# Application of Queuing Theory to the Treatment of Patients in Hospitals

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**Abstract:** Emergency Department of any hospital deals with the treatment of emergency patients at any time of a day. Because of over Population in India and in Particular any town or city in India, overcrowding occurs in the emergency department. This may affect the quality and access to the treatment of the patients. For any emergency department, it is must to decrease the patients' waiting time to give proper and timely treatment to the patients. To analyze this situation, we propose to use queuing models to the Lion's Hospital in Mehsana as a particular case. We use the data of patients of the year 2018 in this hospital. Total 69,120 patients were registered during that year. This analysis indicates to minimize the gap between service time of human resources or doctors and waiting time of the patients and also suggests remedies in the future.

**Keywords:** Queue, Queuing theory, Patients, Doctors, Waiting time.

**Introduction:** Overcrowding in emergency department is common problem in India and worldwide. The number of patients in emergency department is growing and the Emergency Medical Treatment must be provided in short period of time. Delays in the emergency medical treatment may cause the deaths of the patients. It may also cost them severely.

The patients' flow in the ED can be studied by using the waiting lines. The waiting lines are effective tools to measure waiting time of patients and number of the patients in queues to analyze queues in the hospitals. The queuing theory models can be useful in the queuing situations in the hospital. When Utilization in the queuing situations in the hospital is increased, waiting time can be decreased. By measuring waiting times, delays in the services can be decreased in many ways. Such as by adding more number of server or by decreasing service times of the patient or by giving priority to those patients who require shorter service time. In this paper queuing theory model is used to manage the patients flow in optimal way in Lions' Hospital Mehsana.

## M/M/n Queuing Model:

We consider $M/M/n$ queuing model to estimate number of doctors needed. The arrivals are considered to follow a Poisson process and the service time follows an exponential distribution.

Poisson distribution is a distribution which shows the probability of arrivals in the given period of time where the mean and variance of Poisson distribution are same.

Using $M/M/n$ model, we assume that

$\frac{\lambda}{n\mu} < 1$, for steady-state conditions. [by Ref 1]

Where $\lambda$ = arrival rate,

$\mu$ = service rate,

$n$ = number of server,

$\rho$ = system utilization,

$\frac{1}{\mu}$ = service time.

$p_0$= probability of 0 patients in system,

$p_k$ = probability of k patients in system.

To calculate these probabilities we will use the following relations,

$$p_k = \frac{1}{k!}\left(\frac{\lambda}{\mu}\right)^k p_0\,, 0 \le k < n \qquad \text{[by Ref 1]}$$

$$p_k = \frac{1}{n!n^{k-n}}\left(\frac{\lambda}{\mu}\right)^k p_0\,, k \ge n \qquad \text{[by Ref 1]}$$

We know that total probability is always equal to one, There for

$$\sum_{k=0}^{\infty} p_k = 1$$

Using this equation we will calculate $p_0$ and the probability $p_k$.

If the number of patients $k$ is less than the number of servers $n$ then we have no waiting line.

If the number of patients exceeds the number of servers then we must reduce the waiting time in emergency department.

## Queuing Theory Model for Emergency Department:

In Mehsana, the Lions' hospital provides the emergency treatment to the patients. This emergency department is the one of the most equipped and the most modern hospital in Mehsana. It provides the latest Medical treatment for the patients having minor or major emergency like accidental emergency, pediatrics emergency, critical emergency etc.

We used, in the analysis, the detailed information of the patients over the period, January to December 2018. The arrival and departure of emergency patients is analyzed by using queuing theory models. Depending on patient in flow, we used the queuing theory to estimate the required number of human resources in emergency department and to estimate average waiting time. The result of this analysis helps in management of emergency department. It also provides the optimal decisions to manage the patients flow in future. Moreover, it gives the idea to manage the patients flow in major hospitals in the nearby cities and big cities of India.

In the specified period, total of 69,120 patients were registered. Thus, the daily average is considered as 192 patients.

