BIPOLAR VALUED FUZZY GRAPH

R. Prema Sumathi, M. Palanivelrajan & K. Arjunan

Department of Mathematics, Central Institute of Plastics Engg. and Technology, CIPET: CSTS, Thiruvathavur, Madurai - 625110, Tamilnadu, India.

Email: premasumathisas@gmail.com

Department of Mathematics, Government Arts College, Melur-625106, Tamilnadu, India.

Email: palanivelrajan1975@gmail.com

Department of Mathematics, Alagappa Government Arts College, Karaikudi-630003, Tamilnadu, India.

Email: arjunan.karmegam@gmail.com

ABSTRACT: In this paper, some basic definitions, results and theorems of bipolar valued fuzzy graph are studied and proved. Fuzzy graph is the generalization of the crisp graph; bipolar valued fuzzy graph is the generalization of fuzzy graph. A new structure of bipolar valued fuzzy graph is introduced.

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KEY WORDS. Bipolar valued fuzzy subset, strong bipolar valued fuzzy relation, bipolar valued fuzzy graph, bipolar valued fuzzy loop, bipolar valued fuzzy pseudo graph, bipolar valued fuzzy spanning subgraph, bipolar valued fuzzy induced subgraph, bipolar valued fuzzy underling graph, level set, degree of bipolar valued fuzzy vertex, order of the bipolar valued fuzzy graph, size of the bipolar valued fuzzy graph.

INTRODUCTION. In 1965, fuzzy set was introduced independently by Zadeh [11]. Lee [4] introduced the notion of bipolar valued fuzzy sets. Bipolar valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree (0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0) indicates that elements somewhat satisfy the implicit counter property. Bipolar valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [4, 5]. After that the generalized of crisp graph is fuzzy graphs which was introduced by Rosenfeld [10]. Fuzzy graphs are useful to represent relationships which deal with uncertainty. The fuzzy graph has numerous applications in the field of computer science, electrical engineering system analysis, operations research, economics, networking routing, transportation, etc. NagoorGani.A [8, 9] introduced a fuzzy graph and regular fuzzy graph. Mardeson.J.N and C.S.Peng [6] introduced the operations on fuzzy graph. After that the fuzzy graph has been generalized with fuzzy loop and fuzzy multiple edges, this type of concepts was introduced by K.Arjunan and C.Subramani[1, 2]. The intuitionistic fuzzy graph with multiple edges and self loops has been introduced by K.Arjunan and C.Subramani[3]. In this paper a new structure is introduced that is bipolar valued fuzzy graph with loops and multiple edges and some results of bipolar valued fuzzy graph are stated and proved.

1.PRELIMINARIES.

Definition 1.1. ([11]) Let X be a non-empty set. A **fuzzy subset A** of X is a function A: X→ [0, 1].

Definition 1.2. ([4]) A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^+(x), A^-(x) >/ x \in X\}$, where $A^+ : X \to [0, 1]$ and $A^- : X \to [-1, 0]$. The positive membership degree $A^+(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

Example 1.3. $A = \{ < a, 0.75, -0.3 >, < b, 0.41, -0.7 >, < c, 0.5, -0.84 >\}$ is a bipolar valued fuzzy subset of $X = \{a, b, c \}$.

Definition 1.4. ([4]) Let $A = \langle A^+, A^- \rangle$ and $B = \langle B^+, B^- \rangle$ be two bipolar valued fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subset B$ if and only if $A^+(x) \leq B^+(x)$ and $A^-(x) \geq B^-(x)$ for all $x \in X$.

(ii) $A = B$ if and only if $A^+(x) = B^+(x)$ and $A^-(x) = B^-(x)$ for all $x \in X$.

(iii) $A \cap B = \{ \langle x, \min (A^+(x), B^+(x)), \max (A^-(x), B^-(x)) \rangle / x \in X \}$.

(iv) $A \cup B = \{ \langle x, \max (A^+(x), B^+(x)), \min (A^-(x), B^-(x)) \rangle / x \in X \}$.

2.BIPOLAR VALUED FUZZY GRAPH.

Definition 2.1. Let $A = \langle A^+, A^- \rangle$ be a bipolar valued fuzzy subset in a set V, the strongest bipolar valued fuzzy relation on V, that is a bipolar valued fuzzy relation on A is $S = \{ \langle (x, y), S^+(x, y), S^-(x, y) \rangle / x$ and y in V } given by $S^+(x, y) = \min \{ A^+(x), A^+(y) \}$ and $S^-(x, y) = \max \{ A^-(x), A^-(y) \}$ for all x and y in V.

