

# Solving Fuzzy Transportation Problem using Icosikaoctagonal Fuzzy Numbers

R. Deepa<sup>1</sup> and V. Raju<sup>2</sup>

<sup>1</sup>Research Scholar, Department of Mathematics,  
Vels Institute of Science, Technology and Advanced Studies (VISTAS),  
Chennai-600 117, Tamil Nadu, India.

E-mail: deeparajesh.n7@gmail.com

<sup>2</sup>Assistant Professor, Department of Mathematics  
Vels Institute of Science, Technology and Advanced Studies (VISTAS),  
Chennai-600 117, India.

E-mail: raju.v.sbs@velsuniv.ac.in

## Abstract

The transportation problem is one of the special type and the applications of linear programming problems. The usual transportation problem is unspecified that the decision maker is sure about the accurate values of transportation cost, supply and demand of the product. In some situations decision maker is not in the position to specify the objective precisely but rather than which can be specified in fuzzy concepts. We study fuzzy transportation problem (FTM) using Icosikaoctagonal fuzzy number and its membership function. We have defined the ranking to the Icosikaoctagonal fuzzy numbers to convert the fuzzy valued transportation to crisp valued transportation problem. To illustrate these approaches, a real life problem has been solved using the Least Cost Method

**Keywords:** Fuzzy transportation problem (FTM), Icosikaoctagonal fuzzy number, Alpha cut, least cost method.

## 1. Introduction:

The fuzzy set was presented to the world by Zadeh[1]. The fruits of fuzzy sets are extended to engineering and technology, management and economics by many researchers .The

transportation problem originally developed by Hitchcock. The transportation problems are most useful to the industries and others to reduce the cost and maximize the profit. The transportation problem is a special case of linear programming problem. In a fuzzy transportation problem, costs, supply and demand values are fuzzy values. There are many approaches to solve the fuzzy transportation problem by different Authors. In many real life situations, it is not possible to determine both transportation unit cost and quantities, but the fuzzy numbers give best approximation of them. Raju and Jayagopal [2] introduced the Icosikaioctagonal fuzzy numbers and its membership function. Because there is always no possible to restrict the membership function in a particular form. Icosagonal fuzzy number is complex when it is compared to the triangular and trapezoidal fuzzy number both in form and computation. Using Icosikaioctagonal fuzzy numbers to solve the fuzzy transportation problem gives best optimal value when we compared with triangular and trapezoidal fuzzy numbers. Michael has proposed algorithm for solving transportation problem with fuzzy constraints and has investigated the relationship between the fuzzy algebraic structure of the optimum solution of the deterministic problem and its fuzzy equivalent. In this paper, we have solved fuzzy transportation problem using Icosikaioctagonal fuzzy number. We have illustrated fuzzy transportation problem using ranking technique with Icosikaioctagonal fuzzy numbers.

## 2. PRELIMINARIES

In this section, we give the preliminaries that are required for this study.

**Definition 2.1.** A fuzzy set  $A$  is defined by  $A = \{(x, \mu_A(x)) : x \in A, \mu_A(x) \in [0,1]\}$ . Here  $x$  is crisp set  $A$  and  $\mu_A(x)$  is membership function in the interval  $[0,1]$ .

**Definition 2.2.** The fuzzy number  $A$  is a fuzzy set whose membership function must satisfy the following conditions.

- (i) A fuzzy set  $A$  of the universe of discourse  $X$  is convex
- (ii) A fuzzy set  $A$  of the universe of discourse  $X$  is a normal fuzzy set if  $x_i \in X$  exists
- (iii)  $\mu_A(x)$  is piecewise continuous

**Definition 2.3** An  $\alpha$ -cut of fuzzy set  $A$  is classical set defined as  ${}^\alpha[A] = \{x \in X | \mu_A(x) \geq \alpha\}$

**Definition 2.4** A fuzzy set  $A$  is a convex fuzzy set iff each of its  $\alpha$ -cut  ${}^\alpha A$  is a convex set.

