

SOME MORE RESULTS OF SEMI* PRE- OPEN SET WITH GROUP ACTION

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Abstract:

The subset A of a topological space (X, τ) is called semi* pre open if $A \subset cl^*(Pint(A))$. In this paper we introduced group acting on semi* pre open set, function and semi*irresolute function related theorems by using concept of semi* pre open set.

Keywords: Group acting on semi* open set, semi* -pre open set, semi* -pre open function and semi* irresolute function with topological spaces.

1. Introduction

In 1963, Levine introduced the concept of semi open sets in topological spaces, Andrijevic introduced the class of semi pre open set and semi pre closed sets in topological spaces. Navalagi introduced new class of function semi pre open function, semi pre continuous function. S.Pasunkilipandian introduced new class of semi star pre open sets in topological spaces. S.Pious Missier and A.Robert introduced more function Associated with semi* pre open sets. In this paper we introduce the stronger form of semi-preopen sets, namely semi*-preopen sets, using the generalized closure operator Cl^* due to W. Dunham. We investigate the properties of these sets in the light of existing concepts and results in topology. In this paper, we introduced the concept of Group acting on semi* open set, semi* pre open set, semi* pre open function and semi*irresolute function.

2. Preliminaries:

Definition: 2.1

A subset A of a topological group (X, τ) is called a semi-open set if $A \subseteq cl(int(A))$ and a semi-closed set if $int(cl(A)) \subseteq A$.

Definition: 2.2

A subset A of a topological group (X, τ) is called a pre-open set if $A \subseteq int(cl(A))$ and a pre-closed set if $cl(int(A)) \subseteq A$

Definition: 2.3

The pre closure of a set A of X is the intersection of all pre closed sets that contains A and A is denoted by $pCl(A)$.

The union of all pre open sets which are contained in A is called the pre-interior of A denoted by $pInt(A)$.

Definition: 2.4

A subset A of a topological group (X, τ) is called a β -open set (semi-pre-open set) if $A \subseteq cl(int(cl(A)))$ and a β -closed set (semipre-closed set) if $int(cl(int(A))) \subseteq A$.

Definition: 2.5

The sub set A of a topological spaces (X, τ) is called semi* pre open if $A \subset cl^*(Pint(A))$.

Definition: 2.6

Let G be a topological group this means that G is a topological space and also a group so that the multiplication map $\mu: G \times G \rightarrow G$.

$\mu(g, h) = gh$ and the inverse map $i: G \rightarrow G$

$i(g) = g^{-1}$ are continuous.

Definition: 2.7

A function $g: X \rightarrow Y$ is called semi* pre open, if the image of each open set of X , is semi* pre open in Y . It is denoted by $S^*PO(X)$.

Definition: 2.8

A subset of a topological group (X, τ) is called a group acting on semi* pre – open set if there is a pre open set R in X such that $R \subseteq A \subseteq cl^*(R)$.

The set of all semi*pre open set in (X, τ) is denoted by $S^*PO(X, \tau)$.

3. Group action of Semi* pre open set and functions**Theorem: 3.1**

For a subset R_G of a topological group (X, τ) , the following statements are equivalent.

- (i) R_G is semi* pre-open.
- (ii) $R_G \subseteq cl^*(Pint(R_G))$.
- (iii) $cl^*(Pint(R_G)) = cl^*(R_G)$.

Proof:

(i) \Rightarrow (ii) If R_G is a group acting on semi*pre-open set T_G in X such that

$T_G \subseteq R_G \subseteq cl^*(T_G)$. Now $T_G \subseteq R_G$ implies $T_G = Pint(T_G) \subseteq Pint(R_G)$ which implies that $R_G \subseteq cl^*(T_G) \subseteq cl^*(Pint(R_G))$.

(ii) \Rightarrow (iii) Assume that $R_G \subseteq cl^*(Pint(R_G))$. Then $cl^*(R_G) \subseteq cl^*(cl^*(Pint(R_G))) = cl^*(Pint(R_G))$.

since $Pint(R_G) \subseteq R_G$. we have $cl^*(Pint(R_G)) \subseteq cl^*(R_G)$ and $cl^*(Pint(R_G)) = cl^*(R_G)$.