Definition 2.2. Let V be any nonempty set, E be any set and $f: E \to V \times V$ be any function. Then A is a bipolar valued fuzzy subset of V, S is a bipolar valued fuzzy relation on V with respect to A and B is a bipolar valued fuzzy subset of E such that $B^+(e) \underset{e \in f^{-1}(x,y)}{\leq} S^+(x,y)$ and $B^-(e) \underset{e \in f^{-1}(x,y)}{\geq} S^-(x,y)$. Then the ordered triple F = (A, B, f) is called a **bipolar valued fuzzy graph**, where the elements of A are called **bipolar valued fuzzy points** or **bipolar valued fuzzy vertices** and the elements of B are called **bipolar valued fuzzy lines** or **bipolar valued fuzzy edges** of the bipolar valued fuzzy graph F. If f(e) = (x, y), then the bipolar valued fuzzy points $(x, A^+(x), A^-(x))$, $(y, A^+(y), A^-(y))$ are called **bipolar valued fuzzy adjacent points** and bipolar valued fuzzy point $(x, A^+(x), A^-(x))$, bipolar valued fuzzy line $(e, B^+(e), B^-(e))$ are called **incident** with each other. If two distinct bipolar valued fuzzy lines $(e_1, B^+(e_1), B^-(e_1))$ and $(e_2, B^+(e_2), B^-(e_2))$ are incident with a common bipolar valued fuzzy point, then they are called **bipolar valued fuzzy adjacent lines.**

Definition 2.3. A bipolar valued fuzzy line joining a bipolar valued fuzzy point to itself is called a **bipolar valued fuzzy loop**.

Definition 2.4. Let F = (A, B, f) be a bipolar valued fuzzy graph. If more than one bipolar valued fuzzy line joining two bipolar valued fuzzy vertices is allowed, then the bipolar valued fuzzy graph F is called a **bipolar valued fuzzy pseudo graph**.

Definition 2.5. F = (A, B, f) is called a **bipolar valued fuzzy simple graph** if it has neither bipolar valued fuzzy multiple lines nor bipolar valued fuzzy loops.

Example 2.6. F = (A, B, f), where $V = \{v_1, v_2, v_3, v_4, v_5 \}$, E = {a, b, c, d, e, h, g } and $f: E \to V \times V$ is defined by $f(a) = (v_1, v_2)$, $f(b) = (v_2, v_2)$, $f(c) = (v_2, v_3)$, $f(d) = (v_3, v_4)$, $f(e) = (v_3, v_4)$, $f(h) = (v_4, v_5)$, $f(g) = (v_1, v_5)$. A bipolar valued fuzzy subset $A = \{ (v_1, 0.3, -0.4), (v_2, 0.2, -0.3), (v_3, 0.3, -0.3), (v_4, 0.3, -0.4), (v_5, 0.3, -0.3) \}$ of V. A bipolar valued fuzzy relation $S = \{ ((v_1, v_1), 0.3, -0.4), ((v_1, v_2), 0.2, -0.3),$

((v_1, v_3), 0.3, −0.3), ((v_1, v_4), 0.3, −0.4), ((v_1, v_5), 0.3, −0.3), ((v_2, v_1), 0.2, −0.3), ((v_2, v_2), 0.2, −0.3), ((v_2, v_3), 0.2, −0.3), ((v_2, v_4), 0.2, −0.3), ((v_2, v_5), 0.2, −0.3), ((v_3, v_1), 0.3, −0.3), ((v_3, v_2), 0.2, −0.3), ((v_3, v_3), 0.3, −0.3), ((v_3, v_4), 0.3, −0.3), ((v_3, v_5), 0.3, −0.3), ((v_4, v_1), 0.3, −0.4), ((v_4, v_2), 0.2, −0.3), ((v_4, v_3), 0.3, −0.3), ((v_4, v_4), 0.3, −0.4), ((v_4, v_5), 0.3, −0.3), ((v_5, v_1), 0.3, −0.3), ((v_5, v_2), 0.2, −0.3), ((v_5, v_3), 0.3, −0.3), ((v_5, v_4), 0.3, −0.3), ((v_5, v_5), 0.3, −0.3) } on V with respect to A and a bipolar valued fuzzy subset B = { (a, 0.1, −0.2), (b, 0.1, −0.2), (c, 0.1, −0.2), (d, 0.1, −0.2), (e, 0.2, −0.2), (h, 0.2, −0.2), (g, 0.2, −0.2) } of E.

