MAGNETIC FIELD AND HALL EFFECTS ON FLOW PAST A PARABOLIC ACCELERATED ISOTHERMAL VERTICAL PLATE WITH VARYING MASS DIFFUSION IN THE PRESENCE OF THERMAL RADIATION

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ABSTRACT

This paper deals with Hall currents and magnetic field effects on flow past a parabolic accelerated infinite isothermal vertical plate with variable mass diffusion in the presence of thermal radiation. The Laplace transform technique is used to obtain the expression for velocity, temperature distribution and concentration field of the fluid. The influence of the various parameters occuring in the problem on the velocity field, temperature and concentration field are extensively discussed with the help of graphs.

Key words: Hall effect, magnetic field, thermal radiation, parabolic, isothermal, vertical plate, mass transfer.

1. INTRODUCTION

Study of MHD flow with mass transfer effect in the presence of thermal radiation and Hall currents play an important role in many engineering and industrial applications like heating and cooling of chambers, fossil fuel combustion energy processes etc.

Magneto hydrodynamics plays a vital role in plasma confinement, liquid-metal cooling of nuclear reactors, electromagnetic casting and also Magnetic Drug Targeting.

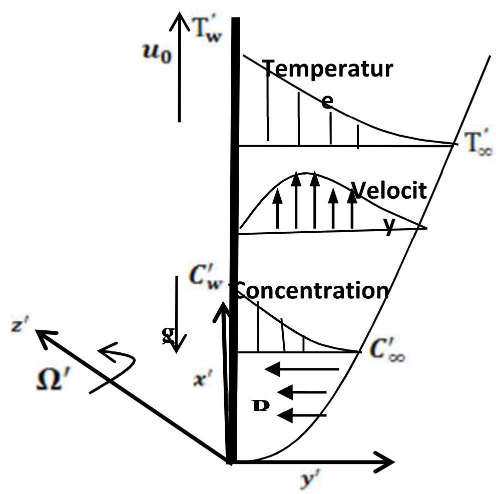
The Hall Effect is the production of a voltage difference across an electrical conductor, transverse to an electric current in the conductor and a magnetic field perpendicular to the current. Hall Effect sensors are used in rotating speed sensors, fluid flow sensors, current sensors and pressure sensors. Using the principle of Hall currents the Brushless DC motors based on motion sensing is functioning. Hall Effect joysticks find its application in mining trucks, cranes, diggers and scissor lifts etc.

Deka (2008) depicted graphically the effect of Hall currents and skin friction over a linearly accelerated plate. Dileep Singh Chauhan (2012) analyzes the Hall effects in a rotating system using Crank-Nicolson method. Sarkar et al (2013) considered Hall effects on moving vertical plate and concluded that fluid velocity, temperature raises due to viscous and joule dissipations. Effects of Hall current on a sheet which is exponentially stretching in radius, in cylindrical co-ordinates using homotopy analysis was carried out by Haider Zaman et al (2014). The problem of Hall effects on an exponentially accelerated vertical plate without chemical reaction and radiation was discussed by Thamizhsudar et al (2014).

The study reported herein considers the thermal radiation effects on flow past a parabolic accelerated infinite isothermal vertical plate with MHD and Hallcurrents.This type of problem has some relevance in geophysical, astrophysical and space vechile re-entry etc.

2. Mathematical Analysis

Initially, the plate and the fluid are in stationary medium and with same temperature. Here, the x' -axis is taken along the vertical plate in the vertically upward direction and the y'-axis is taken normal to the plate. Initially the plate and the fluid were rotating with a constant angular velocity Ω' about the z'-axis normal to the plate, which is perpendicular to x' -axis and y'-axis. Here the unsteady flow of a viscous incompressible fluid past an parabolic accelerated isothermal verticle plate with variable mass diffusion is considered. A magnetic field of uniform strength B_0 is applied transversely to the plate.The viscous dissipation and induced magnetic field has been neglected due to its small effect. At time $t' \le 0$, the plate and the fluid are maintained at the same constant temperature T'_∞ and concentration of the fluid is C'_∞. At time $t' > 0$, the plate is parabolic accelerated with a velocity $u = u_0 t'^2$ in its own plane against gravitational field. At time $t' > 0$, temperature of the plate is raised at a constant rate and the concentration level near the plate is raised linearly with respect to time.



Flow model of the problem

By usual Boussinesq's approximation, the unsteady flow is governed by the following:
Momentum Equation:

$$\frac{\partial u}{\partial t'} = \upsilon \frac{\partial^2 u}{\partial z'^2} + 2\Omega' v - \frac{\sigma \mu_e^2 B_0^2}{\rho\left(1+m^2\right)}\left(u + mv\right) + g\beta\left(T' - T'_\infty\right) + g\beta^*\left(C' - C'_\infty\right) \tag{1}$$

$$\frac{\partial v}{\partial t'} = \upsilon \frac{\partial^2 v}{\partial z'^2} - 2\Omega' u + \frac{\sigma \mu_e^2 B_0^2}{\rho\left(1+m^2\right)}\left(mu - v\right) \tag{2}$$

Energy Equation:

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial z^2} - \frac{\partial q_r}{\partial z} \quad (3)$$

Mass diffusion Equation:

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial z^2} \quad (4)$$

Where u is the axial velocity and v is the transverse velocity.