### Definition 2.5 Mathematical formulation of a fuzzy transportation problem

The general form of Transportation problem is

$$\text{Minimize (total cost) } z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

$$\text{Subject to the constraints } \sum_{j=1}^n x_{ij} = a_i, i=1,2,3,\dots,m$$

$$\sum_{i=1}^m x_{ij} = b_j, j=1,2,3,\dots,n$$

$$x_{ij} \geq 0 \text{ For all } i \text{ and } j$$

## 2.6 Ranking of Icosikaioctagonal fuzzy number:

Let P be a normal Icosikaioctagonal fuzzy number. The value  $M(L)$ , called as measure of L is calculated as

$$M(P) = \frac{1}{2} \int_1^{k_1} (J_1 + J_2) dJ + \frac{1}{2} \int_{k_1}^{k_2} (A_1 + A_2) dA + \int_{k_2}^{k_3} (S_1 + S_2) dS + \int_{k_3}^{k_4} (T_1 + T_2) dT + \int_{k_4}^{k_5} (I_1 + I_2) dI + \int_{k_5}^{k_6} (N_1 + N_2) dN + \int_{k_6}^{k_1} (R_1 + R_2) dR$$

$$\text{where } 0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq 1$$

$$M(P) = \frac{1}{4} \left[ \begin{aligned} &(r_1 + r_2 + r_{27} + r_{28})k_1 + (r_3 + r_4 + r_{25} + r_{26})(k_2 - k_1) + (r_5 + r_6 + r_{23} + r_{24})(k_3 - k_2) + \\ &(r_7 + r_8 + r_{21} + r_{22})(k_4 - k_3) + (r_9 + r_{10} + r_{19} + r_{20})(k_5 - k_4) + (r_{11} + r_{12} + r_{17} + r_{18})(k_6 - k_5) \\ &+ (r_{13} + r_{14} + r_{15} + r_{16})(1 - k_6) \end{aligned} \right]$$

$$\text{where } 0 \leq k_1 \leq k_2 \leq k_3 \leq k_4 \leq k_5 \leq k_6 \leq 1$$

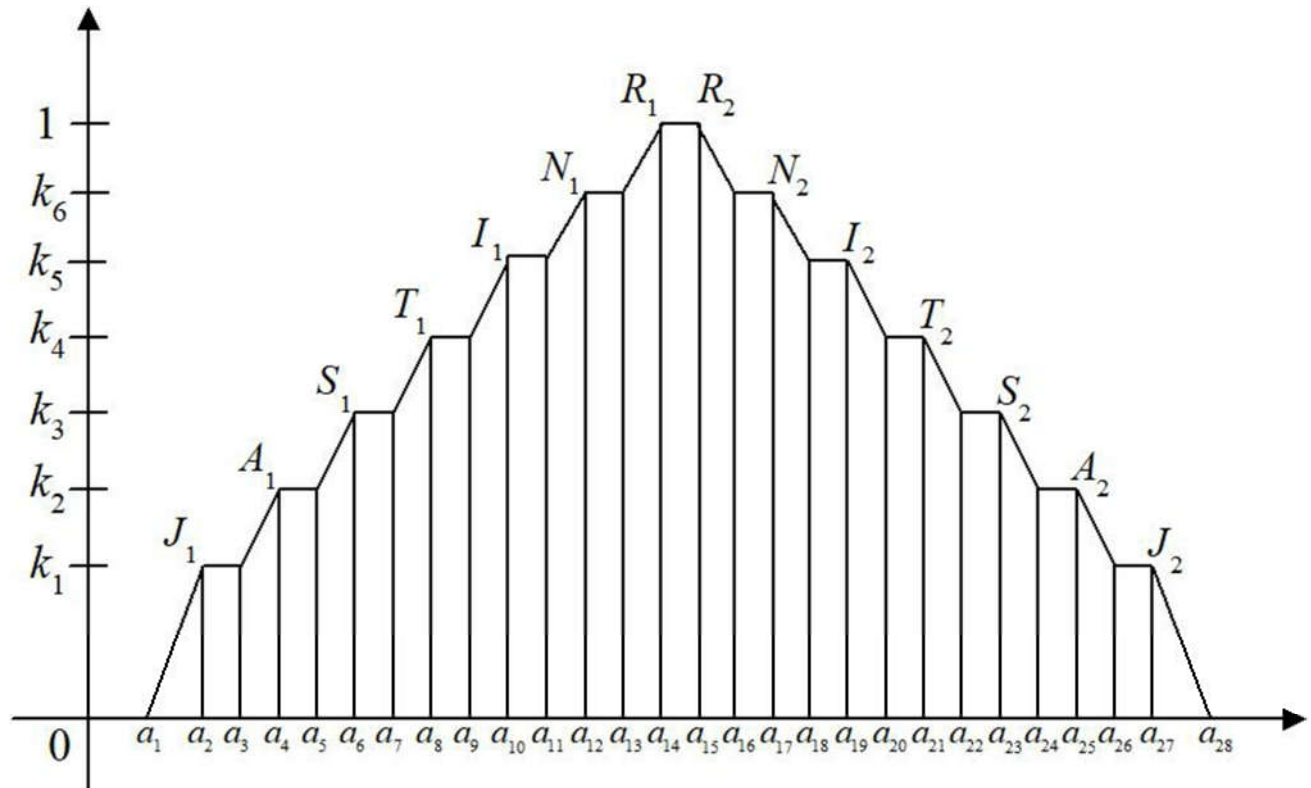
$$\text{we take the values for } k_1 = \frac{1}{7}, k_2 = \frac{2}{7}, k_3 = \frac{3}{7}, k_4 = \frac{4}{7}, k_5 = \frac{5}{7}, k_6 = \frac{6}{7}$$