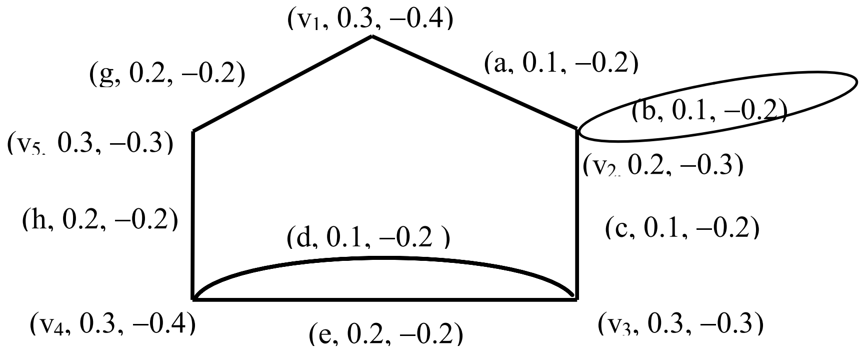

Fig. 1.1

In figure 1.1, (i) (v_1, 0.3, −0.4) is a bipolar valued fuzzy point. (ii) (a, 0.1, −0.2) is a bipolar valued fuzzy edge. (iii) (v_1, 0.3, −0.4) and (v_2, 0.2, −0.3) are bipolar valued fuzzy adjacent points. (iv) (a, 0.1, −0.2) join with (v_1, 0.3, −0.4) and (v_2, 0.2, −0.3) and therefore it is incident with (v_1, 0.3, −0.4) and (v_2, 0.2, −0.3) .(v) (a, 0.1, −0.2) and (g, 0.2, −0.2) are bipolar valued fuzzy adjacent lines. (vi) (b, 0.1, −0.2) is a bipolar valued fuzzy loop. (vii) (d, 0.1, −0.2) and (e, 0.2, −0.2) are bipolar valued fuzzy multiple edges. (viii) The given graph is not a bipolar valued fuzzy simple graph. (ix) The given graph is a bipolar valued fuzzy pseudo graph.

Definition 2.7. The bipolar valued fuzzy graph H = (C, D, f), where $C = \langle C^+, C^- \rangle$ and $D = \langle D^+, D^- \rangle$ is called a **bipolar valued fuzzy subgraph** of F = (A, B, f) if $C \subseteq A$ and $D \subseteq B$.

Definition 2.8. The bipolar valued fuzzy subgraph H = (C, D, f) is said to be a **bipolar valued fuzzy spanning subgraph** of F = (A, B, f) if C = A.

Definition 2.9. The bipolar valued fuzzy subgraph H = (C, D, f) is said to be a **bipolar valued fuzzy induced sub graph** of F = (A, B, f) if H is the maximal bipolar valued fuzzy subgraph of F with bipolar valued fuzzy point set C.

Definition 2.10. Let F = (A, B, f) be a bipolar valued fuzzy graph with respect to the sets V and E. Let $C = \langle C^+, C^- \rangle$ be a bipolar valued fuzzy subset of V, the bipolar valued fuzzy subset $D = \langle D^+, D^- \rangle$ of E is defined as $D^+(e) = \min\{C^+(u), C^+(v), B^+(e)\}$, $D^-(e) = \max\{ C^-(u), C^-(v), B^-(e) \}$, where f(e) = (u, v) for all e in E. Then H = (C, D, f) is called **bipolar valued fuzzy partial subgraph** of F.

Definition 2.11. Let F = (A, B, f) be a bipolar valued fuzzy graph. Let H is a bipolar valued fuzzy sub graph of F obtained by removing the bipolar valued fuzzy point $(x, A^+(x), A^-(x))$ and all the bipolar valued fuzzy lines incident with $(x, A^+(x), A^-(x))$. It is denoted $H = F-(x, A^+(x), A^-(x))$. Thus $F-(x, A^+(x), A^-(x)) = (C, D, f)$, where $C = A-\{(x, A^+(x), A^-(x)) \}$ and $D = \{(e, B^+(e), B^-(e)) / (e, B^+(e), B^-(e)) \in B$ and $(x, A^+(x), A^-(x))$ is not incident with $(e, B^+(e), B^-(e))\}$. Then $H = F-(x, A^+(x), A^-(x))$

is called a **bipolar valued fuzzy induced subgraph** of F. Let (e, B^+(e), B^-(e))∈B. Then F−(e, B^+(e), B^-(e)) = (A, D, f) = H is called bipolar valued fuzzy sub graph of F obtained by the removal of the bipolar valued multi fuzzy line (e, B^+(e), B^-(e)), where D = B−{ (e, B^+(e), B^-(e)) }. Then H = F −(e, B^+(e), B^-(e)) is called a **bipolar valued fuzzy spanning sub graph** of F which contains all the lines of F except (e, B^+(e), B^-(e)).

Definition 2.12. By deleting from a bipolar valued fuzzy graph F all bipolar valued fuzzy loops and in each collection of bipolar valued fuzzy multiple edges all bipolar valued fuzzy edge but one bipolar valued fuzzy edge in the collection we obtain a bipolar valued fuzzy simple spanning subgraph F, is called **bipolar valued fuzzy underling simple graph of F.**

Example 2.13.