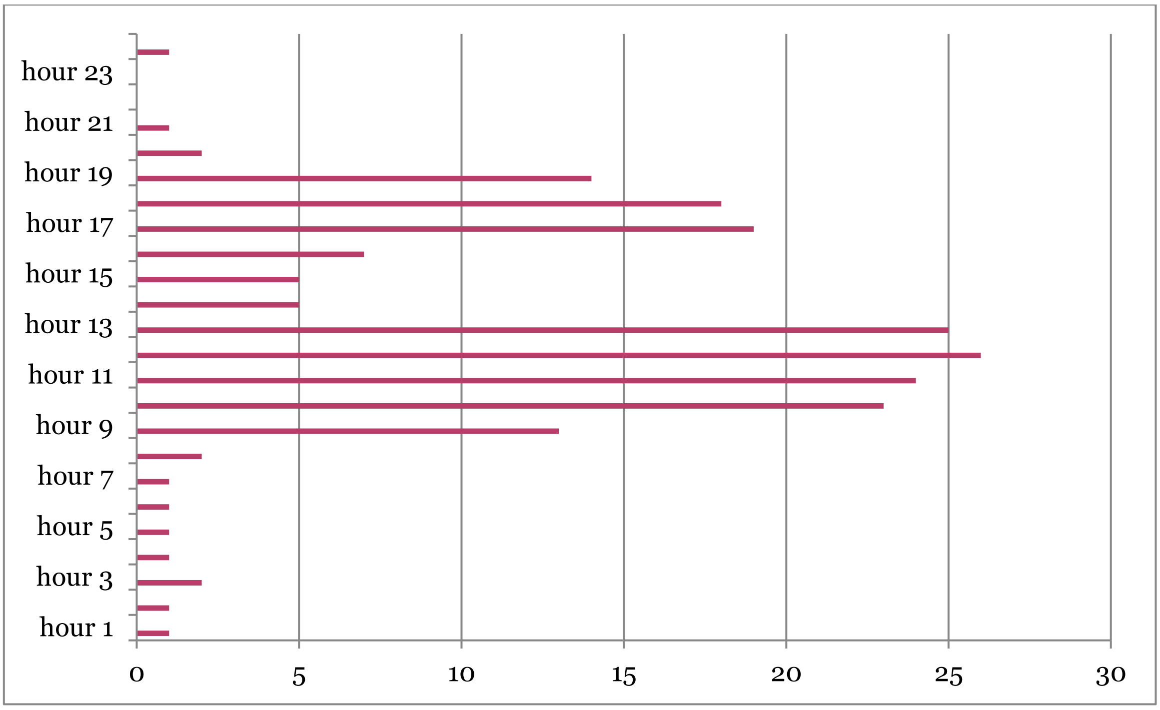
We denote annual average of number of arrivals of the patients in the time interval $[t_{i-1}, t_i]$ by $x_i$, where $t_0 = 0$ means the zero hours, the start of the day. We denote the number of $k$ patients arrived on the day $d$ in the time interval $[t_{i-1}, t_i]$ by ${p_k}^{d,i}$. The average number of patient arrivals per day is,

$$ {x_i}^d = \frac{\sum_{M=1}^{12} \sum_{d=1}^{\{30,31\}} {p_k}^{d,i}}{12} $$

where M denotes the months & the sum over d refers to number of days of each month.

Thus, average number of the patient arrivals in interval $[t_{i-1}, t_i]$ is given by the relations, $x_i = \frac{{x_i}^d}{24}$.

**<u>Bar graph showing arrivals of patients</u>**




Thus, the average number of arrivals as shown in Bar graph above will be: $\lambda = \frac{\sum x_i}{24}$

We obtained $⅄ = 8$, that is, the arrivals of patients occur at a rate 8/hour. If each doctor takes 15 minutes to check patients or prioritize the patients who need 15 minutes to be checked, then we get service rate $\mu = 4$. For steady- state condition it fulfills,

$$\frac{⅄}{n\mu} < 1$$

where $n$ is number of human resources or doctors in Lions' hospital. Using this relation, we derived the minimum number of physicians in hospital. Physician takes average 15 minutes to check patients. So, the number of arrivals of patients per hour $⅄$ can determine the minimum number of physicians $n$ required to serve the arrived patients as shown in the table given below.

| $⅄$ | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|
| $n$ | 3 | 3 | 3 | 3 | 4 | 4 | 4 | 4 | 5 |

We used $M/M/3$ queuing theory to determine different parameters of the queue and different characteristics of the hospital.

We take $⅄ = 8, \mu = 4, n = 3, \rho = \frac{8}{4} = 2$ and $\frac{\rho}{n} = 2/3$.