### Definition 2.7 [2]

A fuzzy number  $A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, \dots, a_{28})$  is Icosikaioctagonal fuzzy number and its membership function is given by

$$\mu_A(x) = \begin{cases} 0, & \text{for } x < a_1 \\ k_1 \left( \frac{x - a_1}{a_2 - a_1} \right), & \text{for } a_1 \leq x \leq a_2 \\ k_1, & \text{for } a_2 \leq x \leq a_3 \\ k_1 + (k_2 - k_1) \left( \frac{x - a_3}{a_4 - a_3} \right), & \text{for } a_3 \leq x \leq a_4 \\ k_2, & \text{for } a_4 \leq x \leq a_5 \\ k_2 + (k_3 - k_2) \left( \frac{x - a_5}{a_6 - a_5} \right), & \text{for } a_5 \leq x \leq a_6 \\ k_3, & \text{for } a_6 \leq x \leq a_7 \\ k_3 + (k_4 - k_3) \left( \frac{x - a_7}{a_8 - a_7} \right), & \text{for } a_7 \leq x \leq a_8 \\ k_4, & \text{for } a_8 \leq x \leq a_9 \\ k_4 + (k_5 - k_4) \left( \frac{x - a_9}{a_{10} - a_9} \right), & \text{for } a_9 \leq x \leq a_{10} \\ k_5, & \text{for } a_{10} \leq x \leq a_{11} \\ k_5 + (k_6 - k_5) \left( \frac{x - a_{11}}{a_{12} - a_{11}} \right), & \text{for } a_{11} \leq x \leq a_{12} \\ k_6, & \text{for } a_{12} \leq x \leq a_{13} \\ k_6 + (1 - k_6) \left( \frac{x - a_{13}}{a_{14} - a_{13}} \right), & \text{for } a_{13} \leq x \leq a_{14} \\ 1, & \text{for } a_{14} \leq x \leq a_{15} \\ k_6 + (1 - k_6) \left( \frac{a_{16} - x}{a_{16} - a_{15}} \right), & \text{for } a_{15} \leq x \leq a_{16} \\ k_6, & \text{for } a_{16} \leq x \leq a_{17} \\ k_5 + (k_6 - k_5) \left( \frac{a_{18} - x}{a_{18} - a_{17}} \right), & \text{for } a_{17} \leq x \leq a_{18} \\ k_5, & \text{for } a_{18} \leq x \leq a_{19} \\ k_4 + (k_5 - k_4) \left( \frac{a_{20} - x}{a_{20} - a_{19}} \right), & \text{for } a_{19} \leq x \leq a_{20} \\ k_4, & \text{for } a_{20} \leq x \leq a_{21} \\ k_3 + (k_4 - k_3) \left( \frac{a_{22} - x}{a_{22} - a_{21}} \right), & \text{for } a_{21} \leq x \leq a_{22} \\ k_3, & \text{for } a_{22} \leq x \leq a_{23} \\ k_2 + (k_3 - k_2) \left( \frac{a_{24} - x}{a_{24} - a_{23}} \right), & \text{for } a_{23} \leq x \leq a_{24} \\ k_2, & \text{for } a_{24} \leq x \leq a_{25} \\ k_1 + (k_2 - k_1) \left( \frac{a_{26} - x}{a_{26} - a_{25}} \right), & \text{for } a_{25} \leq x \leq a_{26} \\ k_1, & \text{for } a_{26} \leq x \leq a_{27} \\ k_1 \left( \frac{a_{28} - x}{a_{28} - a_{27}} \right), & \text{for } a_{27} \leq x \leq a_{28} \\ 0, & \text{for } x > a_{28} \end{cases}$$