The initial and boundary conditions are

$$\begin{aligned} &t' \le 0: \; u = 0, \quad v = 0, \quad T' = T'_\infty, \quad C' = C'_\infty \quad \text{for all z} \\ &t' > 0: \; u = u_0 t'^2, \; v = 0, \quad T' = T'_w, \quad C' = C'_\infty + (C'_w - C'_\infty)At' \quad \text{at z=0} \\ &t' > 0: \; u \to 0, \quad v \to 0, \quad T' \to T'_\infty, \; C' \to C'_\infty \text{ as } z \to \infty \end{aligned} \quad (5)$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial z} = -4a^* \sigma(T'^4_\infty - T'^4) \quad (6)$$

It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher-order terms, thus

$$T'^4 \cong 4T'^3_\infty T' - 3T'^4_\infty \quad (7)$$

By using equations (6) and (7), equation (3) reduces to

$$\rho C_p \frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y^2} + 16a^* \sigma T'^3_\infty (T'_\infty - T') \quad (8)$$

On introducing non-dimensional quantities:

$$U = u\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}}, \; t = \left(\frac{u_0{}^2}{\nu}\right)^{\frac{1}{3}} t', \; Z = z'\left(\frac{u_0}{\nu^2}\right)^{\frac{1}{3}},$$

$$M = \frac{\sigma B_0^{\,2}}{\rho}\left(\frac{\nu}{u_0^{\,2}}\right)^{\frac{1}{3}}, \quad Gc = \frac{g\beta(C' - C'_\infty)}{(\nu.u_0)^{1/3}}, \quad Gr = \frac{g\beta(T - T_\infty)}{(\nu.u_0)^{1/3}}, \quad R = \frac{16a^*\sigma.T_\infty^{\,3}}{k}\left(\frac{\nu^2}{u_0}\right)^{\frac{2}{3}},$$

$$C = \frac{C' - C'_\infty}{C'_w - C'_\infty}, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}, \quad \Pr = \frac{\mu C_p}{K}, \quad Sc = \frac{\upsilon}{D}, \quad \Omega = \Omega'\left(\frac{\nu}{u_0^{\,2}}\right)^{\frac{1}{3}}$$

equations (1) to (5) are reduced to the non-dimensional form as:

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U}{\partial Z^2} + 2\Omega V - \frac{2M^2}{1+m^2}\left(U + mV\right) + Gr\theta + GcC \tag{9}$$

$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial Z^2} - 2\Omega U + \frac{2M^2}{1+m^2}\left(mU - V\right) \tag{10}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Z^2} - \frac{R}{\Pr}\theta \tag{11}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} \tag{12}$$

The initial and boundary conditions in non-dimensional form are

$$t' \le 0: \; U = 0, \; V = 0, \; \theta = 0, \quad C = 0 \quad \text{for all Z}$$

$$t > 0: \; U = t^2, \; V = 0, \; \theta = 1, \quad C = t \quad \text{at} \quad Z = 0 \tag{13}$$

$$t > 0: \; U \to 0, \; V \to 0, \; \theta \to 0, \quad C \to 0 \; \text{as} \quad Z \to \infty$$

The above equations (9)-(12) and the boundary conditions (13) can be combined as

$$\frac{\partial F}{\partial t} = \frac{\partial^2 F}{\partial Z^2} - aF + G_r\theta + G_cC \tag{14}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Z^2} - \frac{R}{\Pr}\theta \tag{15}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Z^2} \tag{16}$$

Where $a = \frac{2M^2}{1+m^2} + 2i\left[\Omega - \frac{M^2 m}{1+m^2}\right]$

With boundary conditions

$$t' \le 0: \; F = 0, \; \theta = 0, \; C = 0 \quad \text{for all } Z$$
$$t > 0: \; F = t^2, \; \theta = 1, \; C = t \quad \text{at } Z = 0$$
$$t > 0: \; F \to 0, \; \theta \to 0 \quad C \to 0 \quad \text{as } Z \to \infty \tag{17}$$

where F=U+iV, U represents the axial velocity(primary velocity), V represents the transverse velocity(secondary velocity). All the physical variables are defined in the nomenclature.

3. Discussion of Solution

Equations (14) to (16), subject to the boundary conditions (17), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta(z,t) = \frac{1}{2}\left[\exp(2\eta\sqrt{Rt})\, erfc(\eta\sqrt{\Pr} + \sqrt{at}\,) + \exp(-2\eta\sqrt{Rt})\, erfc(\eta\sqrt{\Pr} - \sqrt{at}\,)\right] \tag{18}$$

$$C(z,t) = t\left[(1 + 2\eta^2 Sc)\; erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2\; Sc)\right] \tag{19}$$

$$F(z,t) = \begin{cases} \left[\dfrac{\eta^2 t}{a} + t^2\right] S + \left[\dfrac{1}{4a} - t\right] 2\eta\sqrt{t}T - \dfrac{\eta t}{a\sqrt{\pi}}\, exp(-\eta^2 - at) + (e+g)A_1 \\ - e\exp(ct)A_2 - g\exp(dt)A_3 - eA_4 + e\exp(ct)A_5 - 2gA_6 \\ + g\exp(dt)A_7 \;\; - 2ftA_8 + ftA_9 - \dfrac{f\sqrt{t}}{\sqrt{a}}\eta A_{10} \end{cases} \tag{20}$$

where

$$S = \frac{1}{2}\left[\exp(2\eta\sqrt{at})\, erfc(\eta + \sqrt{at}\,) + \exp(-2\eta\sqrt{at})\, erfc(\eta - \sqrt{at}\,)\right]$$

$$T = \frac{1}{2\sqrt{a}}\left[\exp(-2\eta\sqrt{at})\, erfc(\eta - \sqrt{at}\,) - \exp(2\eta\sqrt{at})\, erfc(\eta + \sqrt{at}\,)\right]$$

$$A_1 = \exp(2\eta\sqrt{at})\, erfc(\eta + \sqrt{at}\,) + \exp(-2\eta\sqrt{at})\, erfc(\eta - \sqrt{at}\,)$$

$$A_2 = \exp(2\eta\sqrt{(a+c)t})\; erfc(\eta + \sqrt{(a+c)t}) + \exp(-2\eta\sqrt{(a+c)t})\; erfc(\eta - \sqrt{(a+c)t})$$

$$A_3 = \exp(2\eta\sqrt{(d+a)t})\; erfc(\eta + \sqrt{(d+a)t}) + \exp(-2\eta\sqrt{(d+a)t})\; erfc(\eta - \sqrt{(a+d)t})$$

