BIPOLAR VALUED VAGUE SUBSEMIRINGS OF A SEMIRING

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ABSTRACT. In this paper, bipolar valued vague subsemiring of a semiring is introduced and some properties are discussed. Bipolar valued vague subsemiring of a semiring is a generalized form of vague subsemiring of the semiring, vague subsemiring of the semiring is a generalized form of fuzzy subsemiring of the semiring and fuzzy subsemiring of the semiring is a generalized form of semiring.

KEY WORDS. Fuzzy subset, vague subset, bipolar valued fuzzy subset, bipolar valued vague subset, bipolar valued vague subsemiring, intersection, product, strongest, height.

INTRODUCTION: In 1965, Zadeh [12] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval valued fuzzy sets, vague sets, soft sets etc. Grattan-Guiness [6] discussed about fuzzy membership mapped onto interval and many valued quantities. Vague set is an extension of fuzzy set and it is appeared as a unique case of context dependent fuzzy sets. The vague set was introduced by W.L.Gau and D.J.Buehrer [5]. Lee [7] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [7, 8]. Fuzzy subgroup was introduced by Azriel Rosenfeld [2]. RanjitBiswas [10] introduced the vague groups. Cicily Flora. S and Arockiarani.I [4] have introduced a new class of generalized bipolar vague sets. Anitha.M.S., et.al.[1] defined as bipolar valued fuzzy subgroups of a group and Balasubramanian.A et.al[3] have defined the bipolar interval valued fuzzy subgroups of a group. K.Murugalingam and K.Arjunan[9] have discussed about interval valued fuzzy subsemiring of a semiring and then bipolar valued multi fuzzy subsemirings of a semiring have been introduced by Yasodara.B and KE.Sathappan[11]. Here, the concept of bipolar valued vague subsemiring of a semiring is introduced and estaiblished some results.

1.PRELIMINARIES.

Definition 1.1. [12] Let X be any nonempty set. A mapping M : X → [0, 1] is called a fuzzy subset of X.

Definition 1.2. [5] A vague set A in the universe of discourse U is a pair [t_A, $1-f_A$],where t_A : U→ [0, 1] and f_A : U→ [0, 1] are mappings, they are called truth membership function and false membership function respectively. Here $t_A(x)$ is a lower bound of the grade of membership of x derived from the evidence for x and $f_A(x)$ is a lower bound on the negation of x derived from the evidence against x and $t_A(x) + f_A(x) \leq 1$, for all x∈U.

Definition 1.3. [5] The interval [$t_A(x), 1-f_A(x)$] is called the vague value of x in A and it is denoted by $V_A(x)$, i.e., $V_A(x) = [\, t_A(x), 1-f_A(x)\,]$.

Example 1.4. A = { < a, [0.4, 0.6] >, < b, [0.6, 0.8] >, < c, [0.3, 0.9] >} is a vague subset of X = {a, b, c }.

Definition 1.5. [7] A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form A = { < x, $A^+(x)$, $A^-(x)$ >/ x∈X}, where $A^+ : X\to [0, 1]$ and $A^- : X\to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.6. A = { < a, 0.4, −0.2 >, < b, 0.6, −0.5 >, < c, 0.3, −0.7 >} is a bipolar valued fuzzy subset of X = {a, b, c }.

Definition 1.7. [4] A bipolar valued vague subset A in X is defined as an object of the form A = { ⟨ x, [$t_A^+(x), 1-f_A^+(x)$], [$-1-f_A^-(x), t_A^-(x)$] ⟩ / x∈X }, where $t_A^+: X\to [0, 1]$, $f_A^+ : X\to [0, 1]$, $t_A^- : X\to [-1, 0]$ and $f_A^- : X\to [-1, 0]$ are mapping such that $t_A(x) + f_A(x) \le 1$ and $-1 \le t_A^- + f_A^-$. The positive interval membership degree [$t_A^+(x), 1-f_A^+(x)$] denotes the satisfaction region of an element x to the property corresponding to a bipolar valued vague subset A and the negative interval membership degree [$-1-f_A^-(x), t_A^-(x)$] denotes the satisfaction region of an element x to some implicit counter-property corresponding to a bipolar valued vague subset A. Bipolar valued vague subset A is denoted as A = { ⟨ x, $V_A^+(x)$, $V_A^-(x)$⟩ / x∈X }, where $V_A^+(x),= [\, t_A^+(x), 1-f_A^+(x)\,]$ and $V_A^-(x) = [-1-f_A^-(x), t_A^-(x)]$.