### 3. Diagram of Icosikaioctagonal fuzzy number:



#### 3.1 Balanced transportation problem:

The general form of Transportation problem is

$$\text{Minimize (total cost) } z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

$$\text{Subject to the constraints } \sum_{j=1}^n x_{ij} = a_i, i=1,2,3,\dots,m$$

$$\sum_{i=1}^m x_{ij} = b_j, j=1,2,3,\dots,n$$

$$x_{ij} \geq 0 \text{ For all } i \text{ and } j$$

If total supply from all the sources is equal to the total demand in all destinations, then it is called as balanced transportation problem

### 3.2 Unbalanced transportation problem:

The general form of Transportation problem is

$$\text{Minimize (total cost) } z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} X_{ij}$$

$$\text{Subject to the constraints } \sum_{j=1}^n x_{ij} = a_i, i = 1, 2, 3, \dots, m$$

$$\sum_{i=1}^m x_{ij} = b_j, j = 1, 2, 3, \dots, n$$

$$x_{ij} \geq 0 \text{ For all } i \text{ and } j$$

If in a transportation problem total supply from all the sources is not equal to the total demand in all destinations, then it is called as unbalanced transportation problem. But for a feasible solution to exist, total supply must be equal to the total demand thus it is necessary convert unbalanced problem into balanced problem.

### 3.3 Procedure for solving Least cost method (LCM)

**Step 1:** Select the cell having minimum unit cost  $C_{ij}$  and allocate as much as possible,

$$\text{i.e., } \min(s_i, d_j)$$

**Step 2:** Subtract this minimum value from supply  $S_i$  and demand  $d_j$

**Step 3:** If the supply is  $S_i$  is zero then strike out that row and if the demand  $d_j$  is zero then strike that column

**Step 4:** If minimum unit cost cell is not unique, then select the cell where maximum allocation can be possible

**Step 5:** Repeat this steps for all uncrossed rows and columns until all supply and demand values are zero

#### 4. Numerical example:

Consider the following fuzzy transportation problem

	Destination			Supply
Origin	(-11,-7,-6,-5, -4,-3,-2,-1, 0,1,2,3, 4,5,6,7, 8,9,10,11, 12,13,14,15, 16,17,18,20)	(-10,-9,-8,-7, -6,-5,-4,-3, -2,-1,0,1, 2,3,4,5, 6,7,8,9, 10,11,12,13, 14,15,16,17)	(1,2,3,4, 5,6,7,8, 9,10,11,12, 13,14,15,16, 17,18,19,20, 21,22,23,24, 25,26,27,28)	(-6,-5,-4,-3, -2,-1,0,1, 3,5,6,7, 8,10,12,13, 15,17,19,20, 21,22,24,25, 26,27,28,29)
	(-9,-7,-6,-5, -4,-3,-2,-1, 0,1,2,3, 4,5,6,7, 8,9,10,11, 12,13,14,15, 16,18,20,22)	(-13,-12,-11,-10, -9,-7,-6,-5, -4,-3,-2,-1, 0,1,2,3, 4,5,6,7, 8,9,10,11, 12,13,14,15)	(0,1,2,4, 5,6,7,8, 9,10,11,12, 13,14,16,18, 20,22,24,26, 27,28,29,30, 32,33,34,36)	(-12,-11,-10,-9, -8,-7,-6,-4, -2,-1,0,1, 2,3,4,5, 6,7,9,10, 11,12,14,16, 18,20,22,24)
	(0,1,2,3, 4,5,6,7, 9,10,11,13, 14,15,17,19, 21,22,24,25, 26,27,28,29, 30,31,32,33)	(1,2,3,6, 8,9,10,12, 13,15,16,17, 19,20,22,23, 25,28,30,32, 34,35,37,39, 40,42,44,46)	(-7,-6,-5,-4, -3,-2,-1,0, 1,2,3,4, 5,6,7,8, 9,10,11,12, 13,14,15,16, 17,18,19,20)	(2,4,5,6, 8,10,12,13, 15,17,18,19, 20,22,23,24, 25,26,28,30, 31,32,33,34, 35,36,37,38)
Demand	(2,3,4,6, 7,8,9,10, 11,12,13,14, 15,16,17,18, 19,20,21,22, 23,24,25,26, 27,28,29,30)	(-10,-9,-8,-7, -6,-4,-3,-2, -1,0,1,2, 3,5,6,7, 8,9,10,11, 12,13,14,15, 16,17,18,19)	(-9,-8,-7,-6, -5,-4,-3,-2, -1,0,1,2, 3,4,5,6, 7,8,9,10, 11,12,14,16, 18,20,22,24)	

This problem is solved by taking the values for  $k_1 = \frac{1}{7}, k_2 = \frac{2}{7}, k_3 = \frac{3}{7}, k_4 = \frac{4}{7}, k_5 = \frac{5}{7}, k_6 = \frac{6}{7}$ .