$$A_4 = \exp(2\eta\sqrt{Rt}) erfc(\eta\sqrt{Pr} + \sqrt{bt}) + \exp(-2\eta\sqrt{Rt}) erfc(\eta\sqrt{Pr} - \sqrt{bt})$$

$$A_5 = \exp(-2\eta\sqrt{Pr(c+b)t})\; erfc(\eta\sqrt{Pr} - \sqrt{(c+b)t}) + \exp(2\eta\sqrt{Pr(c+b)t}) erfc(\eta\sqrt{Pr} + \sqrt{(c+b)t})$$

$$A_6 = erfc(\eta\sqrt{Sc})$$

$$A_7 = \exp(-2\eta\sqrt{Scdt})\; erfc(\eta\sqrt{Sc} - \sqrt{dt}) + \; \exp(2\eta\sqrt{Scdt}) erfc(\eta\sqrt{Sc} + \sqrt{dt})]$$

$$A_8 = (1 + 2\eta^2 Sc) erfc(\eta\sqrt{Sc}) - \frac{2\eta\sqrt{Sc}}{\sqrt{\pi}}\exp(-\eta^2\; Sc)$$

$$A_9 = t\left[\exp(2\eta\sqrt{at})erfc(\eta+\sqrt{at}\,)+\exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at}\right]$$

$$A_{10} = \frac{\eta\sqrt{t}}{\sqrt{a}}\left[\exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at}\,)-\exp(2\eta\sqrt{at})erfc(\eta+\sqrt{at})\right]$$

where

$$b=\frac{R}{Pr},\ c=\frac{R-a}{1-Pr},d=\frac{a}{Sc-1},e=\frac{Gr}{2c(1-Pr)}\ f=\frac{Gc}{2d(1-Sc)}\ ,\ g=\frac{Gc}{2d^2(1-Sc)}$$

erfc is the complementary error function.

In order to get the physical insight into the problem, the numerical values of F have been computed from (20). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$erf(a+ib)=erf(a)+\frac{\exp(-a^2)}{2a\pi}\left[1-\cos(2ab)+i\sin(2ab)\right]\ +\frac{2\exp(-a^2)}{\pi}\sum_{n=1}^{\infty}\frac{\exp(-\frac{n^2}{4})}{n^2+4a^2}\left[f_n(a,b)+ig_n(a,b)\right]$$

where

$f_n = 2a-2a\cosh(nb)\cos(2ab)+n\sinh(nab)\sin(2ab)$ and

$g_n = 2a\cosh(nb)\sin(2ab)+n\sinh(nab)\cos(2ab)$

$\left|\in(a,b)\right|\approx 10^{-16}\left|erf(a+ib)\right|$

PROFILES FOR CONCENTRATION, VELOCITY (AXIAL AND TRANSVERSE), AND TEMPERATURE

Fig 1 depicts the concentration profiles for Hydrogen(0.16), Ethyl Benzene(2.01), Water vapour(0.6). Schmidt number(Sc) gives the relationship between momentum diffusivity and mass diffusivity. Concentration in the presence of Hydrogen is high when compared to Ethyl Benzene and Water vapour. As Schmidt number increases, the concentration boundary layer decreases. Fig 2 gives the concentration profiles for different values of time t, which shows that concentration increases when time increases.

Temperature profiles for different values of thermal raidation parameter (R) is shown in Fig 3. It is clear that the temperature decreases for increasing values of thermal radiation parameter.

Axial velocity(U) and transverse velocity(V) increases when there is an increase in thermal Grashof number or mass Grashof number which is shown Fig 4 & Fig 5.Increase in the value of thermal radiation parameter (R) results in an decrease of axial velocity(U)and increase in transverse velocity(V) as shown in Fig 6& Fig 7 respectively.

The presence of magnetic field retards axial velocity but the trend is just reversed in the case of transverse velocity, whenever Magnetic field parameter is increased. It is shown in Fig 8 and 9 respectively.

From Fig 10 & 11, it is observed that the increase in the hall parameter(m) increases axial and decreases transverse velocity.