In $M/M/3$ model, to estimate the probability of no patients in the queue, we use the condition that the total Probability is always 1. So, we can write

$$p_0 + p_1 + p_2 + p_3 + p_4 + p_5 + \dots\dots\dots\dots\dots = 1$$

$$p_0 + p_0\frac{⅄}{\mu} + p_0\left(\frac{⅄}{\mu}\right)^2\frac{1}{2!} + p_0\left(\frac{⅄}{\mu}\right)^3\frac{1}{3!\,3^0} + p_0\left(\frac{⅄}{\mu}\right)^4\frac{1}{3!\,3^1} + \dots. = 1$$

$$p_0 + p_0\frac{⅄}{\mu} + p_0\left(\frac{⅄}{\mu}\right)^2\frac{1}{2!} + p_0\left(\frac{⅄}{\mu}\right)^3\frac{1}{3!}\left[\frac{1}{3^0} + \left(\frac{⅄}{\mu}\right)\frac{1}{3} + \left(\frac{⅄}{\mu}\right)^2\frac{1}{3^2} + \dots..\right] = 1$$

By geometric series, it is convergent and sum of the series,

$$p_0 + p_0\frac{⅄}{\mu} + p_0\left(\frac{⅄}{\mu}\right)^2\frac{1}{2!} + p_0\left(\frac{⅄}{\mu}\right)^3\frac{1}{3!}\frac{3\mu}{3\mu - ⅄} = 1$$

$$p_0 = \frac{1}{1 + \frac{⅄}{\mu} + \frac{1}{2}\left(\frac{⅄}{\mu}\right)^2 + \frac{1}{6}\left(\frac{⅄}{\mu}\right)^3\frac{3\mu}{3\mu - ⅄}}$$

$$p_0 = \frac{1}{1 + 2 + 2^2\frac{1}{2} + \frac{1}{6}2^3\frac{3\times 4}{3\times 4 - 8}}$$

$$= 0.111.$$

The Probability of no patients that is, no queue is;

$$p_0 = 0.111$$

In $M/M/3$ Queuing model, when 3 or more patients are already in system, an incoming patient must wait in queue. Therefore, we can calculate the probabilities of 3 or more patients in the system. That is,

$$\sum_{k=3}^{\infty} p_k = 1 - \sum_{k=0}^{2} p_k$$

$$= 1 - (p_0 + p_1 + p_2)$$

$$= 0.445.$$

This is the probability of 3 or more patients in the system that is, out of 1000 instances 445 times a new patient has to wait in the queue.

The Length of the queue of the is

$$L_q = \sum_{k=n}^{\infty} (k-n) p_k \qquad \text{[by Ref 1]}$$

$$= \sum_{m=0}^{\infty} m p_{m+n}, \quad where\ (k-n) = m$$

$$= \sum_{m=0}^{\infty} m \frac{\rho^{m+n}}{n^m n!} p_0$$

$$= \frac{\rho^{n+1}}{n!n} p_0 \sum_{m=0}^{\infty} m \left(\frac{\rho}{n}\right)^{m-1}$$

$$= \frac{\rho^{n+1}}{n!n} p_0 \frac{d}{d\left(\frac{\rho}{n}\right)} \sum_{m=0}^{\infty} \left(\frac{\rho}{n}\right)^{m}$$

$$= \frac{\rho^{n+1}}{n!n} p_0 \frac{d}{d\left(\frac{\rho}{n}\right)} \left(\frac{1}{1-\frac{\rho}{n}}\right)$$

$$= \frac{\rho^{n+1}}{n!n} p_0 \frac{1}{(1-\frac{\rho}{n})^2}$$

$$= \frac{\rho^{n+1}}{(n-1)!(n-\rho)^2} p_0, \quad where\ \frac{\rho}{n} < 1 \qquad \text{[by Ref 1]}$$

$$= 0.888 \approx 1.$$

This indicates the least probability to wait in the queue for a new patient.

$L_s$ = Length of the system = $L_q + \rho = 0.888 + 2 = 2.888 \approx 3.$

The waiting time in queue $w_q = \frac{L_q}{\lambda} = \frac{0.888}{8} = 0.111\ hours = 6.66\ minutes.$

This means that a new patient has to wait in the queue for $6.66\ minutes.$

The waiting time in system $w_s = \frac{L_s}{\lambda} = \frac{2.888}{8} = 0.361\ hours = 21.66\ minutes.$

When $n = 4$, the probability of waiting in the queue and also the waiting time of patients can be decreased to serve more number of patients arriving into the hospital. When $n = 5$, the probability of waiting and the waiting time can be decreased even further. In this way, the queuing theory is applied to the treatment of patients for the betterment of the hospitals.

## Conclusion:

The paper concludes that the number of human resources or doctors in the hospital can be determined as per the flow of patients in the hospital. With the help of this research, the probability of waiting in the queue can be decreased significantly and in addition the waiting time of patients can be reduced. When $\lambda = 12/$ hour, the number of human resources or doctors needed is $n = 4$ and when $\lambda = 16/$ hour, the number needed is $n = 5$. This can be considered as a future plan.

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