(i) \Rightarrow (ii) take $T_G \subseteq Pint(R_G)$. then T_G is a pre open set such that $T_G \subseteq cl^*(R_G) = cl^*(Pint(R_G)) = R_G \subseteq cl^*(T_G)$ therefore R_G is semi*pre-open.

Theorem:3.2

A group acting on union of every semi* pre- open set is semi* pre-open

Proof:

Let $\{R_G\}$ be a collection of semi*pre-open sets in X .

since each R_G is semi* pre-open set, there is a pre open set T_G in X such that

$$T_G \subseteq R_G \subseteq cl^*(T_G). \text{ Then } \cup T_G \subseteq \cup R_G \subseteq cl^*(\cup T_G) \subseteq cl^* \cup (T_G).$$

Hence $\cup T_G$ is semi*pre-open, since $\cup T_G$ is pre open.

Theorem:3.3

A topological group (X, τ) . If R_G is a semi* pre-open set in X and T_G is open in X then $R_G \cap T_G$ is group acted n semi*pre open.

Proof:

R_G is semi* pre open in X , there is a proper set E_G in X

such that $E_G \subseteq R_G \subseteq cl^*(E_G)$. since T_G is open, it is α -open.

Hence $E_G \cap T_G$ is pre open. $E_G \cap T_G \subseteq R_G \cap T_G \subseteq cl^*(E_G) \cap T_G \subseteq cl^*(E_G \cap T_G)$.

Thus $R_G \cap T_G$ is group acted on semi*pre-open.

Theorem:3.4

A topological group (X, τ) then E_G is a subset of X is g acted on semi* pre-open if and only if E_G contains a g acted on semi* pr-open set.

Proof:

Let E_G is g acted on semi* pre-open then E_G contains a semi* pre open set.

E_G itself about each of its points on the other hand let $x \in E_G$.

Then by assumption there is a semi*pre open set T_G containing x such that $T_G \subseteq E_G$.

Then $\cup \{T_G : x \in E_G\} = E_G$.

Therefore E_G is group acted on semi* pre-open.

Definition:3.5

The semi*pre interior of A is defined as the union of all semi* pre open sets of X contained in A and is denoted by $S^*PInt(A)$.

Example:3.6

Let $X = \{1,2,3,4\}$

$\tau = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, X\}$ here $\{1,4\}$ and $\{2,4\}$ are semi*- pre open set but their intersection $\{4\}$ is not semi* pre open.

Theorem:3.7

A topological group (X, τ) then E_G is a subset of X is g acted on semi* pre-open if and only if every point E_G is g acted on semi*-pre interior point of E_G .

Proof:

If E_G is semi*-pre open then every point of E_G is semi*-pre interior point.

Conversely, every point of E_G is semi* pre interior point of E_G implies that E_G is contains a g acted on semi* pre open set about each of its points.

Therefore E_G is semi* pre open.

Theorem: 3.8

If T_G is any subset of X then g acted on $S^*PInt(T_G)$ is semi*pre open. In fact it is the largest semi*pre-open set contained in T_G .

Proof:

$S^*PInt(T_G)$ is the union of semi*pre open subset of T_G .

It is semi* pre open Also it contain all semi* pre open, Also it contains all semi* pre open subset of T_G .

Theorem:3.9

A subset T_G of X is g acted on semi*pre open if and only if $S^*PInt(T_G) = T_G$.

Proof:

T_G is semi*pre open implies $S^*PInt(T_G) = T_G$ follows from definition,

conversely, let $S^*PInt(T_G) = T_G$.

$S^*PInt(T_G)$ is semi*pre open and hence T_G is semi* pre open.

Theorem:3.10

If T_G is a subset of X , then g acted $S^*PInt(T_G)$ is the set of all semi*pre open interior points of T_G .

Proof:

$X \in S^*PInt(T_G)$ if and only if X belongs to semi*pre open subset E_G of T_G

that is if and only if X is a g acted on semi* pre interior point of T_G .

Theorem:3.11

- (i) A group acting on every semi* pre open set is semi pre open.
- (ii) A group acting on every pre open set is semi*pre open.
- (iii) A group acting on every open set is semi*pre open.