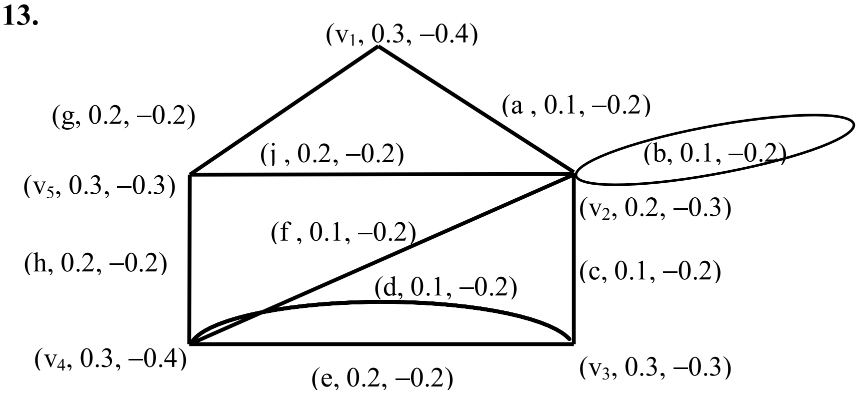

Fig. 1.2 A bipolar valued fuzzy pseudo graph F = (A, B, f)

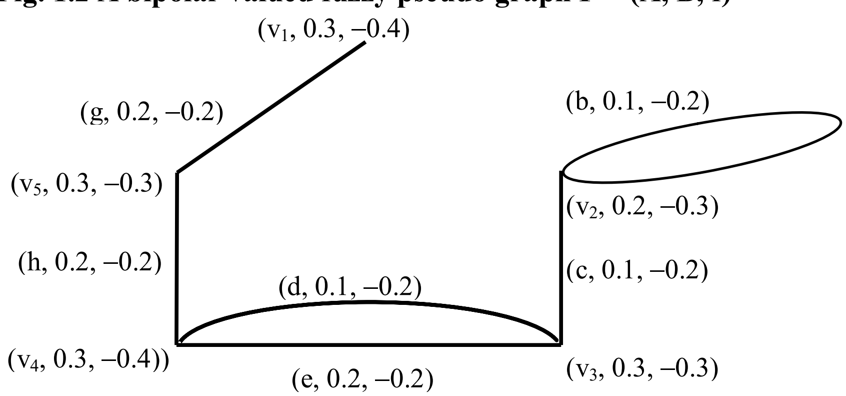

Fig. 1.3 A bipolar valued fuzzy subgraph of F

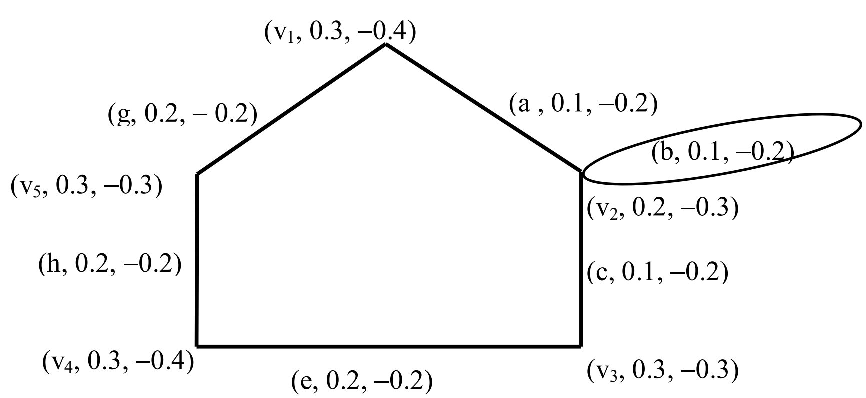

Fig. 1.4 A bipolar valued fuzzy spanning subgraph of F

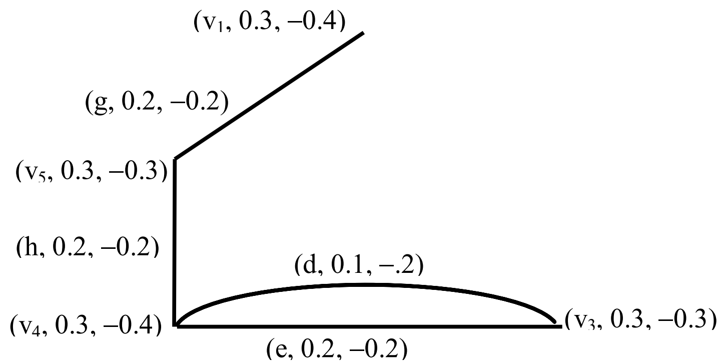

Fig.1.5 A bipolar valued fuzzy subgraph induced by P = { v_1, v_3, v_4,v_5 }

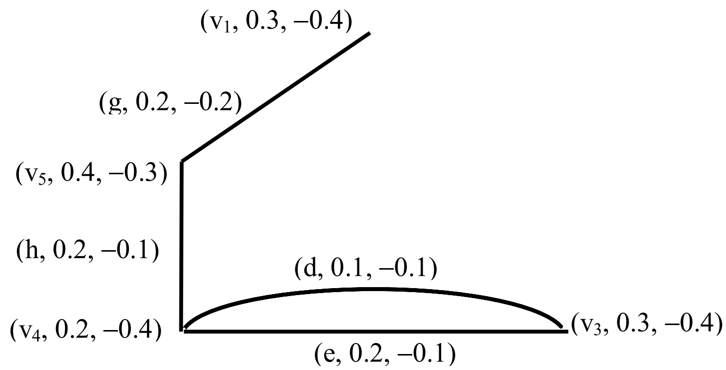

Fig.1.6 A partial bipolar valued fuzzy subgraph induced by C, where
$C(v_1) = (0.3, –0.4)$, $C(v_3) = (0.3, –0.4)$, $C(v_4) = (0.2, –0.4)$, $C(v_5) = (0.4, –0.3)$.

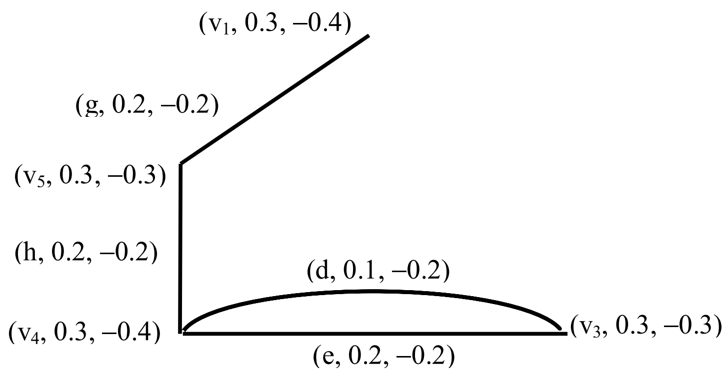

Fig. 1.7 F – (v_2, 0.2, –0.3)

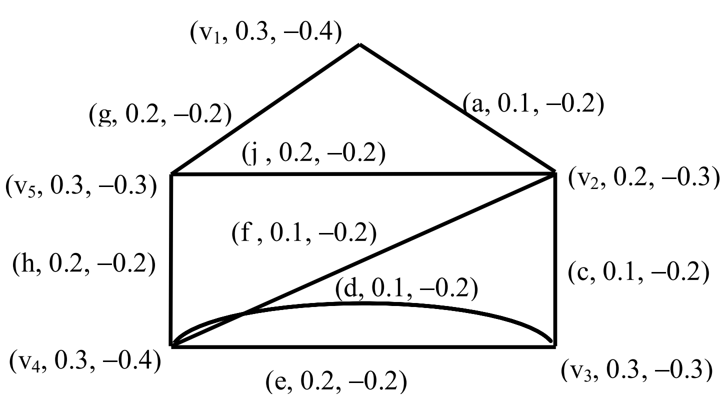

