MULTIVARIABLE H-FUNCTIONS AND ITS APPLICATION
IN THE FIELD OF PHOTOSYNTHESIS

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ABSTRACT

The object of the present paper is to represent the process of Photosynthesis in terms of multivariable H-functions where the results have been performed according to their parameters (these are mainly H₂O (water), CO₂ (Carbon dioxide), sunlight and chlorophyll). Here, the multivariable H-functions are involved in the main result on specialize the parameters, and many know, unknown and new results may be obtained by application of the multivariable H-functions in the field Photosynthesis.

1. INTRODUCTION:

Photosynthesis is one of the single most important physico-biochemical process of the world. This is the process on which the existence of life on earth depends. It is only green plants ability that they utilize the energy of the sun light in producing oxygen contained organic material from stable inorganic matter by Photosynthesis process.

Generally, the Photosynthesis is Physico-Biochemical process, which produces complex carbohydrates by reaction of water and carbon-dioxide in the presence of sun light and chlorophyll.

According to Ruban, Randel and Kamel during the Photosynthesis the reaction between oxygen and heavy isotopes of water O(18) and H₂O(18) produce O₂, which is obtained by H₂O. And complex hydrocarbon's is produced by this reaction with water Tyagi [1].

\[ 12\text{H}_2\text{O}^{(18)} + 6\text{CO}_2 \xrightarrow{\text{Sun light}} \text{C}_6\text{H}_{12}\text{O}_6 + 6\text{H}_2\text{O} + 6\text{O}_2^{(18)} \]

Four parameters complete this natural reaction out of these four parameters two parameters supports internally and two externally. In this way the change happens in these parameters with respect to temperature (6°C to 37°C) these are represented symbolically in the following way and the internal change of variables depends upon these abbreviations given by Lax [4].

\[ \alpha + \gamma \frac{\beta}{\delta} \]

\[ \frac{\Gamma(\alpha+\gamma)\Gamma(\alpha+\delta)\Gamma(\beta+\delta)}{\Gamma(\alpha+\beta+\gamma+\delta)} 6\text{H}_2\text{O} + 6\text{O}_2 \]

(1)
Here water is evaporated and \( \text{O}_2 \) (oxygen) goes to atmosphere we compare it with the formula given by Erdelyi [3] for simplification of temperature. See Bhatnagar [1, p.11-16] and Lax [4].

\[
\frac{1}{2\pi i} \int_{x-i\infty}^{x+i\infty} \left[ \Gamma(\alpha + x)\Gamma(\beta + x)\Gamma(\gamma - x)\Gamma(\delta - x) \right] dx
\]

\[
= \frac{\Gamma(\alpha + \gamma)\Gamma(\alpha + \delta)\Gamma(\beta + \gamma)\Gamma(\beta + \delta)}{\Gamma(\alpha + \beta + \gamma + \delta)} + A,
\]

(2)

Where \( \text{Re}(\alpha) > 0, \text{Re}(\beta) > 0, \text{Re}(\gamma) > 0 \), and \( \text{Re}(\delta) > 0 \). \( \alpha, \beta, \gamma, \delta \) are respectively denoted the parameters water \((\alpha)\), carbon-dioxide \((\beta)\), sunlight \((\gamma)\) and chlorophyll \((\delta)\). \( A = \) constant, when \( \alpha = \beta = \gamma = \delta = 0 \), then \( A \) will be 0.

The multivariable \( H \)-functions given in [4] is defined as follows:

\[
H[z_{(1)}, \ldots, z_{(r)}] = H_{\prod L_1, \ldots, L_r} \psi(\xi_{(1)}, \ldots, \xi_{(r)}) z_{(1)}^{\xi_{(1)}} \ldots z_{(r)}^{\xi_{(r)}} d\xi_{(1)} \ldots d\xi_{(r)},
\]