Proof:

- (i) Let T_G be a semi*pre open set. Then there exists a pre open set E_G

such that $E_G \subseteq T_G \subseteq cl^*(E_G)$. since $cl^*(T_G) \subseteq cl(E_G)$,

$E_G \subseteq T_G \subseteq cl(E_G)$. Thus T_G is semi*pre open.

(ii) If T_G is a pre open set then $PInt(T_G) = T_G$ and $T_G \subseteq cl^*(PInt(T_G))$. Hence T_G is semi*pre open.

(iii) If T_G is an open set. since every open set is pre open Therefore T_G is semi* pre open set.

Example:3.12

In the topological group (X, τ') where $X = \{1,2,3,4\}$ and $\tau' = \{\emptyset, \{1\}, \{2,3,4\}, X\}$

Here $PO(X, \tau') = S^*PO(X, \tau') = SPO(X, \tau')$.

Example: 3.13

In the topological group (X, τ') where $X = \{1,2,3,4\}$ and $\tau' = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}, X\}$.

$$SPO(X, \tau') = S^*PO(X, \tau') \\ = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,4\}, \{2,3\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, \{2,3,4\}, X\}$$

Here $\tau' \subsetneq S^*PO(X, \tau') = SPO(X, \tau')$.

Theorem:3.14

If (X, τ') is a topological group then τ_{s^*p}' is a topology on X and is finer than τ .

Proof:

clearly \emptyset and $X \in \tau_{s^*p}'$. Let $E_i \in \tau_{s^*p}'$ and $E_G = \cup E_i$

since $E_i \in \tau_{s^*p}'$, then $E_i \in S^*PO(X, \tau')$

Let $R \in S^*PO(X, \tau')$, then $E_i \cap R \in S^*PO(X, \tau')$ for each i and

Hence $E_G \cap R = \cup (E_i) \cap R = \cup (E_i \cap R) \in S^*PO(X, \tau')$ Thus $E \in \tau_{s^*p}'$

Now let $E_1, E_2, E_3, \dots \dots E_n \in \tau_{s^*p}'$ then $E_1, E_2, E_3, \dots \dots E_n \in S^*PO(X, \tau')$

by definition of τ_{s^*p}' , $R \cap E_i \in \tau_{s^*p}'$ for every $R \in S^*PO(X, \tau')$.

If $R \in S^*PO(X, \tau')$ then by repeated application of the condition we have

$$(\cap E_i) \cap R \in S^*PO(X, \tau').$$

Hence $\cap E_i \in \tau_{s^*p}'$. so τ_{s^*p}' is a topology on X . Let $T \in \tau'$ by using the theorem every open set is semi*pre open, $T \in S^*PO(X, \tau')$ by theorem 3 $T \cup R \in S^*PO(X, \tau')$

for all $R \in S^*PO(X, \tau')$.

Hence $T \in \tau_{s^*p}'$. Thus τ_{s^*p}' is finer than τ' .

Example :3.15

The topological group (X, τ') where $X = \{1,2,3,4\}$ and $\tau' = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, X\}$.

Hence $S^*PO(X, \tau') = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,4\}, \{2,4\}, \{1,2,3\}, \{1,2,4\}, X\}$ and $\tau_{S^*p'} = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, \{1,2,4\}, X\}$, here $\tau' \subsetneq \tau_{S^*p'}$.

Theorem:3.16

Let R_G be g acted on semi*pre open of $T_G \subseteq X$ such that $R_G \subseteq T_G \subseteq cl^*(R_G)$ then T_G is g acted on semi* pre open.

Proof:

R_G be g acted on semi*pre open then $cl^*(R_G) = cl^*(PInt(R_G))$

since $R_G \subseteq T_G$, $PInt(R_G) \subseteq PInt(T_G)$ and hence $cl^*(PInt(R_G)) \subseteq cl^*(PInt(T_G))$

Now $T_G \subseteq cl^*(R_G) \subseteq cl^*(PInt(R_G)) \subseteq cl^*(PInt(T_G))$.

Hence T_G is g acted on semi*pre open.