Note that. [0] = [0, 0], [1] = [1, 1] and [−1] = [−1, −1].

Example 1.8. [A] = { < a, [0.4, 0,6], [−0.5, −0.2] >, < b, [0.2, 0.4], [−0.6, −0.3] >, < c, [0.1, 0.6], [−0.6, −0.2] > } is a bipolar valued vague subset of X = {a, b, c }.

Definition 1.9. [4] Let A = ⟨V_A^+, V_A^-⟩ and B = ⟨V_B^+, V_B^-⟩ be two bipolar valued vague subsets of a set X. We define the following relations and operations:

(i) [A] ⊂ [B] if and only if $V_A^+(u) \le V_B^+(u)$ and $V_A^-(u) \ge V_B^-(u)$, ∀ u∈X.

(ii) [A] = [B] if and only if $V_A^+(u) = V_B^+(u)$ and $V_A^-(u) = V_B^-(u)$, ∀ u∈X.

(iii) [A]∩[B] = { ⟨ u, rmin ($V_A^+(u)$, $V_B^+(u)$), rmax ($V_A^-(u)$, $V_B^-(u)$) ⟩ / u∈X }.

(iv) [A]∪[B] = { ⟨ u, rmax ($V_A^+(u)$, $V_B^+(u)$), rmin ($V_A^-(u)$, $V_B^-(u)$) ⟩ / u∈X }. Here rmin ($V_A^+(u)$, $V_B^+(u)$) = [min { $t_A^+(x), t_B^+(x)$ }, min {$1-f_A^+(x)$, $1-f_B^+(x)$}], rmax ($V_A^+(u)$, $V_B^+(u)$) = [max { $t_A^+(x), t_B^+(x)$ }, max {$1-f_A^+(x)$, $1-f_B^+(x)$ }], rmin ($V_A^-(u)$, $V_B^-(u)$) = [min {$-1-f_A^-(x)$, $-1-f_B^-(x)$}, min { $t_A^-(x), t_B^-(x)$ }], rmax ($V_A^-(u)$, $V_B^-(u)$) = [max {$-1-f_A^-(x)$, $-1-f_B^-(x)$ }, max { $t_A^-(x), t_B^-(x)$ }].

Definition 1.10. Let R be a semiring. A bipolar valued vague subset A of R is said to be a bipolar valued vague subsemiring of R (BVVSSR) if the following conditions are satisfied,

(i) $V_A^+(x+y) \ge$ rmin{ $V_A^+(x)$, $V_A^+(y)$ }

(ii) $V_A^+(xy) \ge$ rmin{ $V_A^+(x)$, $V_A^+(y)$ }

(iii) $V_A^-(x+y) \le$ rmax{ $V_A^-(x)$, $V_A^-(y)$ }

(iv) $V_A^-(xy) \le$ rmax{ $V_A^-(x)$, $V_A^-(y)$ } for all x and y in R.

Example 1.11. Let R = Z_3 = { 0, 1, 2 } be a semiring with respect to the ordinary addition and multiplication. Then A = { < 0, [0.5, 0.7], [– 0.8, – 0.5] >, < 1, [0.4, 0.6], [– 0.7, – 0.4] >, < 2, [0.4, 0.6], [– 0.7, – 0.4] > } is a bipolar valued vague subsemiring of R.