(3)

where \( i = \sqrt{-1} \),

\[
\psi(\xi_{(1)}, \ldots, \xi_{(r)}) = \frac{\prod_{j=1}^{p} \Gamma \left( 1 - a_j + \sum_{i=1}^{r} \alpha_j(i) \xi_{(i)} \right)}{\prod_{j=n+1}^{q} \Gamma \left( a_j - \sum_{i=1}^{r} \alpha_j(i) \xi_{(i)} \right) \prod_{j=1}^{r} \Gamma \left( 1 - b_j + \sum_{i=1}^{r} \beta_j(i) \xi_{(i)} \right)}
\]

For all \( i \in \{1, \ldots, r\} \) and

\[
\phi_{(i)}(\xi_{(i)}) = \frac{\prod_{j=1}^{m(0)} \Gamma \left( d_j - \delta_j(i) \xi_{(i)} \right) \prod_{j=1}^{n(0)} \Gamma \left( 1 - c_j(i) + \gamma_j(i) \xi_{(i)} \right)}{\prod_{j=m(0)+1}^{q(0)} \Gamma \left( 1 - d_j(i) + \delta_j(i) \xi_{(i)} \right) \prod_{j=n(0)+1}^{p(0)} \Gamma \left( c_j(i) - \gamma_j(i) \xi_{(i)} \right)}
\]

In (3), \( i \) in the superscript \((i)\) stands for the number of primes, e.g., \( b^{(1)} = b', b^{(2)} = b'' \) and so on; and an empty product are interpreted as unity.

Suppose, as usual, that the parameters

\[
\begin{align*}
\{ a_{j(i)} &= 1, \ldots, p; \, c_{j(i)} &= 1, \ldots, p \}, \\
\{ b_{j(i)} &= 1, \ldots, q; \, d_{j(i)} &= 1, \ldots, q \},
\end{align*}
\]

\( \forall i = 1, \ldots, r \),

are associated coefficients and the complex numbers
\[
\begin{align*}
\left\{ \begin{array}{l}
\alpha_j^{(i)} j = 1, ..., p; \\
\beta_j^{(i)} j = 1, ..., q;
\end{array} \right. \\
\forall i \in \{1, ..., r\},
\end{align*}
\]

Where the integers \(n, p, q, m(i), n(i), p(i)\) and \(q(i)\) are constrained by the inequalities 
\(0 \leq n \leq p, q \geq 0, 1 \leq m(i) \leq q(i)\) and \(1 \leq n(i) \leq p(i)\) for all \(i \in \{1, ..., r\}\) and the inequalities holds true in (4) for suitably restricted values of the complex variable \(z_{(1)}, ..., z_{(r)}\). The sequences of parameters in (1) are such that none of poles of the integrand coincide, that is the poles of the integrand coincide, that is, the poles of the integrand in (1) are simple. The contour \(L_{(i)}\) in the complex \(\xi_{(i)}\) plane is of Mellin-Barnes type which runs from \(-\infty, \cdots, \infty\) with indentations, if necessary, to ensure that all the poles of \(\Gamma \left( d_j^{(i)} - \delta_j^{(i)} \xi_{(i)} \right), j = 1, ..., m(i)\) are separated from those of \(\Gamma \left( 1 - c_j^{(i)} + \gamma_j^{(i)} \xi_{(i)} \right), j = 1, ..., n(i)\) and \(\Gamma \left( 1 - a_j + \sum_{i=1}^{r} \delta_j^{(i)} \xi_{(i)} \right), j = 1, ..., n(i)\) \(\forall i \in \{1, ..., r\}\).

2. MAIN RESULT:

In this section the parameters (water \((H_2O)\), carbon-dioxide \((CO_2)\), sunlight and chlorophyll) are complete individually. At that condition the Photosynthesis is represented in multivariable H-functions as follows:

