of L,therefore $\delta(f)(a) = \sup\limits_{t\in\delta^{-1}(a)} f(t) = f(a)$ and hence $\delta(f)(a) = f(x)$.

Theorem.3.5. Let $L$and τ be a homomorphism from L onto M and (δ, A) be an FS left ideal of over Q-s-ring and for each $z \in A, \delta_z$ is a τ invertiable fuzzy left ideal of L,if $\gamma_z = \tau(\delta_z)$ and $z \in A$,then (γ, A) is a FS left ideal Q-s-ring L.

Proof: Let $a, b \in M$,$z \in A$ and $\gamma \in Q$.Then $\exists x, y \in L$ such that $\tau(x) = a$ and $\tau(y) = b, a + b = \tau(x+y), a\gamma b = \tau(a\gamma b)$.since δ_z is τ invertible by above lemma (3.4) we have

$$\gamma_z(a+b) = \tau(\delta_z)(a+b)$$
$$= \delta_z(x+y)$$
$$\geq \min\{\delta_z(x), \delta_z(y)\}$$
$$= \min\{\tau(\delta_z)(a), \tau(\delta_z)(b)\}$$
$$= \min\{\gamma_z(a), \gamma_z(b)\}, \text{ and}$$
$$\gamma_z(a\gamma b) = \tau(\delta_z)(a\gamma b)$$
$$= \delta_z\big(\tau(a\gamma b)\big)$$
$$= \delta_z[(\tau(x)\gamma\tau(y)]$$

$$\geq \gamma_z(\tau(y))$$
$$= \tau(\delta_z)b$$
$$= \gamma_z$$

Therefore γ_z is the left ideal of L.Hence (γ, A) is a FS left ideal Q-s-ring$L$.

4. Conclusion

In this paper, we added the perception of fuzzy soft $Q-$ s-ring homomorphism and studied a few residences of holomorphic image of fuzzy soft $Q-$ s-ring. In the next paper, we have a look at properties of kernel of fuzzy soft $Q-$ s-ring homomorphism, fuzzy soft top principles and fuzzy soft filters over $Q-$s-rings

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