Definition 1.12. Let A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ be any two bipolar valued vague subsets of sets G and H, respectively. The product of A and B, denoted by A×B, is defined as A×B = { $\langle$ (x, y), $V_{A\times B}^+$(x, y), $V_{A\times B}^-$(x, y) $\rangle$ / for all x in G and y in H } where $V_{A\times B}^+$(x, y) = rmin { V_A^+(x), V_B^+(y) } and $V_{A\times B}^-$(x, y) = rmax { V_A^-(x), V_B^-(y) } for all x in G and y in H.

Definition 1.13. Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subset in a set S, the strongest bipolar valued vague relation on S, that is a bipolar valued vague relation on A is V = {$\langle$ (x, y), V_V^+(x, y), V_V^-(x, y) $\rangle$ / x and y in S} given by V_V^+(x, y) = rmin{ V_A^+(x), V_A^+(y) } and V_V^-(x, y) = rmax { V_A^-(x), V_A^-(y) } for all x and y in S.

Definition 1.14. Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subset of X. Then the height H(A) = $\langle$ H(V_A^+), H(V_A^-) $\rangle$ is defined as H(V_A^+) = rsupV_A^+(x) for all x in X and H(V_A^-) = rinfV_A^-(x) for all x in X.

Definition 1.15. Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subset of X. Then $^{\oplus}$A = $\langle {}^{\oplus}V_A^+, {}^{\oplus}V_A^- \rangle$ is defined as $^{\oplus}V_A^+$(x) = V_A^+(x) + [1] – H(V_A^+) and $^{\oplus}V_A^-$(x) = V_A^-(x) – [1] – H(V_A^-) for all x in X.

2. THEOREMS.

Theorem 2.1. Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subsemiring of a semiring R. (i) If V_A^+(x+y) = [0] then either V_A^+(x) = [0] or V_A^+(y) = [0] for x, y in R. (ii) If V_A^+(xy) = [0] then either V_A^+(x) = [0] or V_A^+(y) = [0] for x, y in R. (iii) If V_A^-(x+y) = [0] then either V_A^-(x) = [0] or V_A^-(y) = [0] for x, y in R. (iv) If V_A^-(xy) = [0] then either V_A^-(x) = [0] or V_A^- (y) = [0] for x, y in R.

Proof. Let x, y in R. (i) By the definition V_A^+(x+y) ≥ rmin { V_A^+(x), V_A^+(y) } which implies that [0] ≥ rmin { V_A^+(x), V_A^+(y) }. Therefore either V_A^+(x) = [0] or V_A^+(y) = [0]. (ii) By the definition V_A^+(xy) ≥ rmin { V_A^+(x), V_A^+(y) } which implies that [0] ≥ rmin { V_A^+(x), V_A^+(y) }. Therefore either V_A^+(x) = [0] or V_A^+(y) = [0]. (iii) By the definition V_A^-(x+y) ≤ rmax { V_A^-(x), V_A^-(y) } which implies that [0] ≤ rmax {V_A^-(x), V_A^-(y) }. Therefore either V_A^-(x) = [0] or V_A^-(y) = [0]. (iv) By the definition V_A^-(xy) ≤ rmax { V_A^-(x), V_A^-(y) } which implies that [0] ≤ rmax { V_A^-(x), V_A^-(y) }. Therefore either V_A^-(x) = [0] or V_A^-(y) = [0].

Theorem 2.2. If A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subsemiring of a semiring R, then H = { x∈R | V_A^+(x) = [1], V_A^-(x) = [−1] } is either empty or a subsemiring of R.

Proof. If no element satisfies this condition then H is empty. If x and y in H then V_A^+(x+y) ≥ rmin { V_A^+(x), V_A^+(y) } = rmin { [1], [1] } = [1]. Therefore V_A^+(x+y) = [1]. And V_A^+(xy) ≥ rmin { V_A^+(x), V_A^+(y) } = rmin { [1], [1] } = [1]. Therefore V_A^+(xy) = [1]. Also V_A^-(x+y) ≤ rmax { V_A^-(x), V_A^-(y) } = rmax { [−1], [−1] } = [−1]. Therefore V_A^-(x+y) = [−1]. And V_A^-(xy) ≤ rmax { V_A^-(x), V_A^-(y) } = rmax { [−1], [−1] } = [−1]. Therefore V_A^-(xy) = [−1].