\[
\frac{1}{2\pi i} \int_{x=-\infty}^{x=+\infty} \prod_{p=1}^{n+4m(i)-n_{(i)}} \prod_{q=1}^{\sum_{(i)=1}^{r} n_{(i)}} \prod_{p=1}^{\sum_{(i)=1}^{r} q_{(i)}} \left[ \begin{array}{c}
Z_1 \\
\vdots \\
Z_r
\end{array} \right] \left( \begin{array}{c}
1 - \alpha - x, \sum_{(i)=1}^{r} \alpha_{(i)} \\
\vdots \\
1 - \beta - x, \sum_{(i)=1}^{r} \beta_{(i)}
\end{array} \right) \\
\left( \begin{array}{c}
\gamma_{(i)} \\
\delta_{(i)}
\end{array} \right) \\
\left( \begin{array}{c}
\alpha_j^{(1)} \ldots, \alpha_j^{(r)} \end{array} \right)_{1,p} \\
\left( \begin{array}{c}
\beta_j^{(1)} \ldots, \beta_j^{(r)} \end{array} \right)_{1,q}
\right) dx
\]

\[
= \prod_{p=1}^{n+4m(i)-n_{(i)}} \prod_{q=1}^{\sum_{(i)=1}^{r} n_{(i)}} \prod_{p=1}^{\sum_{(i)=1}^{r} q_{(i)}} \left[ \begin{array}{c}
Z_1 \\
\vdots \\
Z_r
\end{array} \right] \left( \begin{array}{c}
1 - \alpha - x, \sum_{(i)=1}^{r} \alpha_{(i)} + \gamma_{(i)} \\
\vdots \\
1 - \beta - x, \sum_{(i)=1}^{r} \beta_{(i)} + \delta_{(i)}
\end{array} \right) \\
\left( \begin{array}{c}
\alpha_j^{(1)} \ldots, \alpha_j^{(r)} \end{array} \right)_{1,p} \\
\left( \begin{array}{c}
\beta_j^{(1)} \ldots, \beta_j^{(r)} \end{array} \right)_{1,q} \\
\left( \begin{array}{c}
1 - \alpha - \gamma, \sum_{(i)=1}^{r} \alpha_{(i)} + \delta_{(i)} \end{array} \right)_{1,p} \\
\left( \begin{array}{c}
1 - \beta - \gamma, \sum_{(i)=1}^{r} \beta_{(i)} + \delta_{(i)} \end{array} \right)_{1,q}
\]
\[(\epsilon_{j}^{(1)}, \gamma_{j}^{(1)})_{1,p(1)} ; \ldots ; (\epsilon_{j}^{(r)}, \gamma_{j}^{(r)})_{1,p(r)} + A \]

\[|argz_{(i)}| < \frac{1}{2} V_{(i)} \pi, \forall i \in \{1, ..., r\}, \text{where } V_{(i)} \text{ is given in (4)}.\]

3. ANALYSIS AND PROOF OF MATHEMATICAL FORMULA:

To prove (6), we put value of the internal and external parameters for Photosynthesis and for a healthy plant whose surrounding temperature is \(\beta \) to \(\gamma\). Photosynthesis in the absence of water: \((\beta + \gamma)\) (see Tyagi[1] and bhatnagar [2]) in the integral of (2) on both sides and multiplying by \(\frac{1}{(2\pi)^{1}} (\phi_{(r)}(\xi_{(1)}), ..., \phi_{(r)}(\xi_{(r)})) \psi(\xi_{(1)}, ..., \xi_{(r)})) \) both sides and, further integrating in the direction of \(L_{(i)} \) with respect to time and after changing the order of integration on the left hand side, we get the required result (6).

4. APPLICATION:

In this section, we remove various parameters (water \((H_{2}O)\), carbon-dioxide \((CO_{2})\), sunlight and chlorophyll) in the reaction of Photosynthesis respectively and represent their position by formula.

(a). Photosynthesis in the absence of water:

If we put \(\alpha_{(i)} = 0\) in the main result then we get following formula:

\[
\frac{1}{2\pi i} \int_{-\infty}^{+\infty} H_{0,n+3m_{(1)},n_{(1)}} ; \ldots ; m_{(i)}, n_{(i)} \Gamma \left(1 - \beta - x, \sum_{(i)=1}^{r} \beta_{(i)} \right), \left(1 - \gamma - x, \sum_{(i)=1}^{r} \gamma_{(i)} \right), \left(1 - \delta - x, \sum_{(i)=1}^{r} \delta_{(i)} \right), \sum_{(i)=1}^{r} \right) \Gamma(\alpha + x) \, dx
\]