Fig. 1.8 F– (b, 0.1, –0.2)

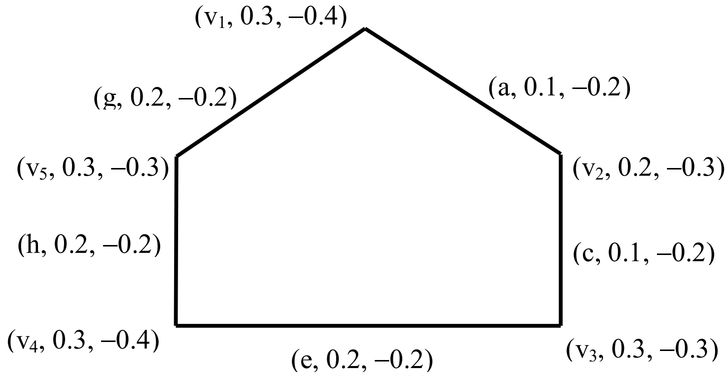

Fig .1.9 Underling bipolar valued fuzzy simple graph of F.

Definition 2.14. Let A be a bipolar valued fuzzy subset of X. Then the **level subset** or **(α,β)-cut** of A is $A_{(\alpha,\beta)} = \{ x \in X \,/\, A^+(x) \geq \alpha \text{ and } A^-(x) \leq \beta \}$, where $\alpha \in [0, 1]$, $\beta \in [-1, 0]$.

Theorem 2.15. Let F = (A, B, f) be a bipolar valued fuzzy graph with respect to the set V and E. Let $\alpha, \beta \in [0,1]$, $\lambda, \eta \in [-1, 0]$ and $\alpha \leq \beta$ and $\lambda \geq \eta$. Then $(A_{(\beta,\eta)}, B_{(\beta,\eta)}, f)$ is a subgraph of $(A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$.