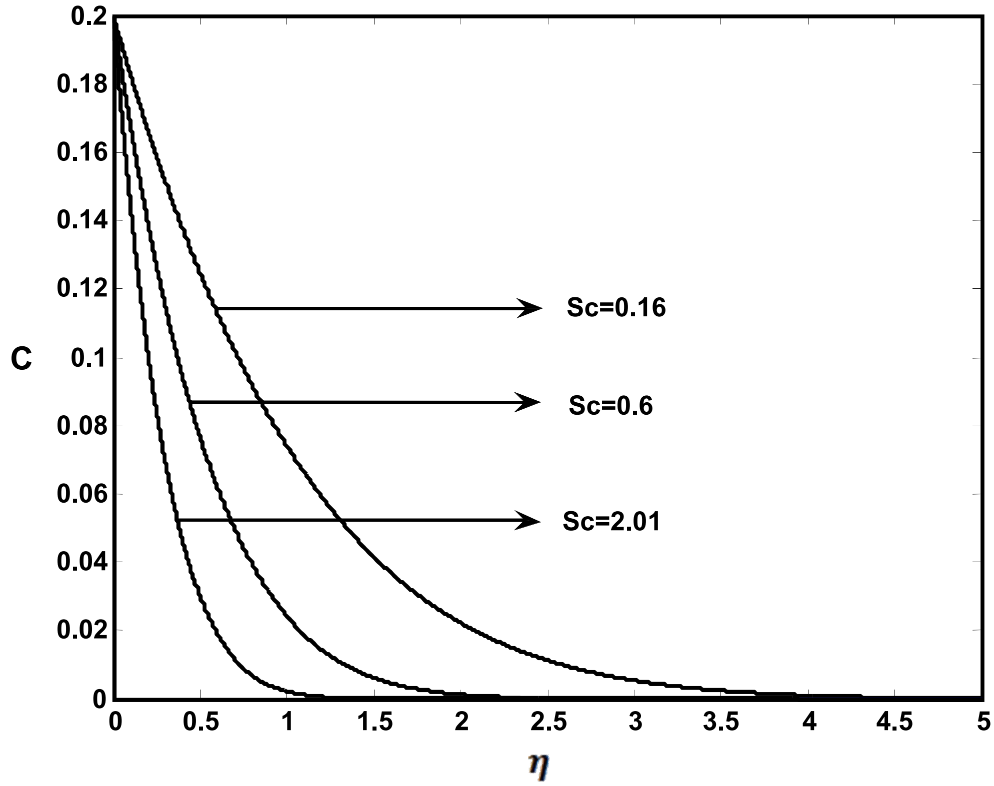


Figure 1:Concentration Profiles for different values of Sc

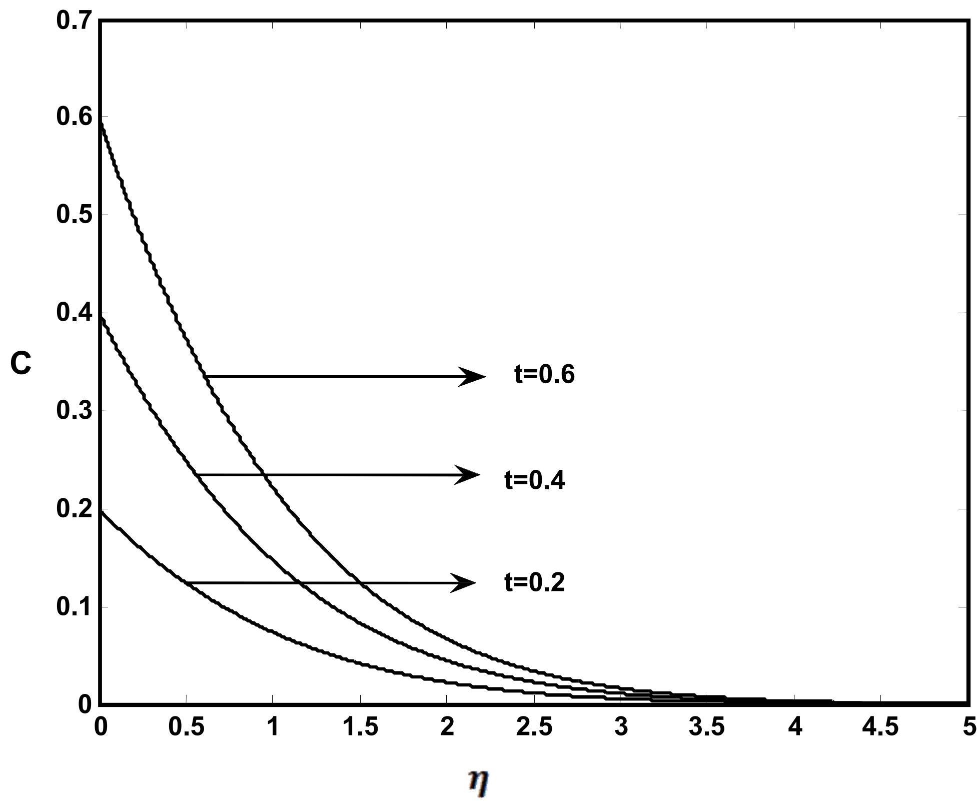


Figure 2:Concentration Profiles for different values of t

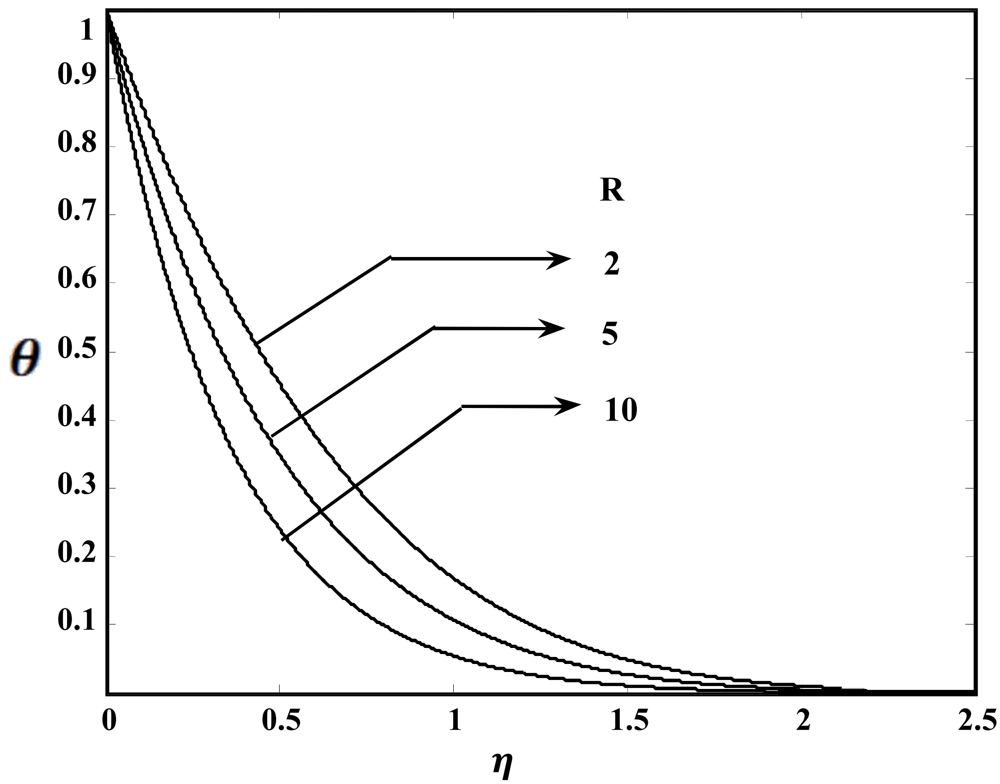


Figure 3: Temperature profiles for different values of R

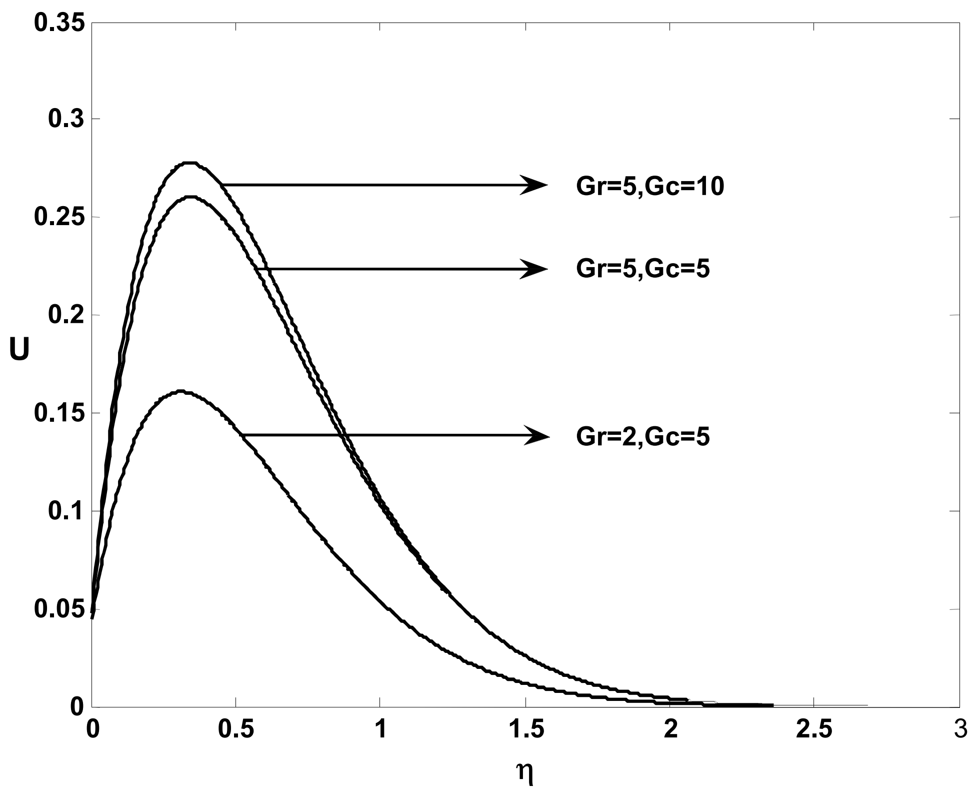


Fig 4. Axial velocity profile for different values of Gr and Gc

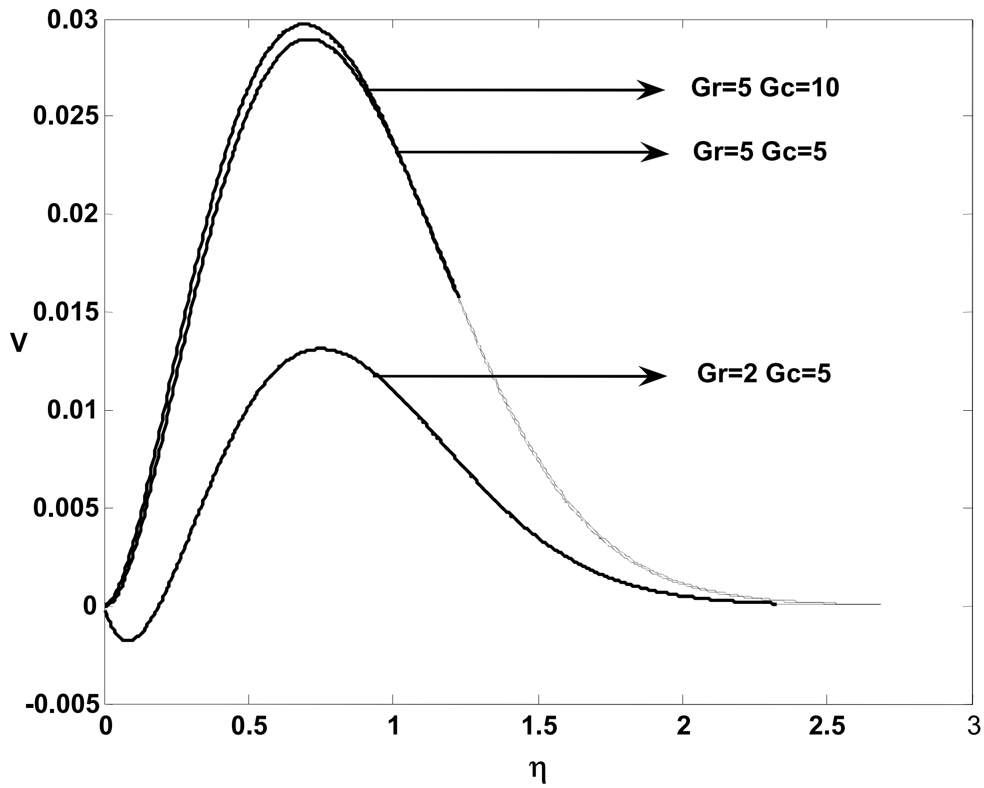


Fig 5. Transverse velocity profile for different values of Gr and Gc

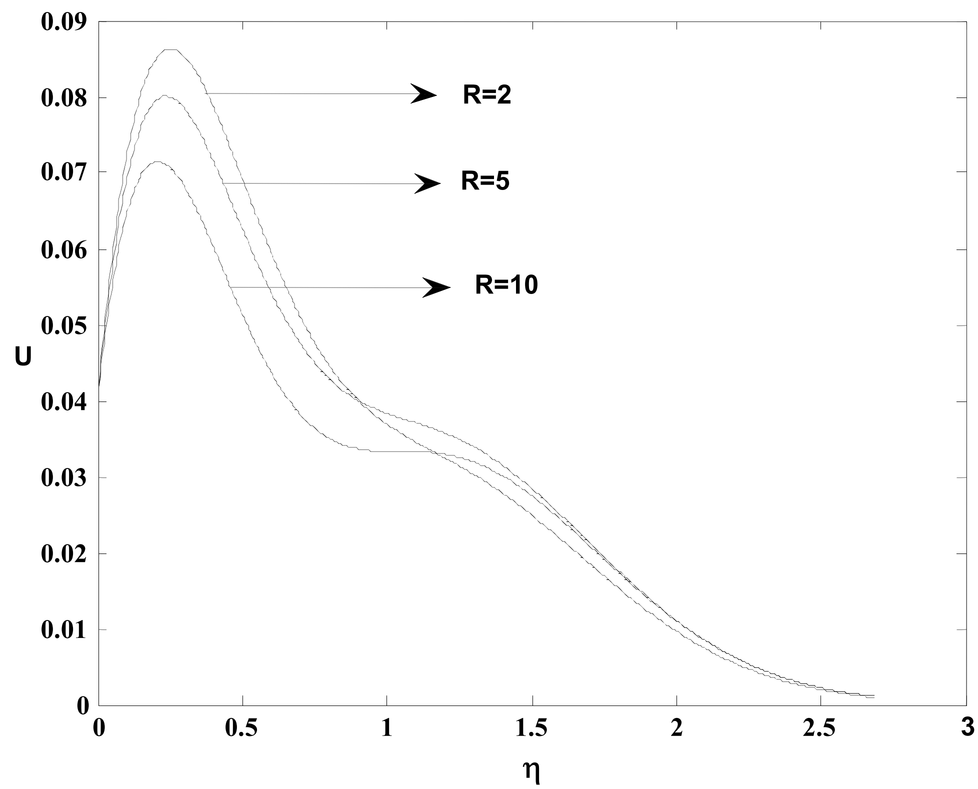


Fig 6.Axial velocity profile for different values of R

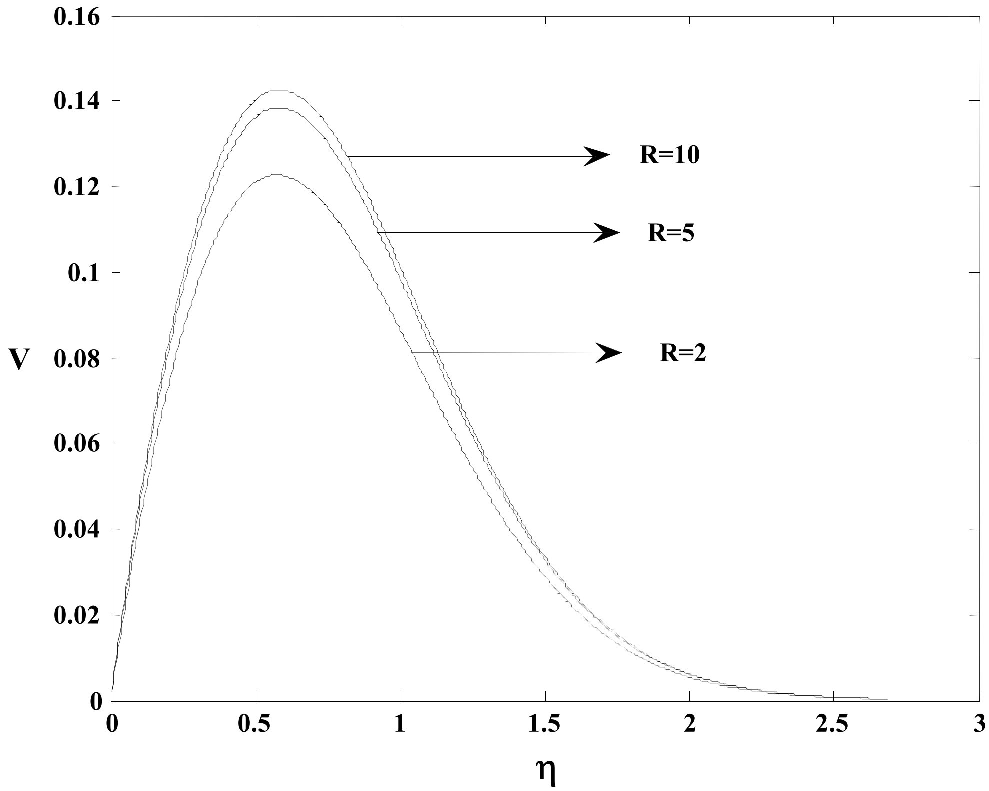


Fig 7: Transverse velocity profile for different values of R