That is x+y∈H and xy∈H. Hence H is a subsemiring of R. Hence H is either empty or a subsemiring of R.

Theorem 2.3. If A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ are two bipolar valued vague subsemirings of a semiring R, then their intersection A∩B is a bipolar valued vague subsemiring of R.

Proof. Let C = A∩B and let x, y in R. Now V_C^+(x+y) = rmin { V_A^+(x+y), V_B^+(x+y) } ≥ rmin { rmin { V_A^+(x), V_A^+(y) }, rmin { V_B^+(x), V_B^+(y) } } ≥ rmin { rmin { V_A^+(x), V_B^+(x) }, rmin {V_A^+(y), V_B^+(y)} } = rmin { V_C^+(x), V_C^+(y) }. Therefore V_C^+(x+y) ≥ rmin{ V_C^+(x), V_C^+(y) }, for all x, y in R. And V_C^+(xy) = rmin { V_A^+(xy), V_B^+(xy) } ≥ rmin { rmin{V_A^+(x), V_A^+(y) }, rmin { V_B^+(x), V_B^+(y) }} ≥ rmin { rmin { V_A^+(x), V_B^+(x) }, rmin { V_A^+(y), V_B^+(y) } } = rmin { V_C^+(x), V_C^+(y) }. Therefore V_C^+(xy) ≥ rmin { V_C^+(x), V_C^+(y) }, for all x, y in R. Also V_C^-(x+y) = rmax { V_A^-(x+y), V_B^-(x+y) } ≤ rmax { rmax {V_A^-(x), V_A^-(y) }, rmax{ V_B^-(x), V_B^-(y) }} ≤ rmax { rmax { V_A^-(x), V_B^-(x) }, rmax{ V_A^-(y), V_B^-(y) } } = rmax { V_C^-(x), V_C^-(y) }. Therefore V_C^-(x+y) ≤ rmax { V_C^-(x), V_C^-(y) }, for all x, y in R. And V_C^-(xy) = rmax { V_A^-(xy), V_B^-(xy) } ≤ rmax { rmax { V_A^-(x), V_A^-(y) }, rmax { V_B^-(x), V_B^-(y) } } ≤ rmax { rmax {V_A^-(x), V_B^-(x) }, rmax {V_A^-(y), V_B^-(y) } }= rmax { V_C^-(x), V_C^-(y) }. Therefore V_C^-(xy) ≤ rmax {V_C^-(x), V_C^-(y) }, for all x, y in R. Hence A∩B is a bipolar valued vague subsemiring of R.

Theorem 2.4. The intersection of a family of bipolar valued vague subsemirings of a semiring R is a bipolar valued vague subsemiring of R.

Proof. The proof follows from the Theorem 2.3.

Theorem 2.5. If A = $\langle V_A^+, V_A^- \rangle$ and B = $\langle V_B^+, V_B^- \rangle$ are any two bipolar valued vague subsemirings of the semirings R_1 and R_2 respectively, then A×B = $\langle V_{A\times B}^+, V_{A\times B}^- \rangle$ is a bipolar valued vague subsemiring of $R_1 \times R_2$.