We obtain the values of Measure of matrix A and is denoted by  $\mu_{Icskoc}(a_{ij})$

a <sub>11</sub>	-11,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4, 5,6,7,8,9,10,11,12,13,14,15,16,17,18,20	$\mu_{Icskoc}(a_{11}) = 5.43$
a <sub>12</sub>	-10,-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1, 2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17	$\mu_{Icskoc}(a_{12}) = 2.36$
a <sub>13</sub>	1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16, 17,18,19,20,21,22,23,24,25,26,27,28	$\mu_{Icskoc}(a_{13}) = 16.57$
a <sub>21</sub>	-9,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5, 6,7,8,9,10,11,12,13,14,15,16,18,20,22	$\mu_{Icskoc}(a_{21}) = 5.67$
a <sub>22</sub>	-13,-12,-11,-10,-9,-7,-6,-5,-4,-3, -2,-1,0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15	$\mu_{Icskoc}(a_{22}) = 9.25$
a <sub>23</sub>	0,1,2,4,5,6,7,8,9,10,11,12,13,14,16,18, 20,22,24,26,27,28,29,30,32,33,34,36	$\mu_{Icskoc}(a_{23}) = 22.43$
a <sub>31</sub>	0,1,2,3,4,5,6,7,9,10,11,13,14,15,17,19, 21,22,24,25,26,27,28,29,30,31,32,33	$\mu_{Icskoc}(a_{31}) = 14.5$
a <sub>32</sub>	1,2,3,6,8,9,10,12,13,15,16,17,19,20,22,23, 25,28,30,32,34,35,37,39,40,42,44,46	$\mu_{Icskoc}(a_{32}) = 17.04$
a <sub>33</sub>	-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6 7,8,9,10,11,12,13,14,15,16,17,18,19,20	$\mu_{Icskoc}(a_{33}) = 6.5$

Fuzzy supplies are noted as follows

S <sub>1</sub>	-6,-5,-4,-3,-2,-1,0,1,3,5,6,7,8,10, 12,13,15,17,19,20,21,22,24,25,26,27,28,29	11.32
S <sub>2</sub>	-12,-11,-10,-9,-8,-7,-6,-4,-2,-1,0,1,2,3,4,5, 6,7,9,10,11,12,14,16, 18,20,22,24	4.07
S <sub>3</sub>	2,4,5,6,8,10,12,13,15,17,18,19,20,22, 23,24,25,26,28,30,31,32,33,34,35,36,37,38	21.54

And Fuzzy demands are depicted as follows

D <sub>1</sub>	2,3,4,6,7,8,9,10,11,12,13,14,15,16,17,18, 19,20,21,22,23,24,25,26,27,28,29,30	16.39
D <sub>2</sub>	-10,-9,-8,-7,-6,-4,-3,-2,-1,0,1,2,3,5,6,7,8, 9,10,11,12,13,14,15,16,17,18,19	4.86
D <sub>3</sub>	-9,-8,-7,-6,-5,-4,-3,-2,-1,0,1,2,3,4,5,6,7,8, 9,10,11,12,14,16,18,20,22,24	5.25



And Total fuzzy supply and the total fuzzy demands are shown as follows

S	-16,-12,-9,-6,-2,2,6,10,16,21,24,27,30,35,39,42,46,50,56,60,63,66,71,75,79,83,87,91	36.93
D	-17,-14,-11,-7,-4,0,3,6,9,12,15,18,21,25,28,31,34,37,40,43,46,49,53,57,61,65,69,73	26.5

The crisp valued transportation is as follows

	Destination			Supply
Source	5.43	2.36	16.57	11.32
	5.67	9.25	22.43	4.07
	14.5	17.04	6.5	21.54
Demand	16.39	4.86	5.25	

Then the solution is explained in the following tables

	Destination						Supply
Source	5.43	2.36	0.89	16.57	0	10.43	11.32
	5.67	4.07	9.25	3.6	22.43	0	4.07
	14.5	12.32	17.04	3.97	6.5	5.25	21.54
Demand	16.39	4.86	5.25	10.43			36.93

The initial basic feasible solution is obtained by least cost method. We get the solution containing 8 non negative independent allocations equal to  $m+n-1$