\[
= \sum_{(i)=1}^{p+4q+1} (\beta_{(i)} + \gamma_{(i)}), (\epsilon_{j}^{(1)}, \gamma_{j}^{(1)})_{1,p(1)} ; \ldots ; (\epsilon_{j}^{(r)}, \gamma_{j}^{(r)})_{1,p(r)} + A
\]

\[|argz_{(i)}| < \frac{1}{2} V_{(i)} \pi, \forall i \in \{1, ..., r\}, \text{where } V_{(i)} \text{ is given in (4)}.\]
(b). Photosynthesis in the absence of carbon-dioxide:

If we put $\beta_{(i)}=0$ in the main result then we get following formula:

$$
\frac{1}{2\pi i} \int_{x=-\infty}^{x=\infty} H_{0,n+3,m_{(1)},n_{(1)};\ldots;m_{(r)},n_{(r)}}^{\beta_{(1)},\ldots,\beta_{(r)}} \left[ \frac{Z_1}{Z_r} \right] \left( 1 - \alpha - x, \sum_{(i)=1}^{r} \alpha_{(i)} \right), \left( 1 - \gamma - x, \sum_{(i)=1}^{r} \gamma_{(i)} \right), \gamma(x) \, dx
$$

$$
\left( 1 - \delta - x, \sum_{(i)=1}^{r} \delta_{(i)} \right), \left( \alpha_{(1)}, \ldots, \alpha_{(r)} \right)_{1,p} : \left( \beta_{(1)}, \ldots, \beta_{(r)} \right)_{1,q} \left[ \frac{Z_1}{Z_r} \right] \left( 1 - \alpha - y, \sum_{(i)=1}^{r} \alpha_{(i)} + \gamma_{(i)} \right), \left( 1 - \alpha - \delta, \sum_{(i)=1}^{r} \alpha_{(i)} + \delta_{(i)} \right), \gamma(y) \, dy
$$

$$
\left( 1 - \beta - y, \sum_{(i)=1}^{r} \gamma_{(i)} \right), \left( 1 - \beta - \delta, \sum_{(i)=1}^{r} \delta_{(i)} \right), \left( \alpha_{(1)}, \ldots, \alpha_{(r)} \right)_{1,p} : \left( \beta_{(1)}, \ldots, \beta_{(r)} \right)_{1,q} \left[ \frac{Z_1}{Z_r} \right] \left( 1 - \alpha - \beta - y - \delta, \sum_{(i)=1}^{r} \alpha_{(i)} + \gamma_{(i)} + \delta_{(i)} \right):
$$

$$
\left( \beta_{(1)}, \ldots, \beta_{(r)} \right)_{1,q} \left[ \frac{Z_1}{Z_r} \right] \left( 1 - \alpha - \beta - y - \delta, \sum_{(i)=1}^{r} \alpha_{(i)} + \gamma_{(i)} + \delta_{(i)} \right):
$$

$$
\Gamma(\beta + x) \, dx
$$

$$
\Gamma(\gamma + x) \, dx
$$

(c). Photosynthesis in the absence of sunlight:

If we put $\gamma_{(i)} = 0$ in the main result then we get following formula:

$$
\frac{1}{2\pi i} \int_{x=-\infty}^{x=\infty} H_{0,n+3,m_{(1)},n_{(1)};\ldots;m_{(r)},n_{(r)}}^{\beta_{(1)},\ldots,\beta_{(r)}} \left[ \frac{Z_1}{Z_r} \right] \left( 1 - \alpha - x, \sum_{(i)=1}^{r} \beta_{(i)} \right), \left( 1 - \beta - x, \sum_{(i)=1}^{r} \beta_{(i)} \right), \gamma(x) \, dx
$$

$$
\left( 1 - \delta - x, \sum_{(i)=1}^{r} \delta_{(i)} \right), \left( \alpha_{(1)}, \ldots, \alpha_{(r)} \right)_{1,p} : \left( \beta_{(1)}, \ldots, \beta_{(r)} \right)_{1,q} \left[ \frac{Z_1}{Z_r} \right] \left( 1 - \alpha - x, \sum_{(i)=1}^{r} \beta_{(i)} \right), \left( 1 - \beta - x, \sum_{(i)=1}^{r} \beta_{(i)} \right), \gamma(x) \, dx
$$