Proof. Let x_1, x_2 be in R_1, y_1 and y_2 be in R_2. Then (x_1, y_1) and (x_2, y_2) are in $R_1 \times R_2$. Now, $V_{A\times B}^+$[(x_1, y_1) + (x_2, y_2)] = $V_{A\times B}^+$(x_1+x_2, y_1+y_2) = rmin { V_A^+(x_1+x_2), V_B^+(y_1+y_2) } ≥ rmin { rmin{ V_A^+(x_1), V_A^+(x_2) }, rmin { V_B^+(y_1), V_B^+(y_2) }} = rmin { rmin { V_A^+(x_1), V_B^+(y_1) }, rmin { V_A^+(x_2), V_B^+(y_2) } } = rmin { $V_{A\times B}^+$(x_1, y_1), $V_{A\times B}^+$(x_2, y_2) }. Therefore $V_{A\times B}^+$[(x_1, y_1)+ (x_2, y_2)] ≥ rmin { $V_{A\times B}^+$(x_1, y_1), $V_{A\times B}^+$(x_2, y_2) }. And $V_{A\times B}^+$[(x_1, y_1)(x_2, y_2)] = $V_{A\times B}^+$(x_1x_2, y_1y_2) = rmin { V_A^+(x_1x_2), V_B^+(y_1y_2) } ≥ rmin { rmin { V_A^+(x_1), V_A^+(x_2) }, rmin { V_B^+(y_1), V_B^+(y_2) } } = rmin { rmin { V_A^+(x_1), V_B^+(y_1) }, rmin { V_A^+(x_2), V_B^+(y_2) } } = rmin { $V_{A\times B}^+$(x_1, y_1), $V_{A\times B}^+$(x_2, y_2) }. Therefore $V_{A\times B}^+$[(x_1, y_1)(x_2, y_2)] ≥ rmin{ $V_{A\times B}^+$(x_1, y_1), $V_{A\times B}^+$(x_2, y_2) }. Also $V_{A\times B}^-$[(x_1, y_1) + (x_2, y_2)] = $V_{A\times B}^-$(x_1+x_2, y_1+y_2) = rmax {V_A^-(x_1+x_2), V_B^-(y_1+y_2) } ≤ rmax { rmax { V_A^-(x_1), V_A^-(x_2) }, rmax { V_B^-(y_1), V_B^-(y_2) }} = rmax { rmax { V_A^-(x_1), V_B^-(y_1) }, rmax {V_A^-(x_2), V_B^-(y_2) }} = rmax {$V_{A\times B}^-$(x_1, y_1), $V_{A\times B}^-$(x_2, y_2) }. Therefore $V_{A\times B}^-$[(x_1, y_1) +(x_2, y_2)] ≤ rmax { $V_{A\times B}^-$(x_1, y_1), $V_{A\times B}^-$(x_2, y_2) }. And $V_{A\times B}^-$[(x_1, y_1)(x_2, y_2)] = $V_{A\times B}^-$(x_1x_2, y_1y_2) = rmax { V_A^-(x_1x_2), V_B^-(y_1y_2) } ≤ rmax { rmax { V_A^-(x_1), V_A^-(x_2) }, rmax { V_B^-(y_1), V_B^-(y_2) } } = rmax { rmax {V_A^-(x_1), V_B^-(y_1) }, rmax { V_A^-(x_2), V_B^-(y_2) } } = rmax { $V_{A\times B}^-$(x_1, y_1),

$V^{-}_{A\times B}$ (x2, y2) }. Therefore $V^{-}_{A\times B}$ [(x1, y1)(x2, y2)] ≤ rmax{ $V^{-}_{A\times B}$ (x1, y1), $V^{-}_{A\times B}$ (x2, y2) }. Hence A×B is a bipolar valued vague subsemiring of $R_1\times R_2$.

Theorem 2.6. Let A = ⟨V^{+}_{A}, V^{-}_{A}⟩ be a bipolar valued vague subset of a semiring R and V = ⟨ V^{+}_{V}, V^{-}_{V} ⟩ be the strongest bipolar valued vague relation of R. Then A is a bipolar valued vague subsemiring of R if and only if V is a bipolar valued vague subsemiring of R×R.