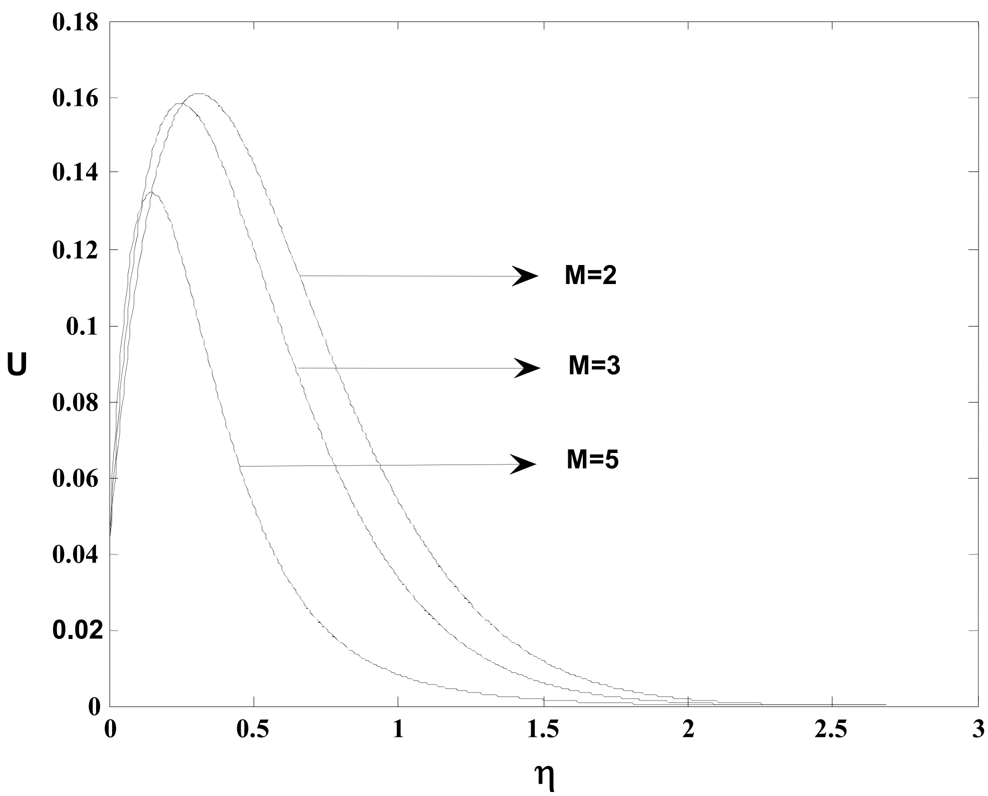


Fig 8 : Axial velocity profile for different values of M

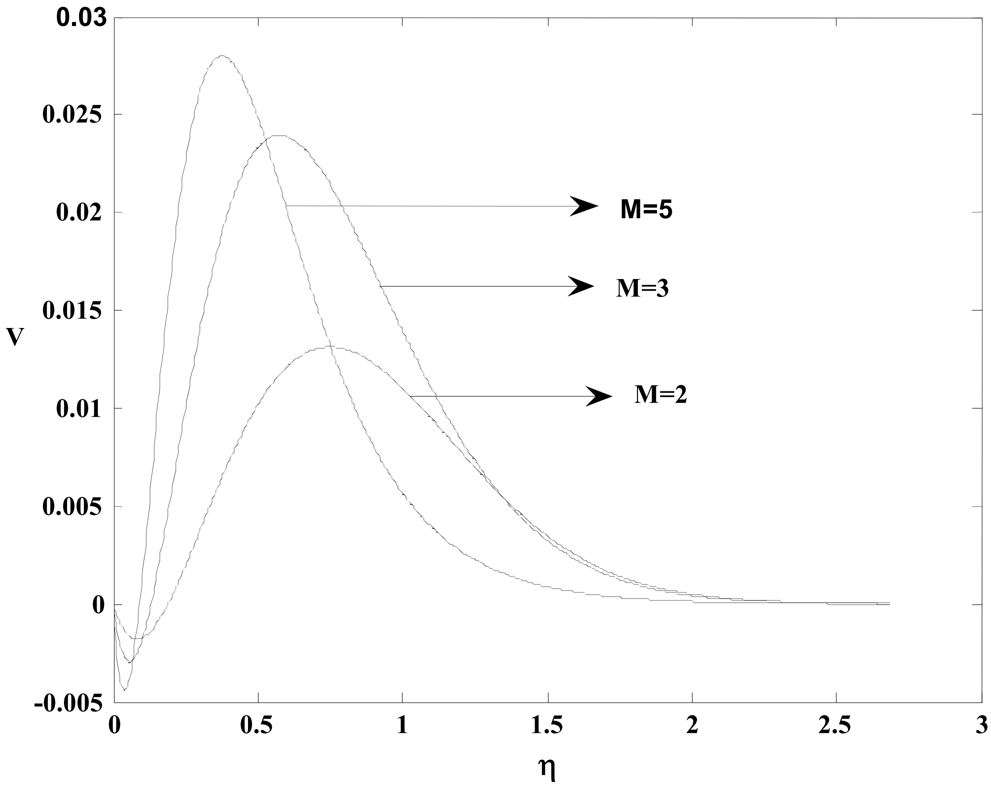


Fig 9: Transverse velocity profile for different values of M

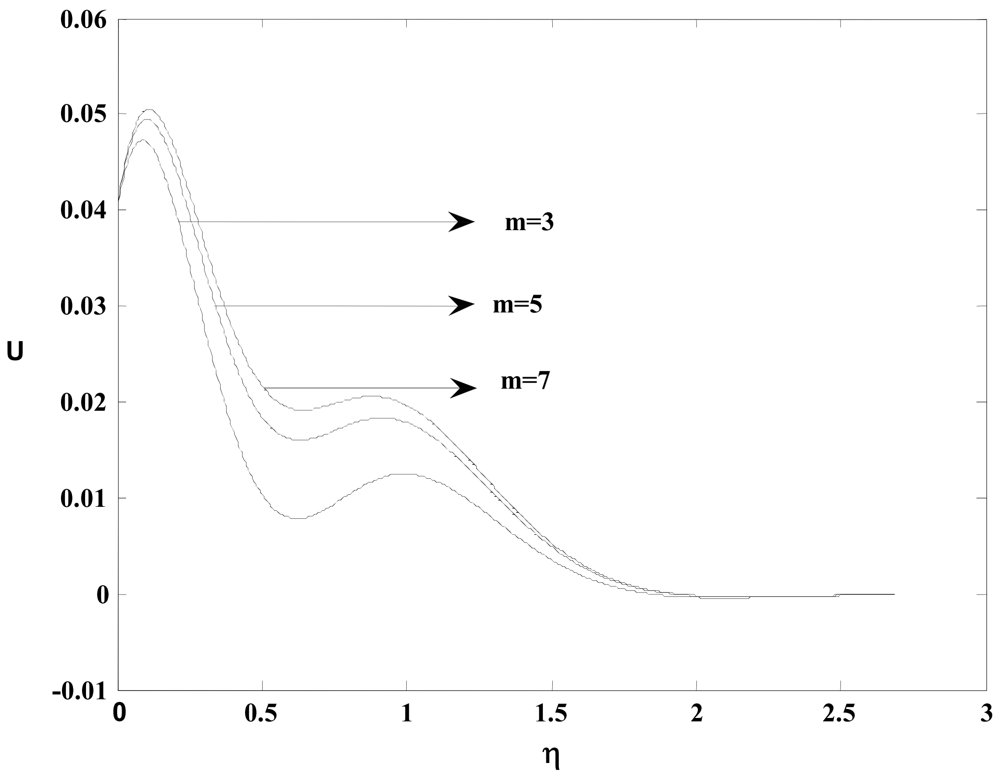


Fig 10: Axial velocity profile for different values m

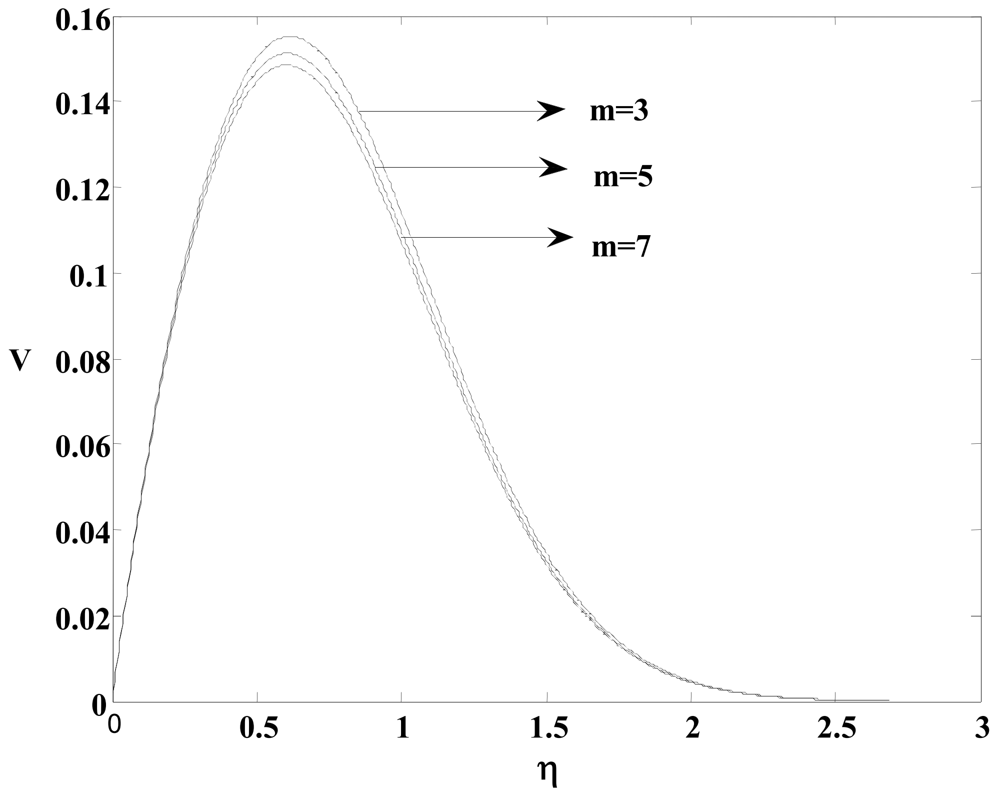


Fig 11:Transverse velocity profile for different values of m

CONCLUSION

An analytical study of profiles for concentration, temperature, axial velocity and transverse velocity on convective flow past an parabolic accelerated vertical plate in the presence of Magnetic field and Hall current has been carried out. In the case of cooling of