Proof. The proof follows from the definition 2.14.

Theorem 2.16. Let F = (A, B, f) be a bipolar valued fuzzy graph with respect to the set V and E, the level subsets $A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}$ of A and B subset of V and E respectively.

Then $F_{(\alpha,\lambda)} = (A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$ is a subgraph of G = (V, E, f).

Proof. The proof follows from the definition 2.14 and the theorem 2.15.

Theorem 2.17. Let H = (C, D, f) be a bipolar valued fuzzy subgraph of F = (A, B, f), $\alpha \in [0,1]$ and $\lambda \in [-1, 0]$. Then $H_{(\alpha,\lambda)} = (C_{(\alpha,\lambda)}, D_{(\alpha,\lambda)}, f)$ is a subgraph of

$F_{(\alpha,\lambda)} = (A_{(\alpha,\lambda)}, B_{(\alpha,\lambda)}, f)$.

Proof. Let H = (C, D, f) be a bipolar valued fuzzy subgraph of F = (A, B, f). That is $C^+(u) \leq A^+(u)$ and $C^-(u) \geq A^-(u)$ for all u in V. Also $D^+(e) \leq B^+(e)$ and $D^-(e) \geq B^-(e)$ for all e in E. We have to prove that ($C_{(\alpha,\lambda)}$, $D_{(\alpha,\lambda)}$, f) is a subgraph of ($A_{(\alpha,\lambda)}$, $B_{(\alpha,\lambda)}$, f). It is enough to prove that $C_{(\alpha,\lambda)} \subseteq A_{(\alpha,\lambda)}$ and $D_{(\alpha,\lambda)} \subseteq B_{(\alpha,\lambda)}$. Let $u \in C_{(\alpha,\lambda)}$ which implies that $C^+(u) \geq \alpha$ and $C^-(u) \leq \lambda$ implies that $A^+(u) \geq C^+(u) \geq \alpha$ and $A^-(u) \leq C^-(u) \leq \lambda$ implies that $u \in A_{(\alpha,\lambda)}$. Therefore $C_{(\alpha,\lambda)} \subseteq A_{(\alpha,\lambda)}$. Let $e \in D_{(\alpha,\lambda)}$ implies that $D^+(e) \geq \alpha$ and $D^-(e) \leq \lambda$ implies that $B^+(e) \geq D^+(e) \geq \alpha$ and $B^-(e) \leq D^-(e) \leq \lambda$ implies that $e \in B_{(\alpha,\lambda)}$. Therefore $D_{(\alpha,\lambda)} \subseteq B_{(\alpha,\lambda)}$. Hence $H_{(\alpha,\lambda)}$ is a subgraph of $F_{(\alpha,\lambda)}$.

Definition 2.18. Let A be a bipolar valued fuzzy subset of X. Then the **strong level subset** or **strong(α,β)-cut** of A is $A_{(\alpha+,\ \beta+)} = \{\ x \in X\ /\ A^+(x) > \alpha$ and $A^-(x) < \beta\ \}$, where $\alpha \in [0,1]$ and $\beta \in [-1, 0]$.

Theorem 2.19. Let F = (A, B, F) be a bipolar valued fuzzy graph with respect to the set V and E. Let α, $\beta \in [0,1]$, λ, $\eta \in [-1, 0]$ and $\alpha \leq \beta$ and $\lambda \geq \eta$. Then $(A_{(\beta+,\eta+)}, B_{(\beta+,\eta+)}, f)$ is a subgraph of $(A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$.

Proof. The proof follows from definition 2.18.

Theorem 2.20. Let F = (A, B, f) be a bipolar valued fuzzy subgraph with respect to the set V and E, the level subsets $A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}$ of A and B subset of V and E respectively. Then $F_{(\alpha+,\lambda+)} = (A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$ is a subgraph of G = (V, E, f).

Proof. The proof follows from the definition 2.18 and the theorem 2.19.

Theorem 2.21. Let H = (C, D, f) be a bipolar valued fuzzy subgraph of F = (A, B, f), $\alpha \in [0,1]$ and $\lambda \in [-1, 0]$. Then $H_{\alpha+} = (C_{\alpha+}, D_{\alpha+}, f)$ is a subgraph of $F_{(\alpha+,\lambda+)} = (A_{(\alpha+,\lambda+)}, B_{(\alpha+,\lambda+)}, f)$.

Proof. The proof follows from the definition 2.18 and the theorem 2.20.

