OPERATIONS ON GRUNDY NUMBERS BETWEEN SOME GRAPHS AND ITS LINE GRAPHS

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Abstract:

Inequalities of Nordhaus –Gaddum analyzes maximal or minimal values of the operations on a graph and its complement. Bounds on the sum and product of Grundy numbers of certain classes of graphs and their line graphs are examined in this paper. Also certain classes of graphs are characterized.

KEYWORDS: line graph, wheel graph, double wheel graph, Tadpole graph, fan graph, Double fan graph, Ladder graph.

PRELIMINARIES:

Many graph coloring problems imply the relationship between the chromatic parameters of a graph. The substantial values of operations of bounds on a graph and its complement are analyzed by graph theorists. Collins.et.al analysed the characteristics of some graphs with chromatic number based on Nordhaus and Goddum Principle[3]. Susanth.et.al, studied the problem on the extreme values of operations of Graphs and their associated graphs[12]. Zaker.et.al,holds Nordhaus-Goddum Principle to explicit Grundy number of the operations of bipartite and complement of bipartite graphs[15].

Grundy number was initiated by Christen and Selkow [10] in 1979. Manouchehr Zaker obtained the results on the Grundy chromatic number of graphs[15] to get the inequalities also. The bounds on the Grundy number of products like direct, strong, lexigographic of graphs were analyzed by Victor campos,et.al [13].

Jensen.et.al state “Grundy coloring of order k of a graph G is a k-coloring of G with colors 1,2…k such that for each vertex x the color of x is the smallest positive integer not used as a color on any neighbor of x in G”. The Grundy number Γ(G) is the largest integer k for which G has a Grundy coloring of order k is defined by Christen and selkow as in page no.170 in [10].

Balakrishnan.et.al states "The Cartesian product $G_1\times G_2$ of two graphs G_1 and G_2 is the simple graph with $V_1\times V_2$ .The dot set (u_1, v_1) and two dots (u_2, v_2) are neighboring in $G_1\times G_2$ if, and only if either $u_1= u_2$ and v_1 is neighboring to v_2 in G_2, u_1 is neighboring to u_2 in G_1 and $v_1= v_2$ " in the page.no.26 in his book [1].

Example:

A ladder graph Ln of order n is a Cartesian product of a path Pn and complete graph K_1.

Also he explored "The join, $G_1\vee G_2$ of G_1 and G_2 is the sub graph of G_1+G_2 in which each dot of G_1 is neighboring to each dot of G_2 where G_1 and G_2 be dot disjoint graphs" in the page no.26[2]

Example; A wheel graph W_n is the join of (n-1) cycle and Complete graph K_1.

By [9] S.P.Hande.et.al explained that Tadpole graph $T_{n,k}$ is a graph constructed by associating a cycle graph C_n to a path of order k.

3.RELATION BETWEEN THE SUM AND PRODUCT OF GRUNDY NUMBER 0F GRAPHS AND ITS LINE GRAPHS

Theorem.3.1.

For a wheel graph W_n , $\Gamma(W_n) + \Gamma[L(W_n)] = n+5$,

and $\Gamma(W_n) . \Gamma[L(W_n)] = 4(n+1)$.

Proof:

Let W_n be a wheel graph of order n and V(G)= $\{w_1, w_2\ldots w_n\}$ with W_n as hub. Since wheel graph is the join of cycle C_{n-1} and complete graph K_1, $\Gamma(C_{n-1} VK_1)= \Gamma(W_n)+ \Gamma(K_1)$ according to the Theorem. in [].

Hence $\Gamma(W_n)=4$.

Let $L(W_n)$ be line graph of wheel graph which induces a complete graph of order n-1 and have a cycle of order n-1 .Here complete graph needs n-1 colors for Grundy coloring but the grundy coloring should be maximum ,as the remaining vertices of cycle needs 2 new colors.
Hence $\Gamma[L(W_n)]=n+1$.

As a result $\Gamma(W_n) + \Gamma(L(W_n)] = n+5$,

and $\Gamma(W_n) . \Gamma[L(W_n)] = 4(n+1)$.

Theorem.3.2.

For n>5, T(n,m) be a Tadpole graph, $\Gamma[T(n,m)] + \Gamma[L(T(n,m))] = 8$

$$\Gamma[T(n,m)]\ .\Gamma[L(T(n,m))] = 16$$

Proof:

Let T(n,m) be a Tadpole graph. Since the Tadpole graph is planar, it is four colorable[1].Upper bound for Grundy coloring is $\Delta(G)+1$. Hence $\Gamma\ [T(n,m)] = 4$.

Since the line graph of Tadpole graph induces a clique of order 3 and $\Gamma(G) \geq \omega(G)$ and $\Delta(G)=3$

When n>5, $\Gamma\ [L\{T(n,m)\}] = 4$.

Hence, $\Gamma\ [T(n,m)]\ + \Gamma[L(T(n,m))] =4+4= 8$,

$\Gamma[T(n,m)]. \Gamma[L\ (T(n,m))] = 4.4=16$.

Theorem.3.3:

If $G= P_2\square P_m$ be a ladder graph, then $\Gamma\ [G]\ + \Gamma L\ (G)] = 9$

$$\Gamma[G]\ .\ \Gamma[L(G)\ ] = 20$$

Proof:

Let $G= P_2\square P_m$ be a ladder graph. Since it is a chordal graph and its girth is 4. Also $\Delta(G)=3$, $\delta(G)=2$. As it contains even cycle of order 4. it is a triangle-free graph and the Grundy color is maximum , $\Gamma\ [G] =4$.

Since the line graph of G is a planar graph which have maximum degree 4, Grundy number is 5 in accordance with Heawood five color theorem.

Hence $\Gamma\ [G]\ + \Gamma L\ (G)] = 9$

$$\Gamma[G]\ .\Gamma[L(G)\ ] = 20.$$

Theorem.3.4:

If $G = P_n \vee K_1$ be a fan graph F_{n+1}, then $\Gamma[G]\ + \Gamma[L(G)] = 2n+3$

$$\Gamma[G]\ .\Gamma[L(G)\ ] = (n+3)(n+1).$$

Proof:

Let $G = P_n \vee K_1$ be a fan graph F_{n+1}. In view of the fact that the upper bound for Grundy coloring is $\Delta(G)+1$, Grundy coloring of G is n+1. Hence $\Gamma(G) =n+1$ as $\Delta(G)=n$.

Meanwhile**,** line graph of G instigates a complete subgraph of order n, Grundy coloring of G must be greater than or equal to n. In the view of Highest degree, $\Gamma(G)=n+3$.

 ISSN:1007-1172

Subsequently, $\Gamma[G] + \Gamma[L(G)] = 2n+3$

$\Gamma[G] .\Gamma[L(G)] = (n+3)(n+1)$.

4. CONCLUSION:

The progress executed from the above work tole out a clear vision into the problem associating Grundy number by justifying the known lower and upper bounds on sums and products of Grundy number of graph G and line graph of G. Many more Chromatic parameters and properties of the operations of graphs have to be discovered and compared.

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