the plate ($Gr > 0$, $Gc > 0$) as convection currents are carried from the plate is focused. It is observed that

- (i) Concentration(C) decreases for increasing values of Schmidt number (Sc).
- (ii) Concentration profiles increases for increasing values of time.
- (iii) Temperature profiles decrease for increasing values of thermal radiation parameter.
- (iv) Axial velocity (U)
  - (a) increases for increasing values of Hall parameter (m),thermal grashof number and mass grashof number (Gr and Gc).
  - (b) decreases for increasing values of thermal radiation parameter(R), magnetic parameter (M).
- (v) Transverse velocity(V)
  - (a) Increases for increasing values of thermal grashof number and mass grashof number (Gr and Gc), magnetic parameter(M) and thermal radiation parameter.
  - (b) decreases for increasing values of Hall parameter (m).

NOMENCLATURE

a, A, a^*	Constants
B_0	Applied magnetic field
C_p	Specific heat at constant pressure
D	Mass diffusion coefficient
Gc	Mass Grashof number
Gr	Thermal Grashof number
g	Acceleration due to gravity
k	Thermal conductivity
M	Hartmann number (Magnetic parameter)
m	Hall parameter
Pr	Prandtl number
Sc	Schmidt number
C^*	Species concentration in the fluid
C_W^*	Concentration of the plate
C_∞^*	Concentration of the fluid far away from the plate
C	Dimensionless concentration
T^*	Temperature of the fluid near the plate
T_W^*	Temperature of the plate
T_∞^*	Temperature of the fluid far away from the plate
θ	Dimensionless temperature
t^*	Time
t	Dimensionless time
u_0^*	Velocity of the plate
$(u' \quad v' \quad w')$	Components of velocity field F
$(\Phi \quad \Psi \quad \Upsilon)$	Dimensionless components of velocity field F
$(x' \quad y' \quad z')$	Cartesian Co-ordinates
Z	Dimensionless coordinate axis normal to the plate
ω	Angular velocity
ω'	Dimensionless angular velocity
μ_e	Magnetic permeability
υ	Kinematic viscosity
ρ	Density of the fluid
β	Volumetric coefficient of thermal expansion
β^*	Volumetric coefficient of expansion

	with concentration
η	Similarity parameter
erfc	Complementary error function

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