Proof. Suppose that A is a bipolar valued vague subsemiring of R. Then for any x = (x_1, x_2), y = (y_1, y_2) are in R×R. Now V^{+}_{V}(x+y) = V^{+}_{V}[(x_1, x_2)+(y_1, y_2)] = V^{+}_{V}(x_1+y_1, x_2+y_2) = rmin { V^{+}_{A}(x_1+y_1), V^{+}_{A}(x_2+y_2) } ≥ rmin { rmin { V^{+}_{A}(x_1), V^{+}_{A}(y_1) }, rmin {V^{+}_{A}(x_2), V^{+}_{A}(y_2) } } = rmin { rmin { V^{+}_{A}(x_1), V^{+}_{A}(x_2) }, rmin { V^{+}_{A}(y_1), V^{+}_{A}(y_2) } } = rmin {V^{+}_{V}(x_1, x_2), V^{+}_{V}(y_1, y_2) } = rmin { V^{+}_{V}(x), V^{+}_{V}(y) }. Therefore V^{+}_{V}(x+y) ≥ rmin { V^{+}_{V}(x), V^{+}_{V}(y) } for all x, y in R×R. And V^{+}_{V}(xy) = V^{+}_{V}[(x_1, x_2)(y_1, y_2)] = V^{+}_{V}(x_1y_1, x_2y_2) = rmin {V^{+}_{A}(x_1y_1), V^{+}_{A}(x_2y_2) } ≥ rmin { rmin {V^{+}_{A}(x_1), V^{+}_{A}(y_1) }, rmin {V^{+}_{A}(x_2), V^{+}_{A}(y_2) } } = rmin { rmin {V^{+}_{A}(x_1), V^{+}_{A}(x_2) }, rmin {V^{+}_{A}(y_1), V^{+}_{A}(y_2) } } = rmin { V^{+}_{V}(x_1, x_2), V^{+}_{V}(y_1, y_2) }= rmin {V^{+}_{V}(x), V^{+}_{V}(y) }. Therefore V^{+}_{V}(xy) ≥ rmin {V^{+}_{V}(x), V^{+}_{V}(y) } for all x and y in R×R. Also we have V^{-}_{V}(x+y) = V^{-}_{V}[(x_1, x_2)+(y_1, y_2)] = V^{-}_{V}(x_1+y_1, x_2+y_2) = rmax {V^{-}_{A}(x_1+y_1), V^{-}_{A}(x_2+y_2) } ≤ rmax { rmax { V^{-}_{A}(x_1), V^{-}_{A}(y_1) }, rmax {V^{-}_{A}(x_2), V^{-}_{A}(y_2) } } = rmax { rmax {V^{-}_{A}(x_1), V^{-}_{A}(x_2) }, rmax { V^{-}_{A}(y_1), V^{-}_{A}(y_2) }}= rmax { V^{-}_{V}(x_1, x_2), V^{-}_{V}(y_1, y_2) } = rmax { V^{-}_{V}(x), V^{-}_{V}(y) }. Therefore V^{-}_{V}(x+y) ≤ rmax { V^{-}_{V}(x), V^{-}_{V}(y) } for all x, y in R×R. And V^{-}_{V}(xy) = V^{-}_{V}[(x_1, x_2)(y_1, y_2)] = V^{-}_{V}(x_1y_1, x_2y_2) = rmax { V^{-}_{A}(x_1y_1), V^{-}_{A}(x_2y_2) } ≤ rmax { rmax { V^{-}_{A}(x_1), V^{-}_{A}(y_1) }, rmax {V^{-}_{A}(x_2), V^{-}_{A}(y_2) } } = rmax { rmax {V^{-}_{A}(x_1), V^{-}_{A}(x_2) }, rmax { V^{-}_{A}(y_1), V^{-}_{A}(y_2) } } = rmax { V^{-}_{V}(x_1, x_2), V^{-}_{V}(y_1, y_2) } = rmax { V^{-}_{V}(x), V^{-}_{V}(y) }. Therefore V^{-}_{V}(xy) ≤ rmax { V^{-}_{V}(x), V^{-}_{V}(y) } for all x, y in R×R. This proves that V is a bipolar valued vague subsemiring of R×R. Conversely assume that V is a bipolar valued vague subsemiring of R×R, then for any x = (x_1, x_2) and y = (y_1, y_2) are in R×R, we have rmin { V^{+}_{A}(x_1+y_1), V^{+}_{A}(x_2+y_2)} = V^{+}_{V}(x_1+y_1, x_2+y_2) = V^{+}_{V}[(x_1, x_2)+(y_1, y_2)] = V^{+}_{V}(x+y) ≥ rmin{ V^{+}_{V}(x), V^{+}_{V}(y) } = rmin{ V^{+}_{V}(x_1, x_2), V^{+}_{V}(y_1, y_2) } = rmin{ rmin{ V^{+}_{A}(x_1), V^{+}_{A}(x_2) }, rmin{ V^{+}_{A}(y_1), V^{+}_{A}(y_2) }}. Suppose V^{+}_{A}(x_1+y_1) ≤ V^{+}_{A}(x_2+y_2), V^{+}_{A}(x_1) ≤ V^{+}_{A}(x_2) and V^{+}_{A}(y_1) ≤ V^{+}_{A}(y_2), we get, V^{+}_{A}(x_1+y_1) ≥ rmin { V^{+}_{A}(x_1), V^{+}_{A}(y_1) } for all x_1 and y_1 in R. And rmin { V^{+}_{A}(x_1y_1), V^{+}_{A}(x_2y_2) } = V^{+}_{V}(x_1y_1, x_2y_2) = V^{+}_{V}[(x_1, x_2)(y_1, y_2)] = V^{+}_{V}(xy) ≥ rmin{V^{+}_{V}(x), V^{+}_{V}(y)} = rmin{ V^{+}_{V}(x_1, x_2), V^{+}_{V}(y_1, y_2) } = rmin { rmin { V^{+}_{A}(x_1), V^{+}_{A}(x_2) }, rmin { V^{+}_{A}(y_1), V^{+}_{A}(y_2) } }. Suppose V^{+}_{A}(x_1y_1) ≤ V^{+}_{A}(x_2y_2), V^{+}_{A}(x_1) ≤ V^{+}_{A}(x_2) and V^{+}_{A}(y_1) ≤ V^{+}_{A}(y_2), we get V^{+}_{A}(x_1y_1) ≥ rmin { V^{+}_{A}(x_1), V^{+}_{A}(y_1) } for all x_1 and y_1 in R. Also we have rmax { V^{-}_{A}(x_1+y_1), V^{-}_{A}(x_2+y_2) } = V^{-}_{V}(x_1+y_1, x_2+y_2) = V^{-}_{V}[(x_1, x_2)+(y_1, y_2)] = V^{-}_{V}(x+y) ≤ rmax {V^{-}_{V}(x), V^{-}_{V}(y) } = rmax { V^{-}_{V}(x_1, x_2), V^{-}_{V}(y_1, y_2) }= rmax { rmax { V^{-}_{A}(x_1), V^{-}_{A}(x_2) }, rmax { V^{-}_{A}(y_1), V^{-}_{A}(y_2) } }. Suppose V^{-}_{A}(x_1+y_1) ≥ V^{-}_{A}(x_2+y_2), V^{-}_{A}(x_1) ≥ V^{-}_{A}(x_2) and V^{-}_{A}(y_1) ≥ V^{-}_{A}(y_2), we get V^{-}_{A}(x_1+y_1) ≤ rmax {V^{-}_{A}(x_1), V^{-}_{A}(y_1) } for all x_1 and y_1 in R. And rmax {V^{-}_{A}(x_1y_1), V^{-}_{A}(x_2y_2) }