$$
\left( \beta_{(1)}, \ldots, \beta_{(r)} \right)_{1,q} \left[ \frac{Z_1}{Z_r} \right] \left( 1 - \alpha - x, \sum_{(i)=1}^{r} \beta_{(i)} \right), \left( 1 - \beta - x, \sum_{(i)=1}^{r} \beta_{(i)} \right), \gamma(x) \, dx
$$
\[
\begin{align*}
&(1 - \beta - \gamma, \sum_{(i=1)}^{r} \beta_{(i)}), (1 - \beta - \delta, \sum_{(i=1)}^{r} \beta_{(i)} + \delta_{(i)}), (a_{p}a_{j}^{(1)}, \ldots, a_{j}^{(r)})_{l.p} : \notag \\
&(b_{j}^{(1)}, \ldots, b_{j}^{(r)})_{l.q}, (1 - \alpha - \beta - \gamma - \delta, \sum_{(i=1)}^{r} \alpha_{(i)} + \beta_{(i)} + \delta_{(i)}) : \\
&(c_{j}^{(1)}, y_{j}^{(1)})_{1,p(1)} ; \ldots ; (c_{j}^{(r)}, y_{j}^{(r)})_{1,p(r)} \\
&(d_{j}^{(1)}, \delta_{j}^{(1)})_{1,q(1)} ; \ldots ; (d_{j}^{(r)}, \delta_{j}^{(r)})_{1,q(r)} \bigg]+A \tag{9}
\end{align*}
\]
\[
|ar\cdot gz_{(i)}| < \frac{1}{2} V_{(i)} \pi, \; \forall \; i \in (1, \ldots, r), \text{where } V_{(i)} \text{ is given in (4).}
\]

(d). Photosynthesis in the absence of chlorophyll:

If we put \( \delta_{(i)} = 0 \) in the main result then we get following formula:

\[
\frac{1}{2\pi} \int_{x=+\infty}^{x=-\infty} H_{p+3,q+1;P(1),Q(1)}^{0,n+4,m_{1},n_{1}} \left[ \begin{array}{c}
Z_{1} \\
\vdots \\
Z_{r}
\end{array} \right] \left( 1 - \alpha - x, \sum_{(i=1)}^{r} \alpha_{(i)} \right), \left( 1 - \beta - x, \sum_{(i=1)}^{r} \beta_{(i)} \right), \\
\vdots \\
\left( 1 - \gamma - x, \sum_{(i=1)}^{r} \gamma_{(i)} \right), \left( 1 - \delta - x, \sum_{(i=1)}^{r} \delta_{(i)} \right) \right)
\]

\[
\left(1 - \gamma - \sum_{(i=1)}^{r} \gamma_{(i)}\right), \left(1 - \delta - \sum_{(i=1)}^{r} \delta_{(i)}\right), (a_{p}a_{j}^{(1)}, \ldots, a_{j}^{(r)})_{l.p} : \\
(b_{j}^{(1)}, \ldots, b_{j}^{(r)})_{l.q}, (1 - \alpha - \beta - \gamma - \delta, \sum_{(i=1)}^{r} \alpha_{(i)} + \beta_{(i)} + \gamma_{(i)}): \\
(c_{j}^{(1)}, y_{j}^{(1)})_{1,p(1)} ; \ldots ; (c_{j}^{(r)}, y_{j}^{(r)})_{1,p(r)} \\
(d_{j}^{(1)}, \delta_{j}^{(1)})_{1,q(1)} ; \ldots ; (d_{j}^{(r)}, \delta_{j}^{(r)})_{1,q(r)} \bigg]+A \tag{10}
\]
\[
|ar\cdot gz_{(i)}| < \frac{1}{2} V_{(i)} \pi, \; \forall \; i \in (1, \ldots, r), \text{where } V_{(i)} \text{ is given in (4).}
\]

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