Theorem 2.22. Let F = (A, B, f) be a bipolar valued fuzzy subgraph with respect to the set V and E, let α, $\beta \in [0,1]$, λ, $\eta \in [-1, 0]$ and also $F_{(\alpha,\lambda)}$ and $F_{(\beta,\eta)}$ be two subgraphs of G. Then (i) $F_{(\alpha,\lambda)} \cap F_{(\beta,\eta)}$ is a subgraph of G. (ii) $F_{(\alpha,\lambda)} \cup F_{(\beta,\eta)}$ is a subgraph of G.

Proof. Since $A_{(\alpha,\lambda)}$ and $A_{(\beta,\eta)}$ are subset of V. Clearly $F_{(\alpha,\lambda)} \cap F_{(\beta,\eta)}$ is a subgraph of G. Also $F_{(\alpha,\lambda)} \cup F_{(\beta,\eta)}$ is a subgraph of G.

Definition 2.23. Let F = (A, B, f) be a bipolar valued fuzzy graph. Then the **degree of a bipolar valued fuzzy vertex** is defined by $d(v) = (\ d^+(v), d^-(v)\ )$, where

$$d^+(v) = \sum_{e \in f^{-1}(u,v)} B^+(e) + 2 \sum_{e \in f^{-1}(v,v)} B^+(e) \ \text{and}\ d^-(v) = \sum_{e \in f^{-1}(u,v)} B^-(e) + 2 \sum_{e \in f^{-1}(v,v)} B^-(e).$$

Defination 2.24. The **minimum degree** of the bipolar valued fuzzy graph F = (A, B, f) is $\delta(F) = (\delta^+(F), \delta^-(F)\ )$, where $\delta^+(F) = \min\{\ d^+(v)\ /\ v \in V\ \}$ and $\delta^-(F) = \min\{\ d^-(v)\ /\ v \in V\}$ and the **maximum degree** of F is $\Delta(F) = (\Delta^+(F), \Delta^-(F))$, where $\Delta^+(F) = \max\ \{\ d^+(v)\ /\ v \in V\ \}$ and $\Delta^-(F) = \max\ \{\ d^-(v)\ /\ v \in V\ \}$.

Definition 2.25. Let F = (A, B, f) be a bipolar valued fuzzy graph. Then the **order of bipolar valued fuzzy graph** F is defined to be $o(F) = (o^+(F), o^-(F))$, where $o^+(F) = \sum_{v \in V} A^+(v)$ and $o^-(F) = \sum_{v \in V} A^-(v)$.

Definition 2.26. Let F = (A, B, f) be a bipolar valued fuzzy graph. Then the **size of the bipolar valued fuzzy graph F** is defined to be $S(F) = (S^+(F), S^-(F)\ )$, where $S^+(F) = \sum_{e \in f^{-1}(u,v)} B^+(e)$ and $S^-(F) = \sum_{e \in f^{-1}(u,v)} B^-(e)$.

Example 2.27.

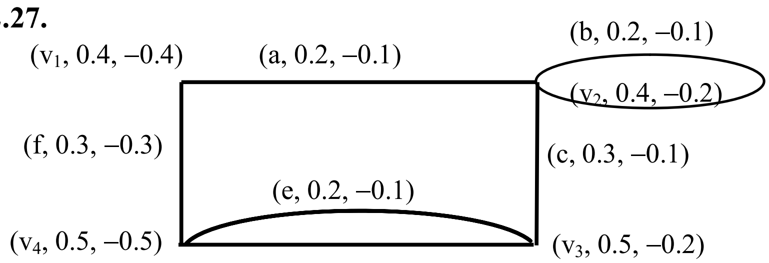

Fig.1.10 Bipolar valued fuzzy graph F

Here $d(v_1) = (0.5, -0.4)$, $d(v_2) = (0.9, -0.4)$, $d(v_3) = (0.7, -0.3)$, $d(v_4) = (0.7, -0.5)$, $\delta(F) = (0.5, -0.5)$, $\Delta(F) = (0.9, -0.3)$, $o(F) = (1.8, -1.3)$, $S(F) = (1.4, -0.8)$.

Theorem 2.28. (i) The sum of the degree of positive membership values of all bipolar valued fuzzy vertices in a bipolar valued fuzzy graph is equal to twice the sum of the positive membership values of all bipolar valued fuzzy edges. i.e., $\sum_{v\in V} d^+(v) = 2S^+(F)$. (ii)The sum of the degree of negative membership values of all bipolar valued fuzzy vertices in a bipolar valued fuzzy graph is equal to twice the sum of the negative membership value of all bipolar valued fuzzy edges. i.e., $\sum_{v\in V} d^-(v) = 2S^-(F)$.

(iii) The sum of the degree of all bipolar valued fuzzy vertices in a bipolar valued fuzzy graph is equal to twice the sum of the all bipolar valued fuzzy edges.i.e., $\sum_{v\in V} d(v) = 2S(F)$.