$= V_V^-(x_1y_1, x_2y_2) = V_V^-[(x_1, x_2)(y_1, y_2)] = V_V^-(xy) \leq \text{rmax}\{V_V^-(x), V_V^-(y)\} = \text{rmax}\{V_V^-(x_1, x_2), V_V^-(y_1, y_2)\} = \text{rmax}\{\text{rmax}\{V_A^-(x_1), V_A^-(x_2)\}, \text{rmax}\{V_A^-(y_1), V_A^-(y_2)\}\}$. Suppose $V_A^-(x_1y_1) \geq V_A^-(x_2y_2)$, $V_A^-(x_1) \geq V_A^-(x_2)$ and $V_A^-(y_1) \geq V_A^-(y_2)$, we get $V_A^-(x_1y_1) \leq \text{rmax}\{V_A^-(x_1), V_A^-(y_1)\}$ for all x_1 and y_1 in R. Hence A is a bipolar valued vague subsemiring of R.

Theorem 2.7. If A = $\langle V_A^+, V_A^- \rangle$ is a bipolar valued vague subsemiring of a semiring R, then $^{\oplus}A = \langle {}^{\oplus}V_A^+, {}^{\oplus}V_A^- \rangle$ is a bipolar valued vague subsemiring of the semiring R.

Proof. Let x and y in R. Now $^{\oplus}V_A^+(x+y) = V_A^+(x+y) + [1] - H(V_A^+) \geq \text{rmin}\{V_A^+(x), V_A^+(y)\} + [1] - H(V_A^+) = \text{rmin}\{V_A^+(x) + [1] - H(V_A^+), V_A^+(y) + [1] - H(V_A^+)\} = \text{rmin}\{{}^{\oplus}V_A^+(x), {}^{\oplus}V_A^+(y)\}$ which implies $^{\oplus}V_A^+(x+y) \geq \text{rmin}\{{}^{\oplus}V_A^+(x), {}^{\oplus}V_A^+(y)\}$ for all x and y in R. And $^{\oplus}V_A^+(xy) = V_A^+(xy) + [1] - H(V_A^+) \geq \text{rmin}\{V_A^+(x), V_A^+(y)\} + [1] - H(V_A^+) = \text{rmin}\{V_A^+(x) + [1] - H(V_A^+), V_A^+(y) + [1] - H(V_A^+)\} = \text{rmin}\{{}^{\oplus}V_A^+(x), {}^{\oplus}V_A^+(y)\}$ which implies $^{\oplus}V_A^+(xy) \geq \text{rmin}\{{}^{\oplus}V_A^+(x), {}^{\oplus}V_A^+(y)\}$ for all x, y in R. Also $^{\oplus}V_A^-(x+y) = V_A^-(x+y) - [1] - H(V_A^-) \leq \text{rmax}\{V_A^-(x), V_A^-(y)\} - [1] - H(V_A^-) = \text{rmax}\{V_A^-(x) - [1] - H(V_A^-), V_A^-(y) - [1] - H(V_A^-)\} = \text{rmax}\{{}^{\oplus}V_A^-(x), {}^{\oplus}V_A^-(y)\}$ which implies $^{\oplus}V_A^-(x+y) \leq \text{rmax}\{{}^{\oplus}V_A^-(x), {}^{\oplus}V_A^-(y)\}$ for all x, y in R. And $^{\oplus}V_A^-(xy) = V_A^-(xy) - [1] - H(V_A^-) \leq \text{rmax}\{V_A^-(x), V_A^-(y)\} - [1] - H(V_A^-) = \text{rmax}\{V_A^-(x) - [1] - H(V_A^-), V_A^-(y) - [1] - H(V_A^-)\} = \text{rmax}\{{}^{\oplus}V_A^-(x), {}^{\oplus}V_A^-(y)\}$ which implies $^{\oplus}V_A^-(xy) \leq \text{rmax}\{{}^{\oplus}V_A^-(x), {}^{\oplus}V_A^-(y)\}$ for all x, y in R. Hence $^{\oplus}A$ is a bipolar valued vague subsemiring of R.

Theorem 2.8. Let A = $\langle V_A^+, V_A^- \rangle$ be a bipolar valued vague subsemiring of a semiring R. Then (i) $H(V_A^+) = [1]$ if and only if $^{\oplus}V_A^+(x) = V_A^+(x)$ for all x in R

(ii) $H(V_A^-) = [-1]$ if and only if $^{\oplus}V_A^-(x) = V_A^-(x)$ for all x in R.

(iii) $^{\oplus}V_A^+(x) = [1]$ if and only if $H(V_A^+) = V_A^+(x)$ for all x in R

(iv) $^{\oplus}V_A^-(x) = [-1]$ if and only if $H(V_A^-) = V_A^-(x)$ for all x in R.

(v) $^{\oplus}({}^{\oplus}A) = {}^{\oplus}A$.

Proof. The proof is trivial.

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