Theorem:3.17

If (X, τ') is a topological group. Let R_G be a collection of subset in (X, τ')

Satisfying (i) $PO(X, \tau') \subseteq R_G$ (ii) If $T_G \in R_G$ and $E_G \subseteq X$ such that $T_G \subseteq E_G \subseteq cl^*(T_G)$.

Then $E_G \in R_G$, $S^*PO(X, \tau') \subseteq R_G$. In fact $S^*PO(X, \tau')$ is the smallest collection satisfying the condition (i) and (ii).

Proof:

Every pre open set is semi*pre open, $S^*PO(X, \tau')$ satisfies condition (i) and (ii)

Suppose R_G is any collection satisfying (i) and (ii). If $T_G \in S^*PO(X, \tau')$ then there is a semi*pre open set E_G in X such that $E_G \subseteq T_G \subseteq cl^*(E_G)$ by (i) $E_G \in R_G$ and

by (ii) $T_G \in R_G$. Hence $S^*PO(X, \tau') \subseteq R_G$ and this concludes the proof.

Example:3.18

In the topological group (X, τ') where $X = \{1,2,3,4\}$ and $\tau' = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, X\}$ the subset $\{1,4\}$ is semi-open but not semi*pre open.

Example:3.19

In the topological group (X, τ') where $X = \{1,2,3,4\}$ and $\tau' = \{\emptyset, \{1,2\}, X\}$ the subset $\{1,3\}$ is semi*-pre open but not semi open.

4. Group Acting of Semi*irresolute function

Definition:4.1

A function $g: X \rightarrow Y$ is said to be semi*-continuous if $f^{-1}(V)$ is semi*- open in X for every open set V in Y .

Definition:4.2

A function $g: X \rightarrow Y$ is said to be semi*-irresolute if $f^{-1}(V)$ is semi*- open in X for every open set V in Y .

Definition:4.3

A function $g: X \rightarrow Y$ is said to be contra semi*-irresolute if $f^{-1}(V)$ is semi*-closure in X for every semi*- open set V in Y .

Theorem:4.4

A group acting on every semi*-irresolute function is semi*-continuous.

Proof:

Let $g: X \rightarrow Y$ be semi*-irresolute . Let R be open in Y .

Then R is semi*-open. since g is semi*-irresolute, $g^{-1}(R)$ is semi*-open in X .

Thus g is semi*- continuous.

Remark: It is not true that group acting of every semi*-continuous function is semi*-irresolute.

Example:4.5

Let $X = \{1,2,3,4\}$ and $\tau = \tau_1 = \{\emptyset, \{1\}, \{2\}, \{1,2\}, \{1,2,3\}, X\}$

Let $g: (X, \tau) \rightarrow (X, \tau_1)$ be definition by $g(1) = 1, g(2) = 2, g(3) = 4, g(4) = 2$. Then g is semi*-continuous Here $\{1,4\}$ is semi*-open (X, τ_1) but $g^{-1}(1,4) = \{1,3\}$ is not semi*-open in (X, τ) . Therefore g is not semi*-irresolute.

Theorem:4.6

A group acting on every constant function is semi*-irresolute .

Proof:

Let $g: X \rightarrow Y$ be a constant function defined by $g(x) = y_1$, for all x in X , where y_1 is a fixed point in Y . Let R be semi*-pre open set in Y . Then $g^{-1}(R) = X$ or \emptyset according as $y_1 \in R$ or $y_1 \notin R$. Thus $g^{-1}(R)$ is semi*-open in X . Hence g is group action on semi*-irresolute.

Theorem:4.7

If X and Y are topological group. Let $g: X \rightarrow Y$ be a function. Then the following are equivalent.

- (i) g is group acted on semi*-irresolute.
- (ii) g is group acted on semi* irresolute at each point of X .
- (iii) $g^{-1}(R_G)$ is group acted on semi*-closed in X for every semi*-closed set R_G in Y .
- (iv) $g(S^*(cl(T_G))) \subseteq S^*cl(g(T_G))$ for every subset T_G of X .
- (v) $S^*cl(g^{-1}(E_G)) \subseteq g^{-1}(S^*cl(E_G))$ for every subset E_G of Y .
- (vi) $Int^*(cl(g^{-1}(R_G))) = Int^*(g^{-1}(R_G))$ for every semi*-closed set R_G in Y .
- (vii) $cl^*(Int(g^{-1}(F_G))) = cl^*(g^{-1}(F_G))$ for every semi*-open set F_G in Y .