Proof. (i) Let F = (A, B, f) be a bipolar valued fuzzy graph with respect to the set V and E. Since degree of a bipolar valued fuzzy vertex denote sum of the positive membership values of all bipolar valued fuzzy edges incident on it. Each bipolar valued fuzzy edge of F is incident with two bipolar valued fuzzy vertices. Hence positive membership value of each bipolar valued fuzzy edge contributes two to the sum of degree of bipolar valued fuzzy vertices. Hence the sum of the degree of all bipolar valued fuzzy vertices in a bipolar valued fuzzy graph is equal to twice the sum of the positive membership value of all bipolar valued fuzzy edges. i.e., $\sum_{v\in V} d^+(v) = 2S^+(F)$.

(ii) Let F = (A, B, f) be a bipolar valued fuzzy graph with respect to the set V and E. Since degree of a bipolar valued fuzzy vertex denote sum of the negative membership values of all bipolar valued fuzzy edges incident on it. Each bipolar valued fuzzy edge of F is incident with two bipolar valued fuzzy vertices. Hence negative membership value of each bipolar valued fuzzy edge contributes two to the sum of degree of bipolar valued fuzzy vertices. Hence the sum of the degree of all bipolar valued fuzzy vertices in a bipolar valued fuzzy graph is equal to twice the sum of the negative membership value of all bipolar valued fuzzy edges. i.e., $\sum_{v\in V} d^-(v) = 2S^-(F)$.

(iii) From (i) and (ii), the sum of the degree of all bipolar valued fuzzy vertices in a bipolar valued fuzzy graph is equal to twice the sum of the all bipolar valued fuzzy edges. i.e., $\sum_{v\in V} d(v) = 2S(F)$.

Theorem 2.29. Let F = (A, B, f) be a bipolar valued fuzzy graph with number of bipolar valued fuzzy vertices n, all of whose bipolar valued fuzzy vertices have degree $s = (s^+, s^-)$ or $t = (t^+, t^-)$. If F has p bipolar valued fuzzy vertices of degree s and $(n-p)$ bipolar valued fuzzy vertices of degree t, then $2S(F) = ps + (n-p)t$.

Proof. Let V_1 be the set of all bipolar valued fuzzy vertices with degree s. Let V_2 be the set of all bipolar valued fuzzy vertices with degree t. Then $\sum_{v\in V} d(v) = \sum_{v\in V_1} d(v) + \sum_{v\in V_2} d(v)$ which implies that $2S(F) = (\sum_{v\in V_1} d^+ v), \sum_{v\in V_1} d^-(v)) + (\sum_{v\in V_2} d^+(v), \sum_{v\in V_2} d^-(v))$ which implies that $2S(F) = p(s^+, s^-) + (n-p)(t^+, t^-)$ which implies that $2S(F) = ps + (n-p)t$.

REFERENCES:

1. Arjunan.K & Subramani.C, "Notes on fuzzy graph", *International Journal of Emerging Technology and Advanced Engineering*, Vol.5, Iss.3 (2015), 425–432.
2. Arjunan.K & Subramani.C, "A study on I-fuzzy graph", *International Journal of Mathematical Archive,* 6(4) (2015), 222–233.
3. Arjunan.K & Subramani.C,"Notes on intuitionistic fuzzy graphs", *Shanlax International Journal of Arts, Science and Humanities,* Vol. 5 (2017), 35–46.
4. K.M.Lee, Bipolar valued fuzzy sets and their operations. *Proc. Int. Conf. on Intelligent Technologies, Bangkok, Thailand,* (2000), 307 - 312.
5. K.M.Lee, Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets and bipolarvalued fuzzy sets. *J. fuzzy Logic Intelligent Systems,* 14 (2) (2004), 125-129.
6. Mardeson.J.N and C.S.Peng, "Operation on Fuzzy graph", *Information science* 19 (1994), 159 – 170.
7. Marichamy.A, K.Arjunan and K.L.Muruganantha Prasad, "Multi fuzzy irregular graph", *Infokara,* Vol. 8, Iss. 8(2019), 636 – 649.
8. NagoorGani, A. and BasheerAhamed.M.,"Order and size in fuzzy graphs", *Bulletin of Pure and Applied Sciences,* Vol 22E (No.1) (2003), 145–148.
9. NagoorGani.A and ShajithaBegum.S, "Degree, order and size in intuitionistic fuzzy graphs", *International Journal of Algorithms and mathematics*, Vol. 3 (2010), 11 – 16.
10. Rosenfeld,A., "Fuzzy Graphs, In:L.A.Zadeh,K.S.Fu, M.Shimura,Eds., Fuzzy Sets and their Applications", *Academic press*(1975), 77–95.
11. L. A. Zadeh, "Fuzzy sets", *Information and Control,* 8 (1965) 338 –353.