	Destination			Supply
Origin	5.43	2.36	0.89	<del>0.89</del> 0
	5.67	9.25	22.43	4.07
	14.5	17.04	6.5	21.54
Demand	16.39	4.86 3.97	5.25	

	Destination				Supply
Origin	5.67	4.07	9.25	22.43	<del>4.07</del> 0
	14.5		17.04	6.5	21.54
Demand	<del>16.39</del> 12.32	3.97	5.25		

	Destination				Supply
Origin	14.5	17.04	6.5	5.25	<del>21.54</del>
	12.32	3.97	<del>5.25</del> 0		16.29
Demand					

	Destination			Supply
Origin	14.5	17.04	3.97	<del>16.29</del>
	12.32	<del>3.97</del> 0		12.32
Demand				

	Destination		Supply
Origin	14.5	12.32	<del>12.32</del>
	<del>12.32</del>		
Demand			

The transportation cost is

$$(2.36 \times 0.89) + (0 \times 10.43) + (5.67 \times 4.07) + (14.5 \times 12.32) + (17.04 \times 3.97) + (6.5 \times 5.25)$$

$$= 2.1004 + 0 + 23.0769 + 178.64 + 67.6488 + 34.125$$

Total cost = 305.5911

## 5. Conclusion:

In this paper, unbalanced transportation problem has been solved with Icosikaioctagonal fuzzy number. The use of Icosikaioctagonal fuzzy number in fuzzy transportation problem is given. Fuzzy transportation problem converted to crisp valued problem and is illustrated by an example. Optimal value obtained using Icosikaioctagonal fuzzy numbers are more optimal than the solution obtained by using hexadecagonal and Icosagonal fuzzy numbers.

**References:**

- [1] L.A. Zadeh, , Fuzzy sets, Information and Control, 8(3) ,1965, 338-353.
- [2] V. Raju and R. Jayagopal “An Approach on Icosikaioctagonal Fuzzy number-Traditional Operations on Icosikaioctagonal fuzzy number” A Journal of composition theory, Vol.XII, Issue X, 2019, Page No: 727-734
- [3] V. Raju and R. Jayagopal “An Arithmetic Operations of Icosagonal fuzzy number using Alpha cut ” International Journal of Pure and Applied Mathematics. Volume 120, No. 8, 2018, 137-145
- [4] V. Raju and R. Jayagopal “ A Rudimentary Operations on Octagonal Fuzzy Numbers “ International Journal of Research in Advent Technology Vol.6, No.6, June 2018, Page No: 1320-1323
- [5] V.Raju and M.ParuvathaVathana “Discourse on Fuzzy Game Problem in Icosagonal Fuzzy Number ‘ International journal of scientific research and review volume 8, Issue 3, 2019 Page no: 1384-1390
- [6] V.Raju and S.Maria Jesu Raja “An Approach on Fuzzy game problem in Icosikaioctagonal Fuzzy number” Journal of Xidian University, Volume 14, Issue 4, 2020, Page no: 1009-1016
- [7] V.Raju , Ranking Function on Icosagonal Fuzzy Number for Solving Fuzzy Transportation Problem, “Journal of Applied Science and Computations “ Volume VI, Issue IV, 2019, Page No: 3631-3640
- [8] V.Raju and M.ParuvathaVathana “An Icosagonal Fuzzy Number for solving Fuzzy Sequence Problem“ International journal of Research in Engineering, IT and Social Sciences” Volume 9, Issue 5, 2019. Page no: 37-40
- [9] V.Raju and M.ParuvathaVathana “ Fuzzy Critical path method with Icosagonal Fuzzy Numbers using Ranking Method” A Journal of Composition Theory, Volume 12, Issue 9, 2019, Page no: 62-69

- [10] V.Raju and S.Arul Amirtha Raja “ Study on fuzzy sequencing problem in Icosikaioctagonal Fuzzy Numbers” Journal of Xidian University, Volume 14, Issue 4, 2020,  
Page no: 3829-3837
- [11] V.Raju and S.Maria Jesu Raja “ Fuzzy decision making problem in Icosikaioctagonal Fuzzy number” Journal of Xidian University, Volume 14, Issue 5, 2020, Page no: 3240-3248
- [12] S.Maria Jesu Raja and V.Raju “ Elucidating Fuzzy Assignment problem Employing Icosikaioctagonal Fuzzy Number ” Journal of Xi’an University of Architecture & Technology, Volume 12, Issue 6, 2020, Page no: 1681-1688