Proof:

(i) \Rightarrow (ii) Let $g: X \rightarrow Y$ be g acted on semi*-irresolute. Let $x \in X$ are F_G be a semi*-open set in Y containing $g(x)$ thus $x \in g^{-1}(F_G)$ since g is semi*-irresolute,

$S_G = g^{-1}(F_G)$ is a semi*-open set containing x such that $g(S_G) \subseteq F_G$. This proves (ii).

(ii) \Rightarrow (iii) Let R_G be a semi*-closed set in Y . Then $F_G = Y \setminus R_G$ is semi*-open in Y . Let $x \in g^{-1}(F_G)$, then $g(x) \in R_G$. by assumption, there is a semi*-open set S_x in X containing x such that $g(x) \in g(S_x) \subseteq F_G$. This implies that $S_x \subseteq g^{-1}(F_G)$.

Hence $g^{-1}(F_G) = \bigcup \{S_x : x \in g^{-1}(F_G)\}$ by using theorem $g^{-1}(F_G)$ is semi*-open in X .

Therefore $g^{-1}(R_G) = g^{-1}(Y \setminus F_G) = X \setminus g^{-1}(F_G)$ is semi*-closed. Thus proves (iii)

(iii) \Rightarrow (iv) Let $T_G \subseteq X$. Let R_G be a semi*-closed set contains $g(T_G)$. then by (iii)

$g^{-1}(R_G)$ is a semi*-closed set that contains T_G this implies that

$S^*cl(T_G) \subseteq g^{-1}(R_G) \Rightarrow g(S^*cl(T_G)) \subseteq R_G$. This implies that $g(S^*cl(T_G)) \subseteq S^*cl(g(T_G))$.

(iv) \Rightarrow (v) Let $E_G \subseteq Y$ and $T_G = g^{-1}(E_G)$, by assumption

$g(S^*cl(T_G)) \subseteq S^*cl(g(T_G)) \subseteq S^*cl(E_G)$ This implies that $S^*cl(T_G) \subseteq g^{-1}(S^*cl(E_G))$.

Hence $S^*cl(g^{-1}(E_G)) \subseteq g^{-1}(S^*cl(E_G))$.

(v) \Rightarrow (vi) Let R_G be a semi*-closed set in Y . Then by (ii) $S^*cl(R_G) = R_G$ and hence by (v) $S^*cl(g^{-1}(R_G)) \subseteq g^{-1}(S^*cl(R_G)) = g^{-1}(R_G)$.

But always $g^{-1}(R_G) \subseteq S^*cl(g^{-1}(R_G))$. Therefore $S^*cl(g^{-1}(R_G)) = g^{-1}(R_G)$

Hence by (ii) $g^{-1}(R_G)$ is semi*-closed. Therefore $Int^*(cl(g^{-1}(R_G))) = Int^*(g^{-1}(R_G))$.

(vi) \Rightarrow (vii) Let F_G be a semi*-open set in Y . Then $Y \setminus F_G$ is semi*-closed in Y .

By assumption $Int^*(cl(g^{-1}(Y \setminus F_G))) = Int^*(g^{-1}(Y \setminus F_G))$

This implies that $Cl^*(Int(g^{-1}(F_G))) = Cl^*(g^{-1}(F_G))$.

(vii) \Rightarrow (i) Let F_G be any semi*-open set in Y .

Then $Cl^*(Int(g^{-1}(F_G))) = Cl^*(g^{-1}(F_G))$ therefore $g^{-1}(F_G)$ is semi*-open in X .

Hence g is group acted on semi*-irresolute

5. CONCLUSION

In this paper we have introduced various set and functions associated with semi*-pre open sets and investigated their properties, concepts and results available in the literature. More over we introduce the concepts of group actions of related to semi*-open set, semi*-pre open set, semi*-pre open functions and semi*-irresolute function of topological spaces. Further study several topics in group actions